Deterministic Two-Stage Transmission Expansion Planning Integrated with Security-Constrained Unit Commitment: An Efficient Hybrid Decomposition Method

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***Abstract*—This paper develops a deterministic, two-stage transmission expansion planning (TEP) model that is tightly integrated with a year-round security-constrained unit commitment (SCUC) formulation. The first-stage problem identifies the economically optimal transmission reinforcements, whereas the second-stage problem executes a chronological SCUC that is comprised of 365 24-hours SCUC to ensure operational feasibility across the whole annual horizon. To efficiently solve this large-scale mixed-integer program, a hybrid Column-and-Constraint Generation (CCG) framework enhanced with Benders Decomposition (BD) is proposed. The nested BD-accelerated CCG scheme substantially improves convergence behavior by reducing the number of oracle evaluations and strengthening the master problem. Numerical studies on the IEEE 118-bus system demonstrate that the method reliably uncovers cost-effective reinforcements, yielding significant annual operating-cost reductions while preserving system-wide security. The results confirm that the proposed hybrid algorithm is computationally tractable and well-suited for long-horizon, high-fidelity SCUC-TEP studies.**

***Index Terms*—Deterministic Two-Stage Problem, Hybrid Decomposition Method, Mixed-Integer Linear Programming, Security-Constrained Unit Commitment, Transmission Expansion Planning.**

NOMENCLATURE

Sets and Indices

IT_{ccg}	Set of the iterations for CCG algorithm
IT_{gbd}	Set of the iterations for BD
j_{ccg}	Iteration index for CCG algorithm
Bus	Set of buses
Node	Alias set for Bus
Slack	Slack bus set
m, n	Indices for the buses
T	Time set
t	Time index
Ω_n^G	Set of the thermal generators G at bus n
i, g	Indices for the thermal generators
$\Omega_{m,n}^{Le}$	Set of the existing TL Le
$\Omega_{m,n}^{Lc}$	Set of the candidate TL Lc
Ω_n^D	Set of the electric demands D at bus n
LD	Demand index

Scalars and Parameters

C_l^{inv}	Investment cost for expanding TL l (\$)
C_n^{ls}	Cost for the unserved energy (\$)
Mc	Big-M scalar for the candidate TL
C_g^V	Variable cost coefficient of TG (\$/MW)
P_g^{min}, P_g^{max}	Min/max of output powers of TG (MW)
R_g^U, R_g^D	Ramp-up/down rate (MW/h)
$T_{i,\min}^{\text{on}}$	Minimum up-time (h)
$T_{i,\min}^{\text{off}}$	Minimum down-time (h)
U_{ini}	Initial on/off status of unit g
$U0_g^0$	Time periods unit g have been on (h)
$D0_g^0$	Time periods unit g have been off (h)
X_{mn}	Line reactance (Ω)
B_{mn}	Line susceptance (Ω^{-1})
P_{mn}^{max}	Max of TL power flow (MW)
$D_{d,t}$	Electric demand (MW)
ϵ	Control error parameter (for CCG)
LB_{bd}	Lower bound of BD
UB_{bd}	Upper bound of BD
LB_{ccg}	Lower bound of CCG algorithm
$UB(IT)_{ccg}$	Upper bound of CCG algorithm
$BigM$	Big-M scalar for slackness conditions

Variables

$x_{l,t}$	Binary variable of expansion decision lc
$s_{n,t}$	Real variable of unserved energy (MW)
$P_{g,t}$	Output powers of TG (MW)
$u_{g,t}$	Binary variable for the unit status (on/off)
$y_{g,t}$	Binary variable for the startup status
$z_{g,t}$	Binary variable for the shutdown status
$P_{mn,t}$	Transmission line power flow (MW)
$\delta_{n,t}$	Bus voltage angles n (rad)

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I. INTRODUCTION

A. Motivation and Problem Statement

With growing electrical demand, aging infrastructure, and increasing operational complexity, modern power systems require strategically planned transmission reinforcements to maintain reliability, security, and economic efficiency. Traditional deterministic TEP approaches often omit detailed operational security constraints, resulting in investment decisions that can be either overly conservative or operationally insufficient. Embedding a complete SCUC model within TEP provides a more realistic representation of system behavior; however, the resulting formulation becomes a large-scale, mixed-integer optimization problem with tight temporal coupling. Existing SCUC-integrated TEP methods face substantial scalability and computational tractability challenges, especially for multi-period and large-network cases. Consequently, there is a critical need for solution frameworks that jointly consider transmission investment and security-constrained operations while remaining computationally viable for realistic system sizes and year-long planning horizons.

B. Approach

To address the computational challenges of the integrated SCUC–TEP formulation, this paper adopts a hybrid solution framework that combines CCG with BD applied to the master problem. The proposed scheme decomposes the large-scale mixed-integer program into tractable subproblems: a first-stage master problem that determines transmission investment decisions, and a second-stage SCUC subproblem that evaluates year-round operational feasibility and cost. BD cuts are embedded within the CCG master problem to strengthen its relaxation, thereby reducing the number of oracle (SCUC) evaluations and accelerating convergence. Through an iterative exchange of optimality and feasibility cuts, the hybrid algorithm progressively refines both investment and operational decisions, ultimately yielding an optimal or near-optimal expansion plan that satisfies all security and operational requirements. This integrated decomposition framework enables efficient transmission-capacity planning while retaining the high-fidelity operational detail of a full 8,760-hour SCUC.

C. Literature Review

Paper [1] introduces an efficient analytical sensitivity method based on the Lagrangian function, which improves on traditional iterative approaches. Ref. [2] proposes a Line-Wise formulation with Quadratic Convex relaxation for globally optimal AC transmission planning, demonstrating scalability and accuracy. Article [3] presents a clustering method that prioritizes extreme values and temporal information, thereby improving the handling of renewable variability. Paper [4] develops a bilevel optimization framework using Cooperative Game Theory to enhance coalition stability and economic efficiency. Article [5] introduces a multi-period hybrid AC/DC model using second-order conic relaxation for accurate, efficient handling of AC constraints. Paper [6] presents a robust, nonlinear, dual-based method that employs convex relaxation and Benders Decomposition to manage renewable and load uncertainties effectively. Ref. [7] offers a mixed-integer second-order cone programming model for co-optimizing transmission and distribution planning, efficiently solved via enhanced Benders decomposition. Paper [8] proposes a hybrid column-and-constraint-generation augmented Lagrangian method for robust security-constrained dynamic planning, validated on the IEEE 118-bus system. Paper [9] introduces a two-stage adaptive robust model capturing operational uncertainties with practical flexibility assessment. Article [10] develops a five-level MILP model with a cutting-plane algorithm for robust TEP under uncertainty, scalable and securing optimal investments. Paper [11] presents compact, deterministic, and multistage robust formulations for wind and load uncertainty. Paper [12] proposes a two-stage adaptive robust optimization model for flexibility assessment in renewable-rich systems. Article [13] provides a scalable decomposition framework that combines progressive hedging, Benders decomposition, and scenario bundling, and demonstrates it on ERCOT systems. Ref. [14] develops a data-driven stochastic approach that robustly optimizes against historical uncertainty distributions via Benders' decomposition and column-and-constraint generation methods. Paper [15] enhances forward selection algorithms for efficient scenario reduction in Benders decomposition, significantly accelerating solution time. Article [16] incorporates heuristics into hierarchical Benders decomposition, considerably reducing computational effort for large-scale transmission planning problems.

D. Contributions

Existing approaches to SCUC-integrated TEP frequently encounter scalability limitations and substantial computational burdens when applied to large-scale systems or long-term horizons. This paper addresses these challenges by introducing a hybrid decomposition framework that integrates CCG with BD to accelerate convergence and enhance tractability under stringent security constraints. The significant contributions of this work are summarized as follows:

1) **A unified (Column and Constraint Generation) CCG–(Benders Decomposition) BD hybrid decomposition algorithm** for solving large-scale, deterministic two-stage (Security-Constrained Unit Commitment) SCUC–(Transmission Expansion Planning) TEP problems. In this framework, BD optimality cuts are embedded within the CCG master problem, thereby strengthening its relaxation, reducing the number of oracle (SCUC) evaluations, and markedly improving convergence behavior and overall runtime.
2) **A full 8,760-hour operational integration** that jointly optimizes planning and detailed hourly (Security-Constrained Unit Commitment) SCUC decisions across an entire year. This long-horizon formulation demonstrates the practicality of the proposed method for high-fidelity, reliability-aware expansion studies.

To the authors' knowledge, this is the first study to apply a BD-enhanced CCG framework to solve a deterministic

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SCUC-integrated TEP model over a complete annual horizon, demonstrating both methodological novelty and computational viability.

E. Paper Organization

In Section II, the mathematical formulation of the SCUC-TEP problem is detailed. Section III formulates the hybrid mixed-integer problem and applies the nested BD-CCG algorithm to solve it. Section IV evaluates the effectiveness of the proposed technique by demonstrating it in a single study. Section V summarizes the article and provides future research.

II. Mathematical Formulation

This section presents the mathematical formulation of the simultaneous SCUC and TEP problem.

A. SCUC-TEP

The objective of the problem is to minimize investment and operating costs under steady-state conditions while satisfying the system reliability requirement. The objective function is formulated as follows.

$$\begin{aligned}\min_{x_{l,t},s_{n,t}} \Big[\sum_{lc\in\Omega_{m,n}^{lc}} C_{lc}^{\text{inv}}\, x_{l,t} \;+\; \sum_{n\in BUS} C_n^{\text{ls}}\, s_{n,t} \Big] \\ +\min_{P_{g,t},P_{mn,t},\delta_{n,t},u_{g,t},y_{g,t},z_{g,t}} \sum_t \sum_g C_g^V\, P_{g,t}\end{aligned} \tag{1}$$

Subject to:

$$0 \;\le\; s_{n,t} \;\le\; D_{d,t}, \quad \forall n\in BUS \tag{2}$$

$$x_{l,t}\in\{0,1\}, \quad \forall lc\in\Omega_{m,n}^{lc} \tag{3}$$

$$y_{g,t}-z_{g,t}=u_{g,t}-u_{g,t-1}, \quad \forall g,\forall t \tag{4}$$

$$y_{g,t}+z_{g,t}\le 1, \quad \forall g,\forall t \tag{5}$$

$$P_g^{\min}\le P_{g,t}\le P_g^{\max} : \mu_{g,t}^{\min},\mu_{g,t}^{\max} \quad \forall g,\forall t \tag{6}$$

$$P_{g,t}-P_{g,t-1}\le R_g^U u_{g,t-1} : \omega_{g,t}^U \quad \forall g,\forall t \tag{7}$$

$$P_{g,t-1}-P_{g,t}\le R_g^D u_{g,t} : \omega_{g,t}^D \quad \forall g,\forall t \tag{8}$$

$$\begin{aligned}\sum_{g\in\Omega_n^G} P_{g,t} = \sum_{d\in\Omega_n^D} D_{d,t} + \\ \sum_{(m,n)\in\Omega_{m,n}^{le}\&\Omega_{m,n}^{lc}} P_{mn,t} : \lambda_{n,t} \quad \forall n,\forall t\end{aligned} \tag{9}$$

$$P_{mn,t}=B_{mn}(\delta_{n,t}-\delta_{m,t}), \quad \forall (m,n),\forall t \tag{10}$$

$$|P_{mn,t}-B_{mn}(\delta_{n,t}-\delta_{m,t})|\le Mc(1-x_{l,t}), \quad \forall (m,n),\forall t \tag{11}$$

$$-P_{mn}^{\max}\le P_{mn,t}\le P_{mn}^{\max} : \nu_{mn,t}^{\min},\nu_{mn,t}^{\max}, \quad \forall (m,n),\forall t \tag{12}$$

$$|P_{mn,t}|\le P_{mn}^{\max} x_{l,t} : \nu_{mn,t}^{\min},\nu_{mn,t}^{\max}, \quad \forall (m,n),\forall t \tag{13}$$

Eq. (1) includes three components: total network investment, a reliability proxy (cost of load shedding), and operational cost. Eq. (2) determines the bounds for the load shedding, and Eq. (3) gives the on/off status of the candidate transmission lines. (4) and (5) ensure logical consistency between generator on/off states. (6) defines physical generation limits. Ramp constraints are presented by (7) and (8). The power balance equation is enforced by (9), whose dual values represent nodal LMPs. (10) represents DC power flow for the existing transmission lines, and (11) represents it for the candidate transmission lines; (12) and (13) constrain line power flows for the existing and candidate transmission lines, respectively.

III. Hybrid deterministic two-stage algorithm

Deterministic two-stage programming partitions decisions into two levels: first-stage (here-and-now) decisions made before operation, and second-stage (wait-and-see) decisions made after the first stage is fixed. In deterministic settings, all parameters are assumed to be known with certainty at the planning stage. In this work, the first stage selects transmission investment decisions and defines the network topology for the study horizon. The second stage solves a full 8,760-hour SCUC to determine hourly operational schedules, capturing both seasonal and diurnal variations. This structure enables consistent coordination between long-term expansion planning and detailed, security-constrained system operations.

A. Deterministic two-stage programming using CCG algorithm

A general deterministic two-stage optimization model can be expressed as follows [18]. Let X be the first-stage (TEP) and U be the second-stage (SCUC) decision variables, respectively. The general form of a two-stage optimization problem is as follows.

$$\min_{\mathbf{X}} \mathbf{c}^\top\mathbf{X} + \min_{\mathbf{U}\in F(\mathbf{X})} \mathbf{b}^\top\mathbf{U} \tag{14}$$

Subject to:

$$\mathbf{AX}\ge\mathbf{d}, \quad \mathbf{X}\in\mathcal{S}_{\mathbf{X}}$$

where

$$F(\mathbf{X})=\{\mathbf{U}\in\mathcal{S}_{\mathbf{U}} : \mathbf{GU}\ge\mathbf{h}-\mathbf{EX}\}$$

with $\mathcal{S}_{\mathbf{X}}\subseteq\mathbb{R}_+^n$ and $\mathcal{S}_{\mathbf{U}}\subseteq\mathbb{R}_+^m$.

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IV. STUDY

The proposed accelerated BD-CCG algorithm was implemented in Pyomo [19] and the Gurobi solver [20] and tested on the IEEE 118-bus system over a complete annual scheduling horizon (8,760 hours). This long-term simulation distinguishes the present study from prior works, which typically examine only 24-hour or week-long horizons. The annual formulation enables simultaneous evaluation of investment cost recovery, operational variability, and congestion patterns across all seasons. For implementation purposes, please refer to [21].

A. IEEE 118-Bus system - Input data [22]

The IEEE 118-Bus system considered in this report is a single-area version comprising 54 generating units and 186 transmission lines. Line resistances are null, and thus, active power losses are disregarded. The load profile for the considered 8760-hour scheduling horizon is based on [23] and is multiplied by 2.5 for supply-and-demand matching purposes.

B. Results & Discussion

In this case study, two parallel circuits are assumed for each transmission line. Then, the maximum capacity decreases from 1 p.u. to 0.1 p.u. in steps of 0.1 p.u. For simplicity, the investment cost of each added line is assumed to be 1 million. Table I investigates how progressively tightening line-capacity limits, expressed through a derating coefficient on the thermal ratings, reshape the combined security-constrained unit-commitment and transmission-expansion plan for the IEEE 118-bus system. For coefficients between 1.0 and 0.8, the incumbent network remains adequate, so no capital actions are triggered and the annual production charge stabilizes at the benchmark \$373.93 million. A further reduction to 0.7 marginally elevates congestion and redispatch, raising operating cost by \$0.09 million, yet still falls short of justifying new construction. Once the coefficient reaches 0.6, however, the optimizer identifies a single strategic reinforcement on corridor 153 whose \$1 million investment expense is exactly offset by an equal cut in operating outlays, restoring total cost to the baseline and revealing a sharp threshold beyond which network adequacy collapses (—line 153—). Strengthening the constraint to 0.5 accelerates this trend: five additional circuits (lines 61, 114, 121, 153, and 158) cost \$5 million but unlock a \$7.48 million dispatch saving, so total expenditure drops \$2.48 million below the reference level (—lines 61, 114, 121, 153, 158—). The most significant net benefit materialises at a coefficient of 0.4, (—lines 8, 9, 26, 35, 38, 51, 61, 96, 117, 145, 150, 153, 154, 158—), where fourteen upgrades reduce operating charges by \$18.70 million at a \$14 million capital cost, yielding the experiment's minimum total cost of \$369.23 million and confirming that, at least for moderate-sized networks, targeted expansion can more than pay for itself when thermal limits are tight. Notably, solution times remained clustered around 4.5 minutes despite the expanding decision space, indicating that the additional binary variables introduced by each candidate corridor had a limited impact on solver scalability at this system size. Overall, the results reveal a non-linear cost frontier in which transmission investment becomes economically attractive only after critical congestion thresholds are crossed, and beyond which each successive dollar of capital delivers diminishing but still appreciable operating savings. It is noted that the results for maximum capacity coefficients of 0.3, 0.2, and 0.1 are infeasible. Although the settings in [8] differ from those employed in this study, the proposed method demonstrates its tractability in computation time, as shown in Table II. Despite the year-long horizon and large decision space, the hybrid CCG-BD structure maintained tractable computational times, proving that BD cuts effectively accelerate CCG convergence, even under extensive temporal coupling in the SCUC problem.

Algorithm 1 BD-CCG

Initialize $LB \leftarrow -\infty$, $UB \leftarrow +\infty$, $k \leftarrow 0$, $\mathcal{O} \leftarrow \varnothing$.
while $UB - LB > \epsilon$ **do**
  Solve MP: $\min_{\mathbf{X},\eta}\ \mathbf{c}^\top \mathbf{X} + \eta$ *s.t.* $\mathbf{AX} \geq \mathbf{d}$; $\eta \geq \mathbf{b}^\top \mathbf{U}^l$ for all $l \in \mathcal{O}$; $\mathbf{EX} + \mathbf{GU}^l \geq \mathbf{h}$ for $l = 1, \ldots, k$; $\mathbf{X} \in \mathcal{S}_\mathbf{X}$, $\eta \in \mathbb{R}$, $\mathbf{U}^l \in \mathcal{S}_\mathbf{U}$.
  Obtain $(\mathbf{X}^*_{k+1}, \eta^*_{k+1}, \{\mathbf{U}^{l*}\}_{l=1}^k)$ and set $LB \leftarrow \mathbf{c}^\top \mathbf{X}^*_{k+1} + \eta^*_{k+1}$.
  Solve SP / oracle: $\mathcal{Q}(\mathbf{X}^*_{k+1}) = \min_{\mathbf{x} \in \mathcal{S}_\mathbf{U}} \{\mathbf{b}^\top \mathbf{x} : \mathbf{Gx} \geq \mathbf{h} - \mathbf{EX}^*_{k+1}$.
  Update $UB \leftarrow \min\{UB,\ \mathbf{c}^\top \mathbf{X}^*_{k+1} + \mathcal{Q}(\mathbf{X}^*_{k+1})\}$.
  if $UB - LB \leq \epsilon$ **then**
    return $\mathbf{X}^*_{k+1}$ and **terminate**.
  else
    if $\mathcal{Q}(\mathbf{X}^*_{k+1}) < +\infty$ **then**
      Add optimality cuts to MP: create $\mathbf{U}^{k+1}$; add $\eta \geq \mathbf{b}^\top \mathbf{U}^{k+1}$; add $\mathbf{Ey} + \mathbf{GU}^{k+1} \geq \mathbf{h}$; set $k \leftarrow k+1$; set $\mathcal{O} \leftarrow \mathcal{O} \cup \{k\}$.
    else
      Add feasibility cut to MP: create $\mathbf{U}^{k+1}$; add $\mathbf{Ey} + \mathbf{GU}^{k+1} \geq \mathbf{h}$; set $k \leftarrow k+1$.
    end if
  end if
end while

V. CONCLUSION

This paper developed a deterministic two-stage TEP model integrated with a full-year (8,760-h) SCUC formulation. A hybrid decomposition framework combining CCG with Benders Decomposition was proposed to solve the resulting large-scale mixed-integer problem efficiently. Embedding BD cuts within the CCG master problem significantly accelerated convergence while preserving optimality. Numerical results for the IEEE 118-bus system demonstrate that the proposed method is scalable, computationally efficient, and well-suited for long-horizon, reliability-aware transmission planning studies.

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TABLE I
THE YEAR-ROUND SCUC-TEP RESULTS FOR THE IEEE 118 BUS SYSTEM

Coefficient of maximum capacity	Number of added lines	Time min	Operation cost	TEP cost	Total cost
1.0	0	4.37	373,929,495.09	–	373,929,495.09
0.9	0	4.35	373,929,495.09	–	373,929,495.09
0.8	0	4.35	373,929,495.09	–	373,929,495.09
0.7	0	4.54	374,020,995.59	–	374,020,995.59
0.6	1	4.52	372,929,495.09	1,000,000.00	373,929,495.09
0.5	5	4.51	366,450,905.19	5,000,000.00	371,450,905.19
0.4	14	4.59	355,233,020.34	14,000,000.00	369,233,020.34

TABLE II
COMPARISON WITH REF. [8]

Item	Current reasearch	Ref. [8]
Method	CCG algorithm + Benders Decomposition	CCG algorithm + Lagrangian algorithm (ADMM)
Case	IEEE 118 Bus	IEEE 118 Bus
Type	deterministic	Including uncertainties
Integer variables	Both master problem and subproblem	Just in master problem
No. of years	1 year	10 years
Load shedding	Not allowed	3%
No. of expanded lines = 5	Half of the coefficient of maximum capacity	Number of candidate lines in the first year
Maximum time	Whole process = 300 seconds	Master problem = 500 seconds

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