BDF2-Based Integration for Electromagnetic Transients Simulation in Modern Power Systems

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***Abstract*—Electromagnetic transient (EMT) simulation is a cornerstone of modern power system analysis, especially with the rapid rise of converter-based grids and nonlinear components. Traditionally, the trapezoidal rule has dominated EMT solvers, despite known limitations in stiff and highly dynamic systems. This paper introduces a paradigm shift toward using the second-order Backward Differentiation Formula (BDF2) for EMT simulation, formulated for three-phase and coupled network elements. We show that the L-stable BDF2 mitigates non-physical numerical oscillations typical of trapezoidal integration, albeit with novel considerations regarding physical oscillatory damping and time step selection. Comparative studies and discussions show its suitability for next-generation EMT simulation tools.**

***Index Terms*—Electromagnetic Transients, Numerical Integration, BDF2, L-stability, Power Electronics, IBR, Stiff Systems**

I. INTRODUCTION

Electromagnetic transient (EMT) simulation has become increasingly critical as power systems incorporate more high-speed switching devices, renewables, and sophisticated control strategies. Based on Prof. H. W. Dommel's pioneering work on the EMTP program in the 1960s [1], the foundation of virtually all modern EMT tools—including ATP-EMTP, PSCAD™, PSS®NETOMAC, EMTP®, and DIgSILENT PowerFactory—rests on the trapezoidal rule as the method of choice for numerical integration, because of its second-order accuracy and A-stability properties.

However, the power system landscape has evolved dramatically since EMTP's inception. Modern grids are characterized by high penetration of Inverter-Based Resources (IBR), fast-switching semiconductor devices, complex control systems with multiple time scales, and grid-forming inverters with rapid response requirements.

These developments introduce significant numerical stiffness into power system models, where the ratio of fastest to slowest time constants can exceed 10^6. While the trapezoidal rule is A-stable (stable for all finite eigenvalues), it lacks L-stability, meaning it cannot properly damp high-frequency numerical oscillations that arise in stiff systems [2].

This limitation manifests as non-physical oscillations in critical scenarios such as circuit breaker operations, modulation in power electronic devices, and fault studies, potentially affecting protection coordination and system design decisions. Unrealistic numerical oscillations for inductive circuits have been described in many early publications, for example in the EMTP Theory Book [3], [4]. Several possible solutions have been suggested, such as re-initializing the history terms, linear interpolating to roll-back the solution to the moment of switching before the corresponding switch action [5], [14], or using parallel damping resistors [6], [7].

We propose a paradigm shift toward using the second-order backward differentiation formula (BDF2) as a valid alternative to the trapezoidal rule for EMT simulation. The BDF2 combines second-order accuracy with L-stability, ensuring non-divergence for any time step and furthermore naturally handling stiff systems without artificial damping. Several other methods for numerical integration of differential equations are known and implemented in mathematical tools, e.g. MATLAB. With special regard to electromagnetic transient simulations, some hybrid methods such as the trapezoidal method combined with BDF2 (TR-BDF2), or other L-stable implicit methods such as two/three-stage diagonally implicit Runge-Kutta (2S-DIRK, 3S-DIRK), have been suggested to improve numerical stability and accuracy [8]–[13]. It should be mentioned that the TR-BDF2 is a two-stage method and thus has a higher computational effort than a BDF2-only solution.

However, none of the mentioned works seem to investigate the effect of the time step size on the accuracy of transient simulations. In this paper, we focus on the BDF2-only method because of its numerical stability, and discuss the possible loss of accuracy due the time step size. We emphasize how a generalized method with variable time step can improve accuracy and performance compared to the trapezoidal rule and other mentioned methods. We present the theoretical foundation (section II), implementation methodology (section III), and discuss its potential for significant improvements in simulating transients in modern power systems (section IV).

II. THEORETICAL MOTIVATION AND NUMERICAL STABILITY

A. Numerical Approximation of Differential Equations

Let us consider a general linear differential equation $\frac{dy}{dt} = \lambda y$, being y a function of the time and $y(0) = y_0$. It is possible to obtain a numerical approximation of this equation by discretization of the time t based on uniform steps of size h, such that $t_n = nh$ with n finite. Thus, the derivative of $y(t)$

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can be approximated by a difference operator, for example we can write the forward difference operator and equate it to the function λy at time t_n or at time t_{n+1} and obtain the explicit (or forward) and implicit (or backward) Euler method, respectively:

$$\frac{y_{n+1} - y_n}{h} = \lambda y_n \tag{1}$$

$$\frac{y_{n+1} - y_n}{h} = \lambda y_{n+1} \tag{2}$$

A numerical method is called *implicit* if the right-hand side of the approximated equation contains a term at time t_{n+1}, otherwise it is called *explicit*. While explicit methods calculate y_{n+1} directly from the state of the system at time $t = t_n$, implicit methods require an extra computation to find y_{n+1} from the current and the future state of the system. Thus, implicit methods are more computationally intensive than explicit ones, and they can be sometimes more challenging to implement. However, they offer considerably greater stability. For stiff systems, explicit methods are impractical due to numerical stability reasons.

The trapezoidal rule is an implicit method as well:

$$\frac{y_{n+1} - y_n}{h} = \lambda \frac{y_{n+1} + y_n}{2} \tag{3}$$

B. BDF2 Mathematical Formulation

Methods that approximate differential equations over one single value of h are called *single-step methods*. Both Euler methods and the trapezoidal rule are single-step methods. In constrast, methods using values at more than time instants t_n and t_{n+1} are called *multi-step methods*.

The BDF2 is an implicit, multi-step method. Its mathematical formulation for fixed-size time steps is given below:

$$\left.\frac{dy}{dt}\right|_{t_{n+1}} \approx \frac{1}{h}\left(\frac{3}{2}y_{n+1} - 2y_n + \frac{1}{2}y_{n-1}\right) \tag{4}$$

C. Stability Analysis

For the general differential equation $\frac{dy}{dt} = \lambda y$, where λ represents system eigenvalues, numerical stability is determined by the amplification factor $G = \frac{y_{n+1}}{y_n}$ of the integration scheme.

The trapezoidal rule yields:

$$G_{trap} = \frac{2 + h\lambda}{2 - h\lambda} \tag{5}$$

For large negative λ (stiff systems), $|G_{trap}| \to 1$, meaning that high-frequency modes are not damped. This implies that for stiff systems, the trapezoidal method shows persistent oscillations around the correct solutions.

For the BDF2 method, instead, we find out that:

$$G_{BDF2} = \frac{2 + \sqrt{1 + 2h\lambda}}{3 - 2h\lambda} \tag{6}$$

For large negative λ, $|G_{BDF2}| \to 0$, which confirms that the BDF2 is L-stable, thus ensuring proper damping of high-frequency components.

Table I summarizes the key numerical properties. The error constants in the last column emphasize that the trapezoidal method has a smaller truncation error, compared to the BDF2. However, the BDF2 method has the advantage of being L-stable, thus preserving overall accuracy for stiff systems:

TABLE I
NUMERICAL INTEGRATION METHOD COMPARISON

Method	Order	A-Stable	L-Stable	Error Constant
Trapezoidal	2	Yes	No	$-1/12$
BDF2	2	Yes	Yes	$1/3$
Backward Euler	1	Yes	Yes	$1/2$

D. Stiffness in Power Systems

A system of differential equations is considered stiff when it has multiple time scales or components that evolve at very different rates. Stiff systems generally force numerical methods to take very small steps to maintain numerical stability, even when larger steps would be sufficient for accuracy. One example of stiff systems are electrical circuits with components having very different time constants.

Power system stiffness arises from multiple sources, for instance power electronics devices with fast switching, complex control strategies, and non-linearities such as arc characteristics in circuit breakers, or transformer saturation. The stiffness ratio in modern power systems can exceed 10^6, well beyond the effective range of A-stable methods without L-stability.

III. BDF2 ALGORITHM IMPLEMENTATION

A. Multi-Phase Circuit Equations

The BDF2 method generalizes naturally to three-phase and coupled networks, improving computational efficiency and scalability for realistic power systems. Applying the fixed-size time step formulation (4), with $h = \Delta t$ the time step chosen for the simulation, to a network equation of type $\mathbf{G}\mathbf{v} + \mathbf{C}\frac{d\mathbf{v}}{dt} = \mathbf{i}$, the numerical integration yields:

$$\left[\mathbf{G} + \frac{3}{2\Delta t}\mathbf{C}\right]\mathbf{v}_{n+1} = \mathbf{i}_{n+1} + \frac{2}{\Delta t}\mathbf{C}\mathbf{v}_n - \frac{1}{2\Delta t}\mathbf{C}\mathbf{v}_{n-1} \tag{7}$$

The above equation can subsequently be re-written in the usual format used by EMT simulation tools as:

$$\mathbf{Y}\mathbf{v}_{n+1} = \mathbf{i}_{n+1} + \mathbf{i}_{hist} \tag{8}$$

with the admittance matrix $\mathbf{Y} = \left[\mathbf{G} + \frac{3}{2\Delta t}\mathbf{C}\right]$, the vector of current sources $\mathbf{i}_{n+1}$ and the history term $\mathbf{i}_{hist} = \frac{2}{\Delta t}\mathbf{C}\mathbf{v}_n - \frac{1}{2\Delta t}\mathbf{C}\mathbf{v}_{n-1}$. Then, the usual algorithms for solving the linear system (for example, LU factorization or iterative methods for large sparse systems) can be used to solve for $\mathbf{v}_{n+1}$.

B. Computational Complexity

BDF2 requires storage of two arrays of values at previous time instants, analogous to the trapezoidal rule implemented in many existing EMT tools. The computational cost per time step and the memory cost are therefore comparable. Other operations such as matrix factorization (roughly $O(n^{1.5})$ for sparse systems) or forward/backward substitution ($O(n)$) have costs that do not depend on the choice of integration method.

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IV. BDF2 FOR POWER SYSTEMS SIMULATION

A. Non-Linearities Causing Numerical Issues

A critical advantage of BDF2 is its natural handling of non-linearities, especially sudden discontinuities that can cause numerical oscillations. For instance, when a switching event occurs at time t_s, the network topology changes, and so does the admittance matrix, requiring matrix re-factorization. Assuming an ideal switch, for which the open state is represented by a very low admittance (e.g., $10^{-9}S$) and the closed state by a very large admittance (e.g., $10^{9}S$), and a simultaneous breaker disconnection of all phases, numerical oscillations with triangular shape will appear on the voltage of the phases where the current interruption takes place when the current is not zero, as shown in Fig. 1.

The comparison shows how the numerical oscillations are damped via in-built artificial damping resistor in PSCAD™ (version 5.0.2), and are undamped in PSS®NETOMAC (version 21.5), which has linear interpolation but no automatic damping resistance. In contrast, BDF2's L-stability ensures rapid damping of numerical oscillations without artificial damping or post-processing required.

It should be noted that such triangular-shape oscillations still appear when the breaker chops a relatively small current.

B. High-Frequency Physical Oscillations

As shown in the previous section, the BDF2 method has the interesting property of damping out naturally high-frequency oscillations. This is a decisive advantage to ensure numerical stability, on the other side also physically meaningful high-frequency oscillations could be damped out. That is, the positive aspect of the L-stability is at the same time a drawback, if the time step Δt is too large relative to the system's fastest modes.

A rule of thumb for EMT simulations is to take a maximum simulation time step $\Delta t \leq \frac{0.1}{f_{max}}$, being f_{max} the maximum frequency of interest. Or, in other words, a minimum of 10 points per cycle (with frequency f_{max}) should be considered. This rule of thumb works quite well with the trapezoidal rule of integration. Taking less than 10 points per cycle in general leads to cutting transient peaks and weakly damped amplitudes of the respective high-frequency component in currents and voltages.

With regard to the BDF2 method for numerical integration, simulations show that it is required to choose a maximum time step $\Delta t \leq \frac{0.05}{f_{max}}$, that is, at least 20 points per cycle referred to f_{max}. Taking less than 20 points per cycle could result in excessive damping of real high-frequency components.

To exemplify this, the simulation of a line energization has been carried out with both methods, whereas the line has been modeled as ten PI-sections in series. The line has a resonance frequency of $f = \frac{1}{2\pi\sqrt{LC}} = 468$Hz. The ideal circuit breaker is closed on all phases simultaneously at $t = 0.2s$. The data for the network equivalent and the line are listed in Table II below.

The following Figures show the source-side voltages and the currents in the closing circuit breaker. As shown in Fig. 2,

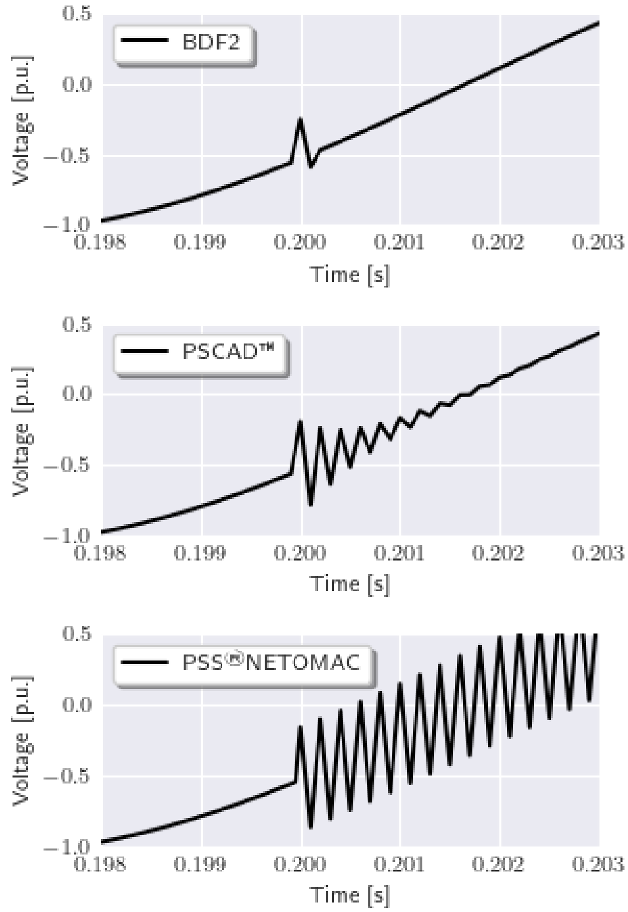

Fig. 1. Numerical oscillations following sudden admittance matrix changes.

a half-size time step is required for the BDF2 to maintain the same high-frequency components compared to the trapezoidal method (Fig. 4). Taking a larger time step results in unwanted, strong damping of the switching transients, leading to inaccurate results, as it can be seen in Fig. 3.

TABLE II
DATA FOR LINE ENERGIZATION EXAMPLE

Element	Parameter	Value	Unit
Source	S_k''	10000.	MVA
	R/X ratio	0.1	-
	f_{rated}	50	Hz
	U_{rated}	380	kV
Line	R_{line}	0.03	Ω/km
	X_{line}	0.2598	Ω/km
	C_{line}	14	nF/km
	l	100	km
	R_0/R_1 ratio	4	-
	X_0/X_1 ratio	2.5	-
	C_0	6	nF/km

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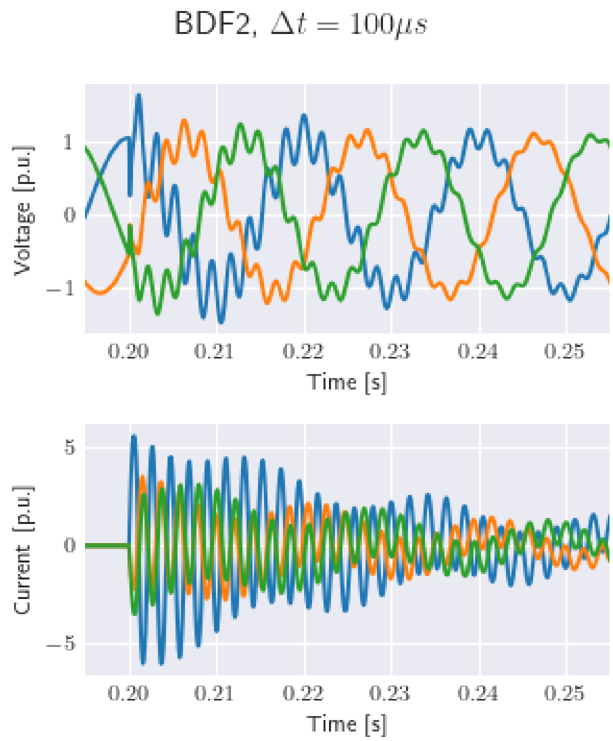

Fig. 2. Line energization with BDF2, smaller time step.

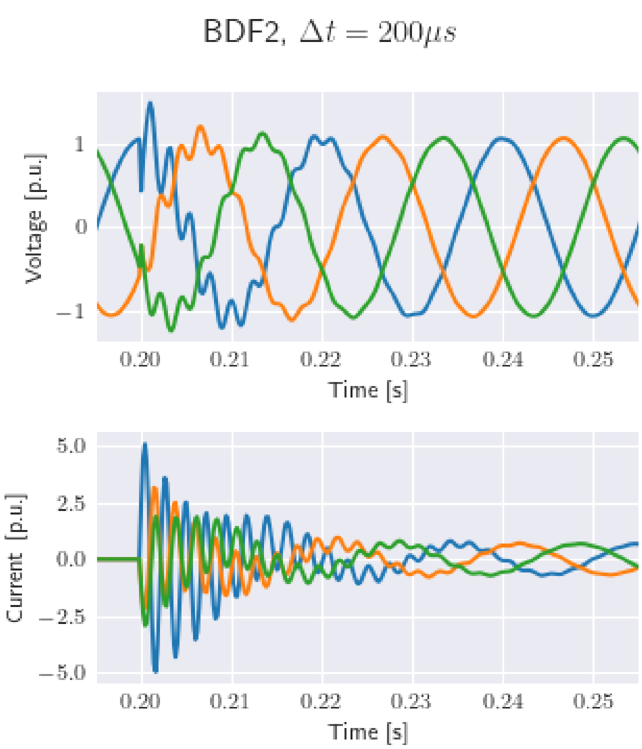

Fig. 3. Line energization with BDF2, too large time step.

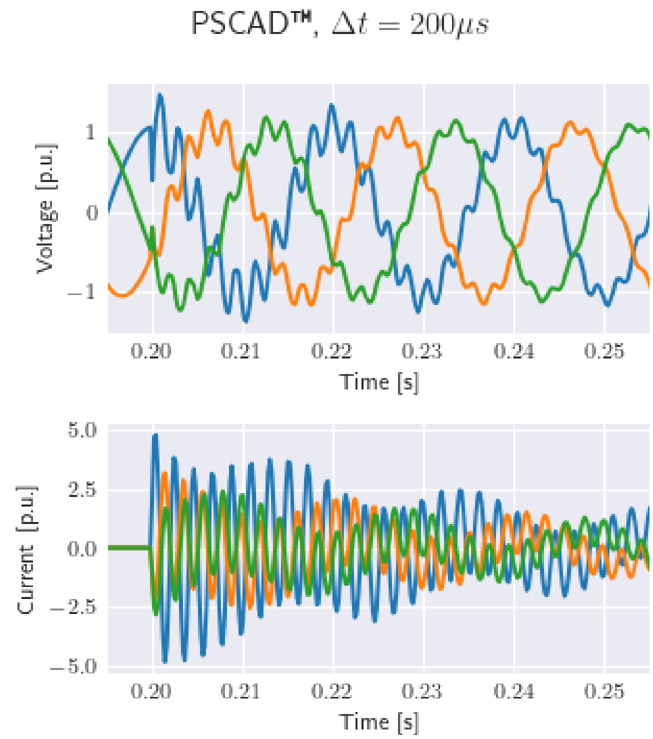

Fig. 4. Line energization with trapezoidal rule.

Although requiring smaller time steps for accuracy, the BDF2 method is not necessarily less performant. In our comparison with PSCAD™ version 5.0.2 carried out on a laptop with an Intel i7-12800H CPU, the line energization simulation with time step 200 μs and end time 1 s and without storing results, thus indicating pure CPU simulation time, has been performed in 171 ms with the GFortran 8.1 compiler, and in 79 ms with the Intel Fortran compiler version 2025.3. The same simulation with our own BDF2 implementation and time step 100 μs has been performed on the same PC in 111 ms. This demonstrates that we can achieve higher accuracy with smaller time steps, all while maintaining the same computation runtime.

C. Adaptive Variable Time Step BDF2

To increase performance and deal with the unwanted filtering effect of high-frequency physical oscillations, a variable time step BDF2 could be used.

The generalized BDF2, which is suitable for a variable time steps, is expressed as follows:

$$\left.\frac{dy}{dt}\right|_{t_{n+1}} \approx \alpha_0 y_{n+1} + \alpha_1 y_n + \alpha_2 y_{n-1} \tag{9}$$

with:

$$\alpha_0 = \frac{2h_n + h_{n-1}}{h_n\left(h_n + h_{n-1}\right)} \quad , \quad \alpha_1 = -\frac{h_n + h_{n-1}}{h_n h_{n-1}}$$

$$\alpha_2 = \frac{h_n}{h_{n-1}\left(h_n + h_{n-1}\right)}$$

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and:

$$h_n = t_n - t_{n-1}$$
$$h_{n-1} = t_{n-1} - t_{n-2}$$

It is easy to verify that when $h_n = h_{n-1}$, the above equation (9) reduces to (4).

An adaptive time step controller could approximately determine the actual frequency of all node voltages and branch currents based on local peak detection and median filtering over 3 instantaneous frequency estimates, choosing the most restrictive time step.

Safety mechanisms could be implemented to increase accuracy at a small penalty in performance, such as:

- Forcing small time steps immediately after events and contingencies, and/or
- Adopting a maximum time step when the signals' frequency decays below a certain threshold.

V. Conclusions

This paper discusses the BDF2 method for numerical integration as possible and valid alternative to the trapezoidal method for the simulation of electromagnetic transients in power systems.

Being L-stable, in contrast to the trapezoidal method, the BDF2 method represents a significant advancement for EMT simulation. Its natural numerical stability makes the need for artificial damping or roll-back and interpolation unnecessary, thus leading to higher performance for equal integration time step. Especially for modern power systems dominated by power electronics and fast control systems, with IBR penetration rates approaching 100%, the need of performant numerical methods that are inherently stable for stiff systems is evident. Implementing the BDF2 into existing EMT simulation tools could be possible alongside the well-established trapezoidal method.

The BDF2 method comes however not only with benefits. The natural filtering of high-frequency oscillations could affect the results accuracy in presence of real high-frequency phenomena, if the simulation time step is not carefully chosen. A feasible countermeasure is the implementation of the generalized BDF2 with adaptive variable time step, which can optimize accuracy and performance at the same time.

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