Modeling Internal Faults in Three-Winding Autotransformers Using the Terminal Duality Method

Meysam Ahmadi
RTDS Technologies Inc.
Winnipeg, Canada
meysam.ahmadi@ametek.com

Ali Dehkordi
RTDS Technologies Inc.
Winnipeg, Canada
ali.dehkordi@ametek.com

Dong Li
RTDS Technologies Inc.
Winnipeg, Canada
dong.li@ametek.com

Yi Zhang
RTDS Technologies Inc.
Winnipeg, Canada
yi.zhang@ametek.com

***Abstract*—This paper introduces a novel electromagnetic approach for modeling internal faults in three-winding autotransformers. The proposed method applies the Terminal Duality Method (TDM) to derive the electrical equivalent circuit from the dual of the magnetic circuit. This approach provides a robust, stable, and physically meaningful representation of the autotransformer, significantly improving the accuracy and stability of internal fault modeling.**

***Index Terms*—Three-winding autotransformer, Internal fault, Terminal Duality Method, Electromagnetic transient modeling.**

I. Introduction

Autotransformers are critical in modern power systems, especially high-voltage transmission networks where efficient voltage regulation and compact design are essential. Unlike conventional transformers, they use a single winding for both primary and secondary functions, enabling energy transfer through electromagnetic induction and direct conduction. This design reduces material use, cost, and losses while improving efficiency and voltage regulation. Consequently, autotransformers are widely applied for interconnecting transmission lines of slightly different voltages, compensating voltage drops in long feeders, and supporting industrial and renewable systems [1]. Despite these advantages, autotransformers pose challenges in modeling and fault analysis. The lack of galvanic isolation between input and output circuits increases fault propagation risk and complicates protection schemes. A tertiary winding—often delta-connected—adds further complexity, particularly in three-winding configurations. It may supply station loads or stabilize the neutral point but significantly affects the zero-sequence network during faults [2], [3].

Simulating internal faults in autotransformers, particularly in three-winding configurations, is a non-trivial task. Conventional models which are mostly based on star-equivalent circuits often fail to capture the nuanced behavior of turn-to-turn faults as these equivalent circuits are only valid from the terminals point of view of healthy transformer. For example, the conversion of measured impedances into a T-model can result in negative impedance values, which applying a fault on negative inductance would complicate fault current calculations. This causes unstable simulations or failure in relay coordination while performing hardware-in-loop (HIL) tests using real time digital simulators (RTDS). These anomalies can lead to malfunction of protection systems if not properly understood and accounted for [4].

Given the critical role of autotransformers and the challenges associated with their fault analysis, there is a pressing need for more accurate, robust and physically meaningful modeling approaches. This paper introduces a novel method based on the Terminal Duality Method (TDM), which leverages the duality of magnetic circuits to construct a stable and accurate electrical equivalent model of three-winding autotransformers. The proposed model aims to enhance fault simulation fidelity and support more reliable protection system design.

II. Developing a TDM-Based Autotransformer Model with Internal Faults

In this section, we first introduce the fundamentals of the Terminal Duality Method (TDM), and then describe how it can be applied to develop an autotransformer model. Furthermore, we explain how turn-to-turn faults can be incorporated into this model for more accurate fault analysis.

A. Terminal Duality Method

In TDM, the electric equivalent circuit of a magneto-electric device is derived by first constructing its magnetic circuit and then obtaining the electric dual of that circuit. This method yields a more realistic equivalent representation compared to conventional approaches, such as the star-equivalent model [5]–[7]. Fig. 1 illustrates an example of this procedure for a two-winding transformer with a loose magnetic coupling configuration.

B. Autotransformer Model Based on TDM

Autotransformers typically consist of a three-winding stack, as shown in Fig. 2. The tertiary winding, which is closest to the core and has the lowest voltage rating, is often used for local loads or connected in delta configuration in three-phase systems. The common winding refers to the section shared

979-8-3315-8138-1

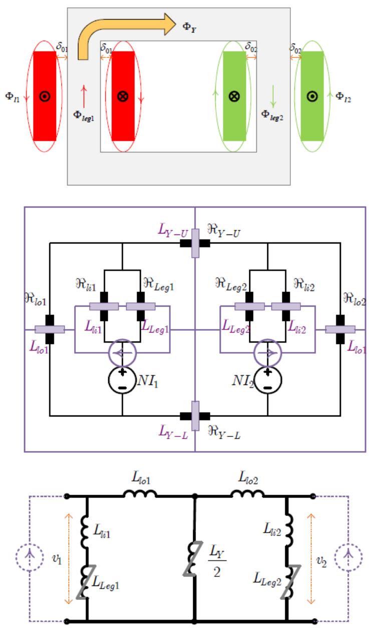

Fig. 1. Duality-Based Equivalent Circuit of a 2-Winding Transformer with Loose Magnetic Coupling: Top—core and winding arrangement; Middle—magnetic circuit and its electric dual; Bottom—simplified equivalent electric circuit.

between the high- and low-voltage terminals, while the series winding is connected to the high-voltage terminal.

Since the autotransformer's galvanic connection is established at the winding terminals, the TDM model of the three-winding transformer must first be derived. Once this model is obtained, the autotransformer configuration can then be constructed by applying the corresponding electrical connections to replicate the physical arrangement. Fig. 3 illustrates the TDM-based equivalent circuit of a three-winding transformer. In this model, the parameters L_{S12}, L_{S23} and M_{12} represent, respectively, the leakage inductance between winding stacks 1 and 2, the leakage inductance between winding stacks 2 and 3, and the mutual inductance associated with these two leakage paths. The mutual inductance is determined using short-circuit impedance tests, as described in (1).

$$M_{12} = \frac{1}{2}(L_{S13} - L_{S12} - L_{S23}) \tag{1}$$

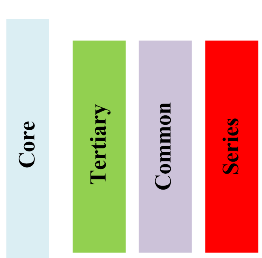

Fig. 2. Three-Winding autotransformer winding stack configuration.

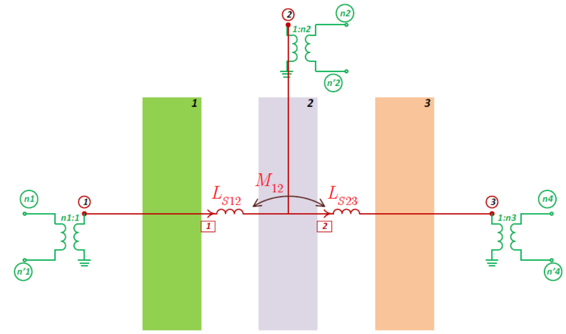

Fig. 3. TDM equivalent model of a three-winding transformer.

Now that the TDM-based model of a three-winding transformer has been developed, it can be reconfigured into an autotransformer by applying the appropriate electrical connections, as illustrated in Fig. 4.

As explained earlier, the equivalent circuit parameters shown in Fig. 4 are derived from the leakage inductances of the three-winding transformer. However, when the transformer is configured as an autotransformer, short-circuit tests are typically conducted differently, providing impedance values between terminals H-L, H-T, and L-T. These measurements differ from the leakage inductances between windings 1-2, 1-3, and 2-3 required for the TDM-based model. Therefore, it is necessary to calculate the corresponding leakage and mutual inductances from the autotransformer short-circuit impedance data.

To begin, consider the branch impedance Z_b as defined in (2).

$$Z_b = \begin{bmatrix} L_{l12} & M \\ M & L_{l23} \end{bmatrix} \tag{2}$$

Then using incidence matrix in (3) and the relation in (4), we find the admittance matrix Y_{ws} of the inductive network, seen from the secondary of the ideal transformers in Fig. 4.

$$A = \begin{bmatrix} 1 & 0 \\ -1 & 1 \\ 0 & -1 \end{bmatrix} \tag{3}$$

$$Y_{ws} = A \cdot Z_b^{-1} \cdot A^T \tag{4}$$

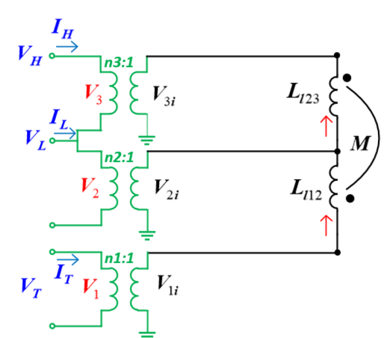

Fig. 4. TDM equivalent model of a three-winding autotransformer.

979-8-3315-8138-1

Then we calculate the admittance matrix seen from the primary side of the ideal transformers by (5).

$$Y_{wp} = \begin{bmatrix} [N]^{-1} \cdot Y_{ws} \cdot [N]^{-1} & -[N]^{-1} \cdot Y_{ws} \cdot [N]^{-1} \\ -[N]^{-1} \cdot Y_{ws} \cdot [N]^{-1} & [N]^{-1} \cdot Y_{ws} \cdot [N]^{-1} \end{bmatrix} \quad (5)$$

in which the matrix N is:

$$N = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{n_2}{n_1} & 0 \\ 0 & 0 & \frac{n_3}{n_1} \end{bmatrix} \quad (6)$$

The matrix Y_{wp} represents the admittance matrix of the three-winding transformer. To obtain the autotransformer configuration, a Kron reduction is applied, since two nodes are merged into a single node to form the electrical connection at terminal L, as it can be seen in Fig. 4.

The reduced admittance matrix, let's call it Y_{auto}, is computed analytically; however, due to space limitations, it is not printed here.

Next, for the short-circuit analysis, we will analytically determine the following short-circuit impedances under the specified operating conditions.

$$Z_{HL} = \frac{V_H}{I_H}\Big|_{(V_L=0,\ I_T=0)} = \frac{Y_{\text{auto},11}}{Y_{\text{auto},11} \cdot Y_{\text{auto},33} - Y_{\text{auto},31}^2} \quad (7)$$

$$Z_{HT} = \frac{V_H}{I_H}\Big|_{(V_T=0,\ I_L=0)} = \frac{Y_{\text{auto},22}}{Y_{\text{auto},22} \cdot Y_{\text{auto},33} - Y_{\text{auto},32}^2} \quad (8)$$

$$Z_{LT} = \frac{V_L}{I_L}\Big|_{(V_T=0,\ I_H=0)} = \frac{Y_{\text{auto},33}}{Y_{\text{auto},33} \cdot Y_{\text{auto},22} - Y_{\text{auto},23}^2} \quad (9)$$

Equations (7), (8), and (9) establish the relationship between the autotransformer short-circuit impedances and the corresponding leakage impedances, Z_{12}, Z_{13} and Z_{23} of the three-winding transformer. Solving these equations analytically yields the following results:

$$Z_{HL} = Z_{23} \cdot \frac{n_3^2}{n_1^2} \quad (10)$$

$$Z_{HT} = Z_{13} \cdot \frac{n_3^2}{n_1^2} + \left(Z_{13} + Z_{13} - Z_{23}\right) \cdot \frac{n_2 n_3}{n_1^2} + Z_{12} \cdot \frac{n_2^2}{n_1^2} \quad (11)$$

$$Z_{LT} = Z_{12} \cdot \frac{n_2^2}{n_1^2} \quad (12)$$

Since the short-circuit test measurements are typically provided in per-unit values, the above mentioned impedances should also be expressed in per-unit on a common base—here, we choose the base of winding 1 of the three-winding transformer. Note that the relationship between the autotransformer terminal voltages and the winding stack voltages is given in (13):

$$V_1 = V_T, \qquad V_2 = V_L, \qquad V_3 = V_H - V_L \quad (13)$$

Furthermore, (14) and (15) define the relationship between the base impedances of the autotransformer and the base impedance of winding 1 of the three-winding transformer.

$$Z_{\text{base},L} = \frac{n_2^2}{n_1^2} \cdot Z_{\text{base},T} = \frac{n_3^2}{n_1^2} \cdot Z_{\text{base},1} \quad (14)$$

$$Z_{\text{base},H} = \frac{(n_2+n_3)^2}{n_1^2} \cdot Z_{\text{base},T} = \frac{(n_2+n_3)^2}{n_1^2} \cdot Z_{\text{base},1} \quad (15)$$

After per-unitizing (10), (11), and (12) using their corresponding base values, we obtain the following relationships, (16), (17) and (18) between the per-unit parameters of the three-winding autotransformer and the transformer.

$$Z_{HL\text{-}pu} = Z_{23\text{-}pu} \cdot \frac{n_3^2}{(n_2+n_3)^2} \quad (16)$$

$$\begin{aligned} Z_{HT\text{-}pu} = {} & Z_{13\text{-}pu} \cdot \frac{n_3^2}{(n_2+n_3)^2} \\ & + \left(Z_{13\text{-}pu} + Z_{13\text{-}pu} - Z_{23\text{-}pu}\right) \cdot \frac{n_2 n_3}{(n_2+n_3)^2} \\ & + Z_{12\text{-}pu} \cdot \frac{n_2^2}{(n_2+n_3)^2} \end{aligned} \quad (17)$$

$$Z_{LT\text{-}pu} = Z_{12} \cdot \frac{n_2^2}{n_1^2} \cdot \frac{1}{Z_{\text{baseH}}} = Z_{12} \cdot \frac{1}{Z_{\text{baseT}}} = Z_{12\text{-}pu} \quad (18)$$

Finally, we can re-write (16), (17), and (18) to obtain the equations for the per-unit leakage impedances of the three-winding transformer in terms of the short-circuit impedances specified for the autotransformer.

$$Z_{12\text{-}pu} = Z_{LT\text{-}pu} \quad (19)$$

$$Z_{23\text{-}pu} = Z_{HL\text{-}pu} \cdot \frac{(n_2+n_3)^2}{n_3^2} \quad (20)$$

$$\begin{aligned} Z_{13\text{-}pu} = {} & Z_{HT\text{-}pu} \cdot \frac{(n_2+n_3)^2}{n_3^2 + n_2 n_3} \\ & - \left(Z_{12\text{-}pu} - Z_{23\text{-}pu}\right) \cdot \frac{n_2 n_3}{n_3^2 + n_2 n_3} \\ & - Z_{12\text{-}pu} \cdot \frac{n_2^2}{n_3^2 + n_2 n_3} \end{aligned} \quad (21)$$

Similar approach was traditionally used in [8] to find the star equivalent parameters for the three-winding transformer from the autotransformer's per-unit short-circuit impedances. Using the star equivalent method, (22), (23), and (24) yield the relationships between the per-unit impedances.

$$Z_{12str\text{-}pu} = Z_{LT\text{-}pu} \quad (22)$$

$$Z_{23str\text{-}pu} = Z_{HL\text{-}pu} \cdot \frac{V_H^2}{(V_H - V_L)^2} \quad (23)$$

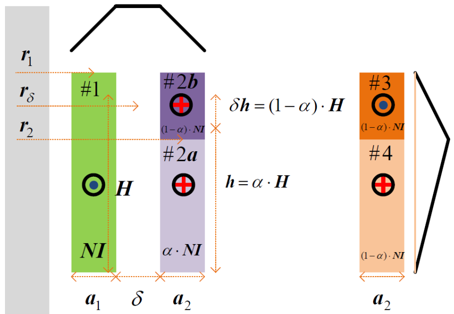

Fig. 5. Resolution of flux to axial and transverse components.

$$Z_{13str\text{-}pu} = Z_{HL\text{-}pu} \cdot \frac{V_H V_L}{(V_H - V_L)^2} + Z_{HT\text{-}pu} \cdot \frac{V_H}{(V_H - V_L)} - Z_{LT\text{-}pu} \cdot \frac{V_H V_L}{(V_H - V_L)} \tag{24}$$

C. Incorporating Internal Faults into the Model

The conventional star equivalent often results in a negative inductance [6]. When an internal fault is applied, changes in the star-connected inductances can lead to a non-passive network, potentially destabilizing the entire simulation [4]. In contrast, one key advantage of TDM-based models is their ability to provide a physically meaningful representation of the magnetic network, ensuring stability even under internal fault conditions. During internal faults, part of the winding stack is lost, altering the geometry and causing an asymmetrical flux distribution composed of axial and transverse components. This phenomenon is modeled using hypothetical auxiliary windings, as shown in Fig. 5. Under these conditions, the short-circuit inductance between winding stacks changes significantly, primarily due to reduced stack height and the presence of a strong transverse flux component. Several methods exist for estimating short-circuit reactance under asymmetrical conditions. A widely adopted approach is the empirical method proposed by Denis-Papin [10], which states that the short-circuit inductance between two windings increases by a factor K, as expressed in (25), (26), and (27).

$$L_{sc12} = K \cdot \mu_0 N^2 \frac{2\pi r_\delta}{H_m} \cdot \left[\delta + \frac{r_1 + r_2}{3}\right] \tag{25}$$

$$H_m = \frac{H - h}{\ln\left(\dfrac{H}{h}\right)} \quad OR \quad \sqrt{H \cdot h} \quad OR \quad \frac{2H \cdot h}{H + h} \tag{26}$$

$$K = 1 + 0.1 \left(\frac{\delta_h}{\delta_e}\right)^{1.6} \tag{27}$$

In the model presented here to calculate the short-circuit inductance between the healthy windings and faulted sub-winding as shown in Fig. 6, an analytical method proposed by H. O. Stephens [11] is employed as the correction factor

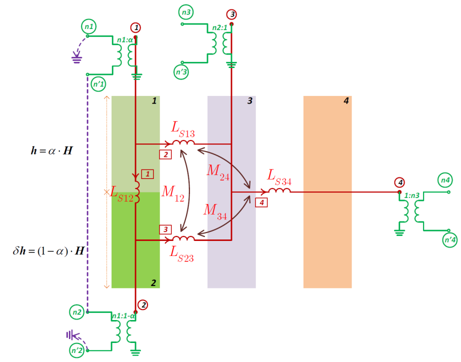

Fig. 6. Three winding transformer model using TDM with fault on winding 1.

is given in (28). More explanation for calculating necessary parameters for the terminal duality approach is given in [4].

$$K_{H.O.S} \simeq \left(1.0 + \frac{\pi}{6} \cdot (1-\alpha)^2 \cdot \frac{h_f}{m_2} \cdot r_f\right) \tag{28}$$

III. Simulation Results

The proposed TDM-based autotransformer model with internal fault is developed in RSCAD-RTDS which is a real-time Electromagnetic Transient (EMT) type simulation tool. To verify this model, an off-line computation and then some steady state and transient studies are performed and compared against conventional star-equivalent model.

a) Comparison in Leakage Inductance Calculation: First of all, is is necessary to numerically verify that the proposed TDM-based approach performs equivalently to the star-equivalent method in calculating the three-winding leakage impedances from the given short-circuit data. To do so, consider an actual autotransformer with a power rating of 250 MVA and high, low, and tertiary voltages of 345 kV, 161 kV, and 37 kV, respectively. The short-circuit impedances of the autotransformer are given as: $Z_{LTpu} = 0.038$, $Z_{HLpu} = 0.1205$, and $Z_{HTpu} = 0.1704$ [9]. Table I presents the results, comparing the two approaches for computing the leakage impedances.

b) Steady State Study: The short-circuit impedance of the autotransformer is verified through steady-state studies. Table II compares the proposed method with the conventional star-equivalent approach. Scenarios with one or two faulted points on the winding stacks, left floating, are also considered. Under these conditions, the transformer should behave like one with healthy windings. To confirm this, short-circuit

TABLE I
LEAKAGE IMPEDANCES CALCULATION USING PROPOSED TDM-BASED APPROACH AND THE STAR-EQUIVALENT METHOD

	Z_{12pu}	Z_{23pu}	Z_{13pu}
TDM based network	0.038	0.423633	0.483945
Star-equivalent network	0.038	0.423633	0.483945

TABLE II
SIMULATION RESULT OF SHORT CIRCUIT IMPEDANCES

	Z_{LTpu}	Z_{HLpu}	Z_{HTpu}
Input data	0.038	0.1205	0.1704
proposed method	0.038054	0.120502	0.170407
Star-equivalent network	0.038043	0.120503	0.170409

impedances were simulated and found to match the healthy winding values, as shown in Table II.

c) Transient Study: The autotransformer described above is simulated with a 1% turn-to-turn fault on the series winding, as shown in Fig. 7, and compared against a similar case using the star-equivalent model. Both autotransformers operate under full resistive load, and the fault is applied for a duration of 0.1 sec. As seen in Fig. 8, both models behave similarly before and after the fault. However, during the fault, the Low and Tertiary voltages (V_L and V_T) remain unchanged in the TDM model, whereas they drop significantly in the star-equivalent model. Likewise, the source-side current at terminal H remains almost unchanged in the TDM model, as expected. In contrast, the star-equivalent model exhibits a large current spike, which is unrealistic.

IV. CONCLUSION

This paper introduces a comprehensive TDM-based three-winding autotransformer model implemented in RSCAD/RTDS for real-time simulation. The model includes functionality for applying internal faults, enabling more robust and stable study frameworks, as compared to conventional star-equivalent modeling approach. Parameter calculations for the TDM model were validated against star-equivalent models through offline computations and simulation studies, confirming its accuracy and reliability. In case of internal faults, it is shown that the proposed method outperforms the star-equivalent modeling approach. The proposed approach not only enhances fault analysis capabilities but also provides a practical foundation for advanced protection applications, such as protection relay HIL testing in real-time environments, which will be done as a future work.

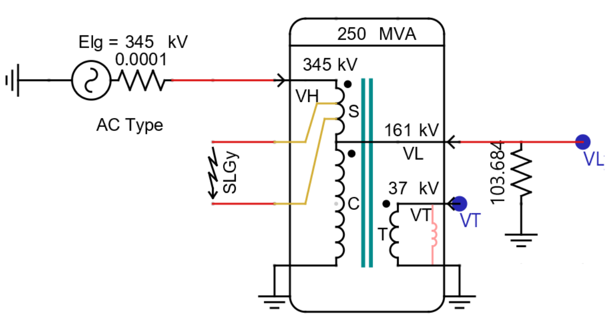

Fig. 7. Transient simulation of a turn-to-turn fault in the proposed three-winding autotransformer in RSCAD/RTDS environment.

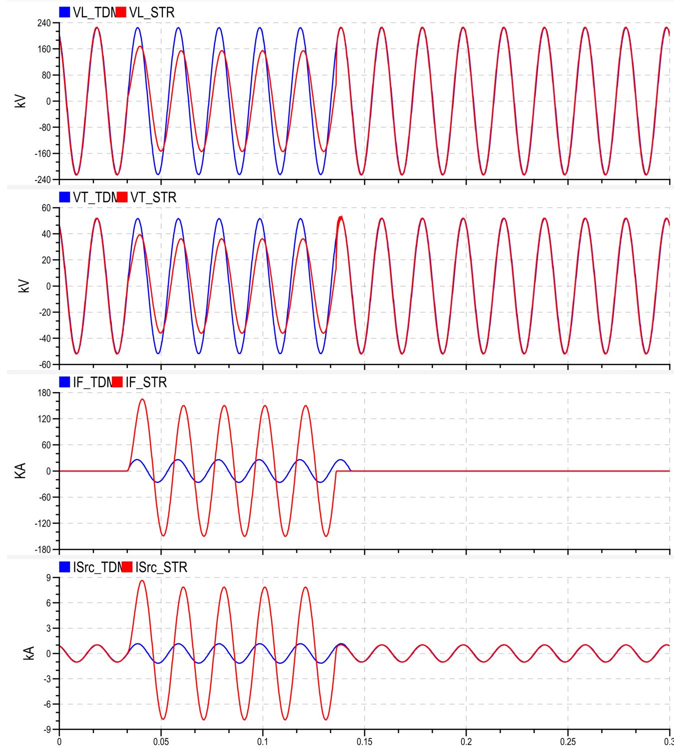

Fig. 8. Transient simulation of a turn-to-turn fault in the proposed TDM-based three-winding autotransformer against star-equivalent model in RSCAD/RTDS environment.

REFERENCES

[1] T. Pavičić, B. Filipović-Grčić, D. Filipović-Grčić, and B. Jurišić, "Effects of Autotransformer's Stabilizing Winding on Current–Voltage Conditions during Unsymmetrical Faults," in *Proc. Int. Conf. Power Systems Transients (IPST)*, Belo Horizonte, Brazil, Jun. 6–10, 2021.

[2] N. Fischer, D. Haas, and D. Costello, "Analysis of an Autotransformer Restricted Earth Fault Application," Schweitzer Engineering Laboratories, Inc., Tech. Paper 6279 (Revised July 2010); originally presented at the 62nd Annual Georgia Tech Protective Relaying Conference, May 2008.

[3] A. Avendaño, B. A. Mork, and H. K. Høidalen, "Transformer Internal Fault Modeling in ATP," in *Proc. Int. Conf. Power Systems Transients (IPST)*, Delft, The Netherlands, Jun. 14–17, 2011

[4] RTDS Technologies Inc., *RSCAD User Manual*, RTDS Technologies Inc., Winnipeg, Canada, Latest Edition. Available at:*https://www.rtds.com/rscad*

[5] F. de León and J. A. Martinez, "Dual Three-Winding Transformer Equivalent Circuit Matching Leakage Measurements," *IEEE Transactions on Power Delivery*, vol. 25, no. 4, pp. 2599–2607, Oct. 2010.

[6] C. Álvarez-Mariño, F. de León, and X. M. López-Fernández, "Equivalent Circuit for the Leakage Inductance of Multiwinding Transformers: Unification of Terminal and Duality Models," *IEEE Transactions on Power Delivery*, vol. 27, no. 1, pp. 353–361, Jan. 2012.

[7] M. Ahmadi, A. Dehkordi, and Y. Zhang, "Enhancements to Terminal Duality-based models for three-phase multi-limb multi-winding transformers," *Electric Power Systems Research*, vol. 251, 112204, Sep. 2025, doi:10.1016/j.epsr.2025.112204.

[8] H. W. Dommel and S. Bhattacharya, *EMTP Theory Book*, Microtran Power System Analysis Corporation, 1992.

[9] F. Zhu and B. Yang, *Power Transformers Design Practices*, CRC Press, 2020.

[10] M. Denis-Papin, *La Pratique Industrielle des Transformateurs*, Editions Albin Michel, 1951.

[11] H. O. Stephens, "Transformer reactance and losses with nonuniform windings," *Electrical Engineering*, vol. 53, pp. 346–349, Feb. 1934.

979-8-3315-8138-1