Data-Driven Post-Event Analysis with Real-World Oscillation Data from Denmark

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***Abstract*—This paper demonstrates how Extended Dynamic Mode Decomposition (EDMD), grounded in Koopman operator theory, can effectively identify the main contributor(s) to oscillations in power grids. We use PMU data recorded from a real 0.16 Hz oscillation event in Denmark for post-event analysis. To this end, the EDMD algorithm processed only voltage and current phasors from nineteen PMUs at different voltage levels across the Danish grid. In such a blind-test setting with no supplementary system information, EDMD accurately pinpointed the location of the main contributor to the 0.16 Hz oscillation. Energinet later confirmed that this was a 145 MW solar photovoltaic (PV) park with control system issues. Notably, conventional approaches, such as the dissipating energy flow (DEF) method used in the ISO-NE OSLp tool did not correctly identify this plant. This joint validation with Energinet, reinforcing earlier studies using simulated IBR-dominated systems and real PMU data from ISO-NE, highlights the potential of EDMD-based post-event analysis for identifying major oscillation contributors and enabling targeted SSO mitigation.**

***Index Terms*—Oscillation source location, Extended Dynamic Mode Decomposition, Data-Driven, Real-world oscillation event**

I. Introduction

Poorly damped oscillations induced by inverter-based resources (IBRs) have become a major concern. Identifying the main contributors to such oscillations is essential for effective mitigation. However, this is challenging in IBR-dominated grids because vendor-specific IBR models are opaque, and perturbation-based model estimation is not feasible for post-event analysis.

The Dissipating Energy Flow (DEF) method performs well for forced oscillations involving synchronous machines (SM) [1] but is less effective for IBR-induced oscillations [2]. Although a modified DEF method can solve this problem, it cannot necessarily identify the source of SM-driven electromechanical oscillations [2]. Using oscillation amplitude at IBR terminals as an indicator of participation [3] lacks theoretical grounding. NREL's GIST tool [4] relies on electromagnetic transient (EMT)-based impedance scans across multiple operating conditions and is therefore unsuitable for post-event analysis.

Dynamic Mode Decomposition (DMD) is a data-driven, model-free method for extracting spatio-temporal patterns and has recently been applied in power systems for oscillation analysis and control [5]–[7]. However, existing DMD-based approaches have not been validated for IBR-driven oscillations, where dynamics are highly nonlinear and less transparent. To address this, an Extended DMD (EDMD) framework was developed to identify major contributors to oscillations [8]. Although [8] validated EDMD with simulated data and real data from synchronous machine-based ISO-NE system, it has not been tested on real IBR-driven events. This paper provides the first validation of EDMD using a real oscillation event from the Danish grid. This demonstrates its effectiveness in identifying the main contributors to poorly damped IBR-driven oscillations for post-event analysis.

II. Methodology

A. Extended Dynamic Mode Decompression (EDMD)

This section provides a brief outline of the proposed EDMD-based methodology and further details can be found in [8]. The Extended Dynamic Mode Decomposition is the core of this framework. Its essential components and properties are summarised here following the developments in [9]. Consider the discrete-time nonlinear system

$$x_{k+1} = F(x_k), \tag{1}$$

where $x_k \in \mathcal{M} \subseteq \mathbb{R}^n$ denotes the system state at time step k and $F : \mathcal{M} \to \mathcal{M}$ represents the nonlinear state-transition map governing the system dynamics. The associated Koopman operator $\mathcal{K}$ acts on observables $\psi : \mathcal{M} \to \mathbb{C}$, where ψ belongs to a chosen library of functions of the state variable x. The Koopman operator is defined as

$$\mathcal{K}\psi = \psi \circ F, \tag{2}$$

yielding the infinite-dimensional linear evolution

$$\psi_{k+1} = \mathcal{K}\psi_k. \tag{3}$$

These representations describe the same dynamics, and future system states may be expressed through the Koopman mode expansion [10]:

$$F(x) = \sum_{j=1}^{N_j} \mu_j v_j \varphi_j(x), \tag{4}$$

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This work was supported by the Engineering and Physical Sciences Research Council (EPSRC), U.K., under Grant EP/Y025946/1.

979-8-3315-8138-1

where μ_j, φ_j, and v_j denote the Koopman eigenvalues, eigenfunctions, and modes, respectively. Note that N_j in many situations could be infinite. Approximating this linear but infinite-dimensional operator from data has become an effective means for analysing nonlinear systems.

EDMD provides such an approximation by projecting the action of $\mathcal{K}$ onto a finite dictionary of observables. To this end, consider a sequence of collected snapshot data x_i, where $i = 1, \ldots, M$, and auxiliary snapshots $y_i = F(x_i)$. Let $\mathcal{D} = \{\psi_1, \ldots, \psi_{n_d}\}$ be a dictionary of observables, such that

$$\Psi(x) = \left[\psi_1(x); \psi_2(x); \ldots; \psi_{n_d}(x)\right]. \tag{5}$$

The finite-dimensional approximation of the Koopman operator can then be obtained as

$$K \triangleq G^{\dagger} H, \tag{6}$$

with

$$G = \frac{1}{M}\sum_{j=1}^{M} \Psi(x_j)^* \Psi(x_j), \quad H = \frac{1}{M}\sum_{j=1}^{M} \Psi(x_j)^* \Psi(y_j). \tag{7}$$

This completes the concise summary of the standard EDMD procedure used to approximate the Koopman operator in a finite-dimensional subspace. An alternative formulation of the Koopman operator K, introduced in [11], defines $M_K = K^{\top}$ and is adopted in this work. This representation, also employed in [12], offers a more convenient structure for computing data-driven participation factors and is used here for consistency.

B. Data-driven algorithm

1) Selection of observables: The EDMD lifts the voltage and current phasors $V\angle\theta_V$ and $I\angle\theta_I$ from PMUs into a higher-dimensional space Ψ using a dictionary of observables to capture nonlinear system behavior [13]. Polynomial and trigonometric functions of voltage and current phasors, particularly active and reactive power, $P = VI\cos(\theta_V - \theta_I), Q = VI\sin(\theta_V - \theta_I)$ at the point of interconnection are used as lifted observables in this study.

2) Filtered mode of interest: The dominant poorly damped frequency f_s is identified from the FFT of the detrended time-series PMU data across all locations. This frequency corresponds to the peak in the FFT amplitude spectrum where bulk of the oscillatory energy is concentrated across multiple locations. The data are band-pass filtered with a 4^{th}-order Butterworth filter at $[0.9, 1.1]f_s$. To reduce boundary effects, only the central 50–80% of the filtered signal is used, improving EDMD accuracy by isolating the main mode and lowering the required model order.

3) Calculation of reduced-order $\tilde{M}_K$: Filtered data are used to compute the finite-dimensional Koopman operator M_K using EDMD. Although an appropriate set of observables spanning a Koopman-invariant subspace would yield an accurate M_K, in practice only an approximately invariant set $\Psi(x)$ can be identified. As a result, inappropriate observable selection may introduce spurious eigenvalues, and the dimension of M_K can be large in power system applications. To address this, M_K is projected onto a reduced subspace via truncated SVD of $G = U\Sigma R^*$. Retaining the r dominant singular values yields $U_r = U(:, 1{:}r), \Sigma_r = \Sigma(1{:}r, 1{:}r), R_r = R(:, 1{:}r)$, and the reduced-order Koopman operator $\tilde{M}_K$ is obtained via

$$\tilde{M}_K = U_r^* H R_r \Sigma_r^{-1}. \tag{8}$$

This reduced-order operator retains the slow dynamical subspace while suppressing fast transients, aligning naturally with the objective of analysing low-frequency oscillations. In general, the singular values are arranged in descending order. The truncation order r is selected by identifying the elbow point in the singular value spectrum, beyond which the singular values exhibit a pronounced decay. This criterion ensures that the dominant dynamical modes are preserved.

The eigen-decomposition

$$\tilde{M}_K \tilde{\Phi} = \tilde{M} \tilde{\Phi} \tag{9}$$

provides the reduced-order eigenvectors matrix ($\tilde{\Phi}$) and diagonal eigenvalue matrix ($\tilde{M}$), from which the full-order right and left eigenvectors of M_K can be reconstructed as

$$\hat{\Phi} = U_r \tilde{\Phi}, \qquad \hat{\Xi} = \hat{\Phi}^{\dagger}. \tag{10}$$

4) Data-Driven modal analysis: The discrete-time eigenvalues μ_i obtained from (9) are converted to continuous-time values using $\lambda_i = \frac{\ln(\mu_i)}{\Delta t}$, from which the damping ratio and oscillation frequency are derived. The participation factors of the observables in mode i are computed from the left and right eigenvectors as [12]

$$p_{si} = \hat{\phi}_{si} \hat{\xi}_{is}, \tag{11}$$

where p_{si} denotes the participation of s^{th} state variable to mode i. The overall participation factors follow from summing over all associated state variables, and is normalized for comparison across plants. Therefore, power plants exhibiting large absolute participation factor values are identified as the primary contributors to the poorly damped oscillation.

III. Case study with EDMD-based method

A. PMU data from oscillation event in Denmark

The oscillation event in Denmark was recorded by 19 PMUs with voltage and current phasors sampled at 50 Hz. All 19 PMUs are located in the western part of Danish transmission grid with 3 PMUs at major transit substations. The rest are placed at connection points of IBR plants (wind farm, solar PV park) or HVDC links. The PMU at location 19 is installed at the connection point of a 145 MW solar PV park.

A 120-second dataset is available, within which pronounced oscillations occur between 60–110 seconds. Figs 1 and 2 show the full recordings, out of which 60–110 second window is selected for data-driven post-event analysis. Based on the FFT spectrum analysis of the detrended PMU data across all locations, the dominant oscillatory spectral peak is found to be at 0.16 Hz as shown later in Fig 4 (in Sec III-C). Accordingly, a low-pass filter with a cutoff frequency of 3 Hz is applied to suppress high-frequency noise while preserving the dynamics around the dominant oscillation frequency.

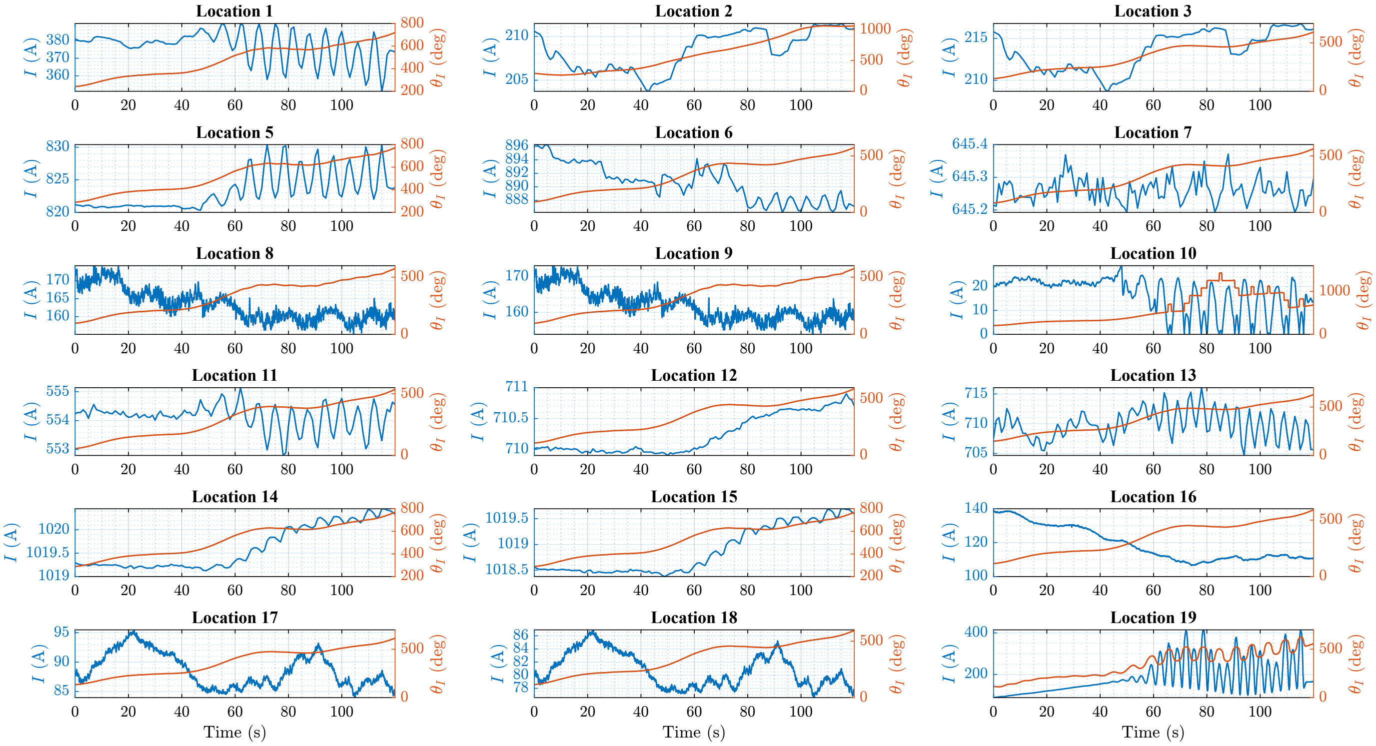

Fig. 1. Time-series phasor current magnitude and angle measurements from PMUs across selected locations in the Danish transmission system.(Location 4 is not included due to bad data quality)

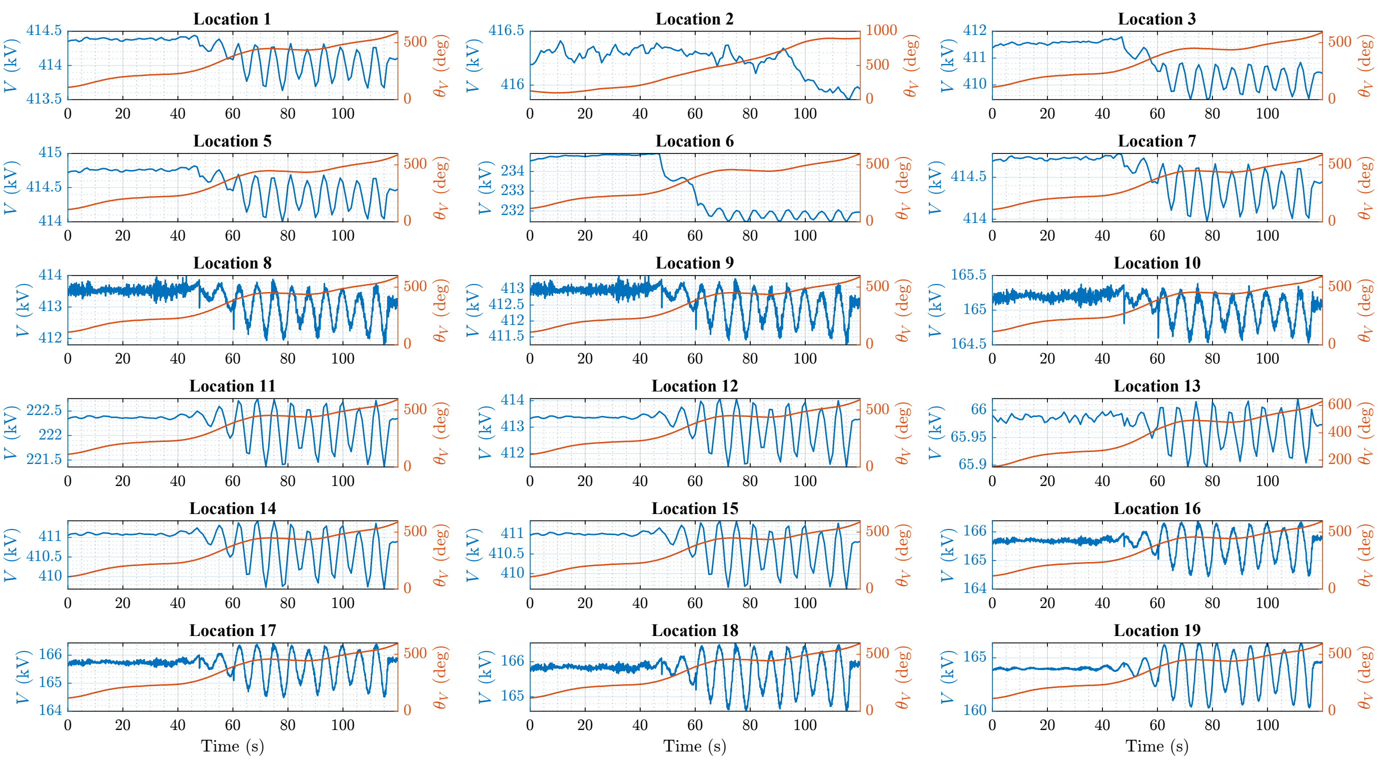

Fig. 2. Time-series phasor voltage magnitude and angle measurements from PMUs across selected locations in the Danish transmission system.

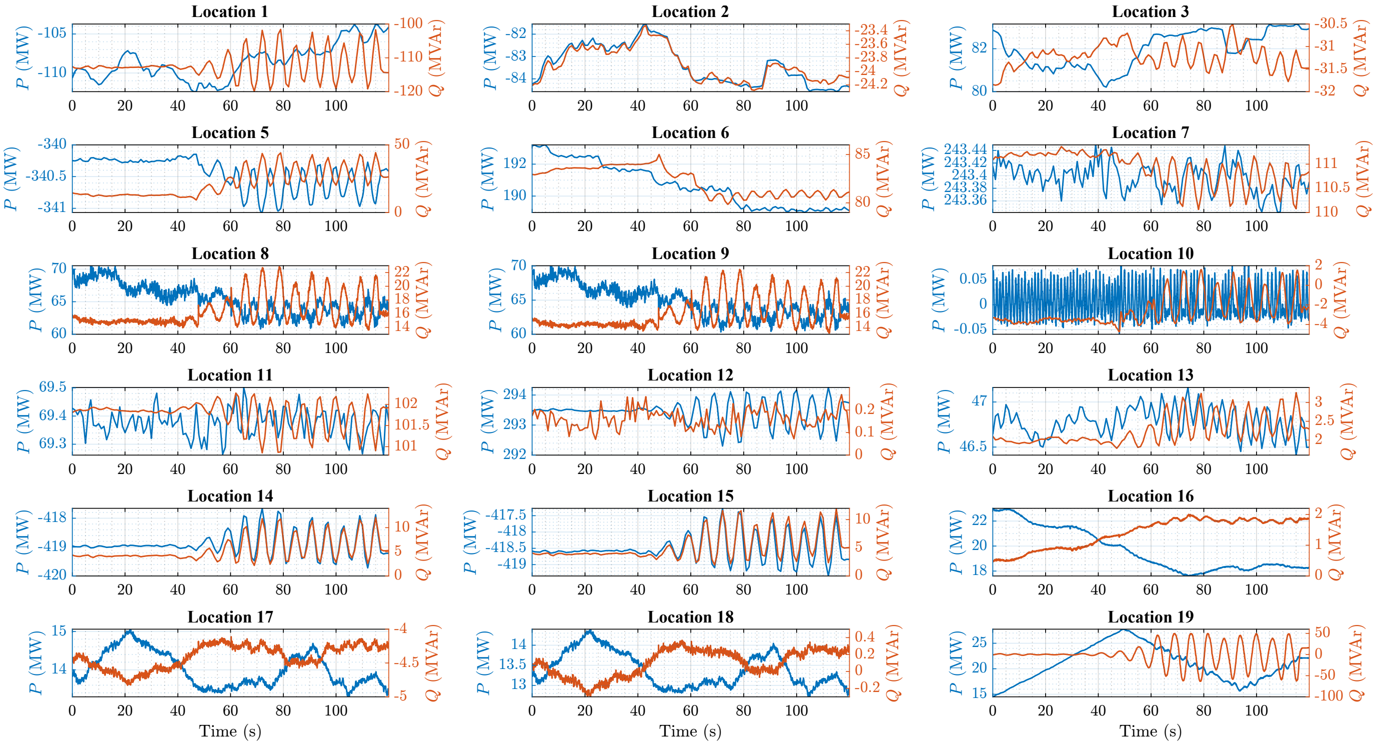

Fig. 3. Active and reactive power P and Q calculated from PMU measurements at selected locations in the Danish transmission system.

B. Observables in EDMD

As discussed in Section II-B1, the active and reactive power components, P and Q, which are polynomial functions of the measured voltage and current phasors from PMU data, are effective in capturing the system nonlinear dynamics. Hence, P and Q are employed in this study. The derived active and reactive power for each location are shown in Fig. 3.

C. Top oscillation contributor

The FFT amplitude spectrum shown in Fig 4(a) reveals a clear spectral peak at 0.16 Hz indicating the dominant oscillatory mode. Notably, the peak occurs at the same frequency for multiple locations. Singular value analysis yields an EDMD truncation order of 7. The EDMD participation analysis (Fig. 4(b)) identifies location 19 as the main contributor. This is consistent with Energinet's post-event finding that an solar PV plant at location 19 caused the oscillation due to improper control settings. This alignment demonstrates the effectiveness of the proposed EDMD method in pinpointing oscillation contributors using PMU data from a real event.

IV. Case study with other methods

A. Dissipating Energy Flow (DEF)

For comparison, the DEF method is applied using ISO New England's OSLp software [14], [15] on the same PMU dataset from Denmark. Fig. 4(c) shows that locations 5 and 1 have the largest positive injected oscillating energy, they would be incorrectly identified as the primary oscillation sources based on DEF. Additionally, location 19, despite having the largest absolute dissipating energy magnitude, would be mistakenly excluded due to its negative sign. This contradicts the actual post-event findings.

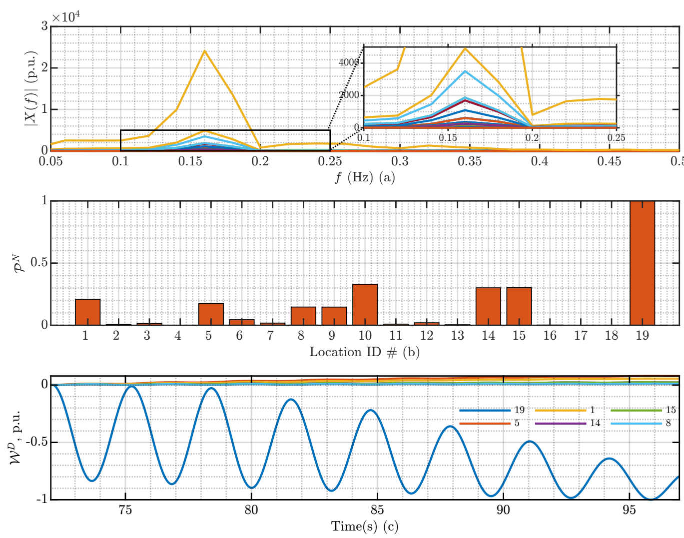

Fig. 4. (a) FFT of detrended time-series PMU data from all 19 locations; (b) Data-driven participation factors obtained via EDMD. Location 19 is the top contributor to 0.16 Hz oscillation. (c) Top six locations in terms of highest dissipating energy flow. Locations 14 and 15 (overlapping traces) show highest positive energy injection, while location 19 has highest energy absorption.

B. Q-V phase-based method

Energinet identifies the main oscillation contributors primarily by determining the phase alignment between Q–V at

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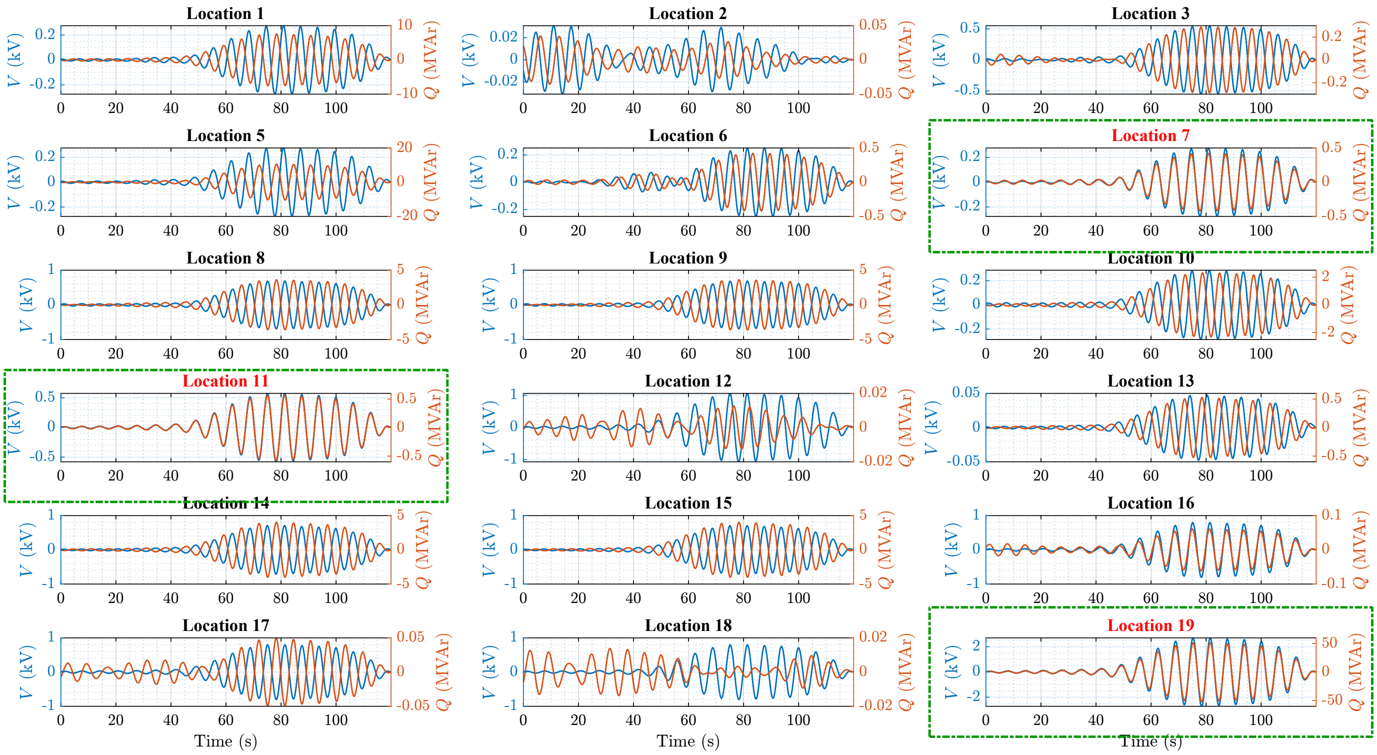

Fig. 5. Phase alignment of reactive power Q and voltage magnitude V at the oscillation frequency (band-pass filtered) for selected Danish transmission system locations. Locations 7, 11, 19 show Q-V in-phase (marked with a green box).

the oscillation frequency. Fig. 5 shows that locations 7, 11, and 19 exhibit in-phase Q–V patterns. When combined with practical information (e.g., new plants undergoing testing), this approach suggests location 19 as the major contributor. While this method is effective for detecting forced oscillations, as in the case presented, it is less reliable for natural oscillations because the results are highly sensitive to the selected window.

V. Conclusion

This paper demonstrates that Extended Dynamic Mode Decomposition (EDMD) can effectively identify the dominant contributor to oscillations using limited PMU data, without any network information. Its successful application to a real 0.16 Hz oscillation event in Denmark, carried out as a blind test, confirms its accuracy in real-world post event analysis. In this particular case, a 145 MW solar PV park was correctly identified as the main contributor to poorly damped 0.16 Hz oscillation. Unlike conventional dissipating energy flow (DEF) or phase-based methods, EDMD correctly reveals the top contributor to the oscillation in IBR-dominated grids. These results highlight EDMD's potential as a powerful data-driven tool for post-event analysis to ensure the effective mitigation.

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