Impact of Security Constraints on Multi-Period AC-DC Transmission Expansion Planning

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***Abstract*—The rapid growth of renewable generation and emerging hybrid AC/DC transmission technologies is reshaping long-term network development. Ensuring that investment decisions remain robust under N-1 contingencies is increasingly critical as operational variability and congestion intensify. This paper presents a multi-period hybrid AC/DC security-constrained transmission expansion planning (SCTNEP) framework that embeds post-contingency feasibility directly into the investment stage using a convex AC network representation. The model captures both inter-temporal reinforcement sequencing and intra-day operational variability, enabling a consistent comparison between secure and non-secure planning configurations at the investment layer. A comparative study on a modified IEEE 6-bus hybrid system shows that enforcing security constraints materially shifts reinforcement timing, technology choices, and long-term cost trajectories. Results demonstrate that secure planning can mitigate future operational risks with only modest increases in capital expenditure, providing a practical and physically grounded pathway for hybrid AC/DC transmission development under growing uncertainty.**

***Index Terms*—Transmission expansion planning, security constraints, HVDC.**

Nomenclature

$\mathcal{T}$	Set of planning periods, indexed by t
$\mathcal{H}$	Set of intra-day operating hours, indexed by h
$\mathcal{S}$	Set of contingency scenarios, indexed by s
$\mathcal{N}$	Set of buses, indexed by i, j
$\mathcal{G}$	Set of generators
$\mathcal{L}^{\mathrm{AC}}$	Set of existing AC transmission branches
$\mathcal{L}^{\mathrm{DC}}$	Set of candidate DC transmission corridors
$y_{k,t}$	Binary investment decision indicating whether candidate corridor k is commissioned by the end t
$P^{\mathrm{G}}_{i,t,h,s}$	Active power generation at bus i in period t, hour h, under contingency s
$Q^{\mathrm{G}}_{i,t,h,s}$	Reactive power generation at bus i
$P^{\mathrm{ls}}_{i,t,h,s}$	Load shedding at bus i
$P^{\mathrm{dc}}_{k,t,h,s}$	Power flow on DC corridor k
$W_{i,t,h,s}$	Squared voltage magnitude at bus i
$W_{ij,t,h,s}$	Voltage cross-product between buses i and j
C^{inv}_k	Investment cost of candidate corridor k
α_t	Discount factor for planning period t
π_s	Weighting factor of contingency scenario s
c_i	Generation cost coefficient at bus i
c^{ls}	Penalty cost for load shedding
η_k	Converter efficiency associated with DC corridor k
S_{ij}	Thermal capacity limit of branch (i, j)

This work is supported by the Australian Research in Power Systems Transition (AR-PST) project number (OD-223474) – This project is led by the Commonwealth Scientific and Industrial Research Organisation (CSIRO).

I. Introduction

The accelerating decarbonization of the energy sector and the rapid proliferation of variable renewable generation are transforming the physical and operational landscape of modern power systems [1]. Large-scale integration of wind and photovoltaic resources has introduced stronger spatial and temporal coupling between generation and load, placing unprecedented stress on existing transmission corridors. At the same time, the electrification of transport and heating creates new demand peaks and cross-sectoral interactions. These trends collectively require transmission systems that are not only economically efficient but also operationally robust against contingencies and forecast errors [2]. Ensuring that network investment decisions remain both cost-effective and secure under multiple futures is thus an emerging priority for planners and regulators.

Traditional TNEP studies were largely conducted under simplified network representations. Garver first introduced a linear programming framework to estimate minimum-cost network expansion under steady-state assumptions, providing a foundation for later mathematical formulations [3]. Latorre *et al*. reviewed these models and classified them by time horizon, uncertainty representation, and solution approach [4]. Then, Alguacil *et al*. [5] developed a mixed-integer linear program that coupled investment and generation dispatch within a single-period DC model. Further, [6] extended this to a market-based framework incorporating congestion pricing. Although computationally tractable, these formulations neglected reactive power effects and could not guarantee feasible operation following line or generator outages. As shown by [7], omitting explicit security enforcement can lead to sub-optimal investment outcomes: their case study on a regional system (i.e., Gansu) demonstrated that incorporating N–1 security reduced total cost by over 5% while avoiding infeasible post-contingency operation.

Recognizing the operational importance of security, subsequent research incorporated contingency analysis directly into planning [8]. [9] formulated an N–1 secure expansion problem and illustrated how enforcing contingency constraints alters

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both the optimal topology and reinforcement timing. Amraee *et al*. introduced a Benders-decomposed structure to address the combinatorial growth of contingency subproblems, achieving scalable computation for large networks [10]. Despite these advances, most security-constrained models remained static, that is optimizing a single representative snapshot of demand and generation, and thus could not represent inter-temporal investment coupling or progressive reinforcement decisions over long horizons.

The growing scale and diversity of transmission technologies have further broadened the planning challenge. Domínguez *et al*. proposed a multistage HVAC/HVDC co-optimization framework under security constraints, showing that hybrid topologies improve renewable integration [11]. Moreover, [12], [13] respectively explored AC/DC conversion options for renewable-rich systems and confirmed the techno-economic advantage of hybrid corridors, yet their formulation was static and did not address multi-period decision sequencing. [14] extended AC/DC planning to multiple periods and incorporated shunt compensation, but the treatment of N–1 feasibility remained external to the planning layer. Likewise, conic relaxations have been applied to approximate AC power flow for expansion problems [15], providing numerical tractability but without evaluating how security embedding affects long-term investment. Existing studies therefore reveal an incomplete integration of three key elements including multi-period coupling, hybrid AC/DC architecture, and embedded N–1 security. Most frameworks optimize over a single stage or rely on simplified DC models, limiting their ability to assess how secure planning influences cumulative capital expenditure and operational cost across successive horizons.

Addressing this methodological gap is crucial for developing transmission systems that are economically efficient yet operationally resilient. To this end, this paper presents a multi-period hybrid AC/DC security-constrained transmission expansion planning (SCTNEP) framework that jointly determines investment and operational decisions under base-case and contingency conditions. The formulation explicitly links long-term reinforcement choices with post-contingency feasibility through a convex AC representation, ensuring consistency between planning and secure operation. A comparative analysis between secure and non-secure planning configurations is carried out to quantify the economic and reliability impacts of embedding security constraints. The results highlight how proactive consideration of security in the planning stage can reduce future operational risks and enhance the overall efficiency of hybrid AC/DC transmission development.

The key contributions of this paper can be summarized as three aspects:

- We develop a general multi-period hybrid AC/DC SCTNEP model that embeds N–1 post-contingency feasibility directly within the investment layer.
- A decomposition-based algorithm is designed to exploit the convex structure of operational subproblems, enabling parallel evaluation and fast convergence while preserving the coupling between investment and security constraints.
- We conduct a comparative study on a benchmark system and demonstrate how embedded N-1 constraints influence the timing and placement of AC/DC reinforcements.

II. Modeling Framework and Formulation

A. Formulation Overview

The proposed study formulates transmission expansion planning as a unified optimization that integrates investment, operation, and security assessment within one decision space. The formulation extends conventional TNEP by embedding both hybrid AC/DC technologies and N-1 operational security constraints directly into the planning layer.

System operation under each prospective network is represented by an AC power flow model that retains voltage and reactive-power characteristics through a convex second order cone programming (SOCP) relaxation. Feasibility under contingencies is modeled via a security-constrained operational layer (SCOPF-type constraints) within the planning problem, ensuring that post-outage operation satisfies thermal and voltage limits without excessive load curtailment. Each candidate reinforcement is therefore evaluated not only by its investment cost but also by its contribution to maintaining feasible and secure operation across representative system states. The optimization further considers both inter- and intra-temporal couplings, allowing investment decisions to accumulate across stages while accounting for hourly operational variations within each stage. This structure captures how early reinforcements alleviate future congestion and ensures that secure operation is maintained over time.

B. Mathematical Formulation

Let $\mathcal{T}$ denote discrete planning periods (e.g., years), $\mathcal{H}$ the set of intra-day hours, $\mathcal{N}$ the bus set, and $\mathcal{L}^{\mathrm{Cd}}$ the set of candidate branches. Each candidate k has an associated binary decision $y_{k,t} \in \{0,1\}$, which indicates whether the facility is commissioned by the end of stage t. For every period t, hour $h \in \mathcal{H}$, and contingency $s \in \mathcal{S}$, continuous variables describe generation dispatch $P^{\mathrm{G}}_{i,t,h,s}$ and $Q^{\mathrm{G}}_{i,t,h,s}$, as well as load shedding $P^{\mathrm{ls}}_{i,t,h,s}$. The decision process must ensure feasibility of every post-contingency operating point given the available infrastructure y_t.

1) Objective Function: The optimization minimizes the present value of investment and operation costs across all periods, hours, and contingencies:

$$\min_{y,x} \sum_{t\in\mathcal{T}} \Bigg[\alpha_t \sum_{k\in\mathcal{L}^{\mathrm{Cd}}} C^{\mathrm{inv}}_k (y_{k,t} - y_{k,t-1}) + \sum_{s\in\mathcal{S}} \pi_s \sum_{h\in\mathcal{H}} \left(C^{\mathrm{G}}_{t,h,s} + C^{\mathrm{ls}}_{t,h,s} \right) \Bigg] \tag{1}$$

Here, C^{inv}_k denotes the investment cost for corridor k, α_t is the discount factor, and π_s is the relative weighting of contingency s. The operational term includes generation cost and load shedding penalty, expressed as

$$C^{\mathrm{G}}_{t,h,s} = \sum_{i\in\mathcal{G}} c_i P^{\mathrm{G}}_{i,t,h,s}, \quad C^{\mathrm{ls}}_{t,h,s} = \sum_{i\in\mathcal{N}} c^{\mathrm{ls}} P^{\mathrm{ls}}_{i,t,h,s}. \tag{2}$$

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2) SOCP Representation of AC Power Flow: Operational subproblems are represented by convex SOCP relaxations of the AC network equations.

Let $W_{i,t,h,s} = |V_{i,t,h,s}|^2$ denote the squared voltage magnitude at bus i and $W_{ij,t,h,s} = V_{i,t,h,s}V^*_{j,t,h,s}$ the cross-term between buses i and j. The active- and reactive-power balance equations for each bus i in period t, hour h, and contingency s are written as:

$$P^{\mathrm{G}}_{i,t,h,s} - P^{\mathrm{L}}_{i,t,h} + P^{\mathrm{ls}}_{i,t,h,s} = \sum_{j:(i,j)\in\mathcal{L}^{\mathrm{AC}}} \Re\{Y^*_{ij}\big(W_{i,t,h,s} - W_{ij,t,h,s}\big)\},$$

$$Q^{\mathrm{G}}_{i,t,h,s} - Q^{\mathrm{L}}_{i,t,h} = \sum_{j:(i,j)\in\mathcal{L}^{\mathrm{AC}}} \Im\{Y^*_{ij}\big(W_{i,t,h,s} - W_{ij,t,h,s}\big)\}. \quad (3)$$

The voltage magnitude and thermal limits of each branch (i, j) are then imposed as:

$$\underline{V}_i^2 \le W_{i,t,h,s} \le \overline{V}_i^2, \quad (4)$$

$$|S_{ij,t,h,s}| \le \overline{S}_{ij}\, y_{ij,t}, \quad (5)$$

and the convex envelope of the power-flow relationship is enforced through

$$|W_{ij,t,h,s}|^2 \le W_{i,t,h,s}W_{j,t,h,s}, \quad (i,j) \in \mathcal{L}^{\mathrm{AC}}. \quad (6)$$

Equations (3)–(6) maintains the voltage-magnitude and reactive-power effects that are often neglected in DC-based TEP models, while avoiding explicit angle variables and ensuring numerical stability for large networks. The resulting convex substructure significantly improves the scalability of the overall formulation.

3) Modeling of DC Corridors: For each candidate HVDC corridor $k \in \mathcal{L}^{\mathrm{DC}}$ connecting converter buses m and n, controllable power transfer is represented as:

$$-\overline{P}^{\mathrm{dc}}_k y_{k,t} \le P^{\mathrm{dc}}_{k,t,h,s} \le \overline{P}^{\mathrm{dc}}_k y_{k,t}, \quad (7)$$

with converter efficiency η_k and active-power coupling at terminals:

$$P^{\mathrm{G}}_{m,t,h,s} - P^{\mathrm{L}}_{m,t,h} + P^{\mathrm{ls}}_{m,t,h,s} = \sum_{j:(m,j)\in\mathcal{L}^{\mathrm{AC}}} P_{mj,t,h,s} + \eta_k P^{\mathrm{dc}}_{k,t,h,s},$$

$$P^{\mathrm{G}}_{n,t,h,s} - P^{\mathrm{L}}_{n,t,h} + P^{\mathrm{ls}}_{n,t,h,s} = \sum_{j:(n,j)\in\mathcal{L}^{\mathrm{AC}}} P_{nj,t,h,s} - P^{\mathrm{dc}}_{k,t,h,s}. \quad (8)$$

Here η_k denotes converter efficiency. a fixed value of $\eta_k = 0.95$ is assumed for all candidate HVDC corridors. These constraints allow AC and DC reinforcements to be optimized together, letting the model endogenously determine the most cost-effective hybrid topology.

4) Security and Contingency Constraints: The planning model enforces $N-1$ security by incorporating contingency-specific subproblems $s \in \mathcal{S}$. When an infrastructure k is outaged in contingency s, its capacity is set to zero:

$$y_{k,t,s} = 0, \quad \forall s \text{ corresponding to outage of } k. \quad (9)$$

All network equations (3)–(8) are re-evaluated under the reduced topology. Load-shedding variables $P^{\mathrm{ls}}_{i,t,h,s} \ge 0$ maintain feasibility under severe conditions and are penalized in the objective.

5) Inter-Temporal Coupling: The temporal aspect of the model links investment decisions across planning stages. Each asset, once built, remains available in all future stages, expressed as

$$y_{k,t} \le y_{k,t+1}, \quad \forall k, t, \quad (10)$$

a cumulative budget constraint can also be imposed to limit total capital budget:

$$\sum_{t,k} C^{\mathrm{inv}}_k (y_{k,t} - y_{k,t-1}) \le \overline{B}^{\mathrm{tot}}. \quad (11)$$

These equations ensure that the multi-period plan reflects both construction sequencing and investment constraints over time.

III. Solution Approach

To ensure tractable and scalable computation, the proposed multi-period hybrid AC/DC SCTNEP is reformulated into a two-level decomposition that separates long-term investment decisions from short-term operational feasibility assessments. The decomposition preserves the physical coupling between these layers through dual information exchange, ensuring that investment updates remain fully informed by both inter-annual and intra-day security constraints.

Combining all relationships established in Section II, the overall problem can be compactly written as:

$$\min_{y,x} \sum_{t\in\mathcal{T}} \left(C^{\mathrm{inv}}_t(y_t) + \sum_{s\in\mathcal{S}} \pi_s \sum_{h\in\mathcal{H}} C^{\mathrm{op}}_{t,h,s}(x_{t,h,s}) \right) \quad (12)$$

subject to

$$x_{t,h,s} \in \mathcal{X}^{\mathrm{SOCP}}(y_t), \quad y_{k,t} \le y_{k,t+1}, \quad y_{k,t} \in \{0,1\}, \quad (13)$$

where $\mathcal{X}^{\mathrm{SOCP}}(y_t)$ denotes the convex operational region defined by (3)–(9) for each (t, h, s) combination. This compact formulation jointly evaluates investment and operational

Algorithm 1: Decomposition-Based Solver for Multi-Period Hybrid AC/DC SCTNEP

Input: Network data, contingency set $\mathcal{S}$, planning horizon $\mathcal{T}$, hourly set $\mathcal{H}$, tolerance ε.
Output: Optimal investment y^*, feasible operation $\{x^*_{t,h,s}\}$, and total cost J^*.

```
Initialize y^0 and cut set C = ∅;
repeat
    Solve master problem: min_{y,θ} Σ_t (C_t^inv(y_t) + θ_t)
      subject to y_t ≤ y_{t+1}, y_t ∈ {0,1}, and cuts C;
    foreach (t,h,s) ∈ (T,H,S) in parallel do
        Solve subproblem min_{x_{t,h,s}} C^op_{t,h,s}(x_{t,h,s}) s.t.
          x_{t,h,s} ∈ X^SOCP(y_t);
        Generate feasibility/optimality cuts from dual
          multipliers and update C;
    end
    Update total cost J and convergence metric ΔJ;
until |ΔJ| ≤ ε;
return y*, x*_{t,h,s}, J*;
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variables across all planning stages, hours, and contingencies while embedding post-contingency feasibility directly into the planning layer.

The resulting problem is a mixed-integer SOCP that couples binary investment variables with continuous SOCP-based operational constraints across multiple planning periods, operating hours, and contingency scenarios. Because the dimensionality increases rapidly with both $|\mathcal{T}|$, $|\mathcal{H}|$, and $|\mathcal{S}|$, direct solution is computationally intractable. To overcome this challenge, a decomposition-based algorithm is adopted: the *master problem* determines candidate investments and approximates the expected operating cost, whereas the *subproblems* solve the SOCP operational models for each (t, h, s) triplet in parallel. Feasibility and optimality cuts derived from subproblem dual multipliers iteratively refine the master approximation until convergence.

Each iteration of Algorithm 1 alternates between solving the investment master problem and validating it through parallel subproblem solutions. The process terminates when all subproblems are feasible and the relative change in total cost, ΔJ, falls below a prescribed tolerance. Because each subproblem is convex and independent, the procedure exhibits excellent parallel scalability on multi-core or high-performance computing environments.

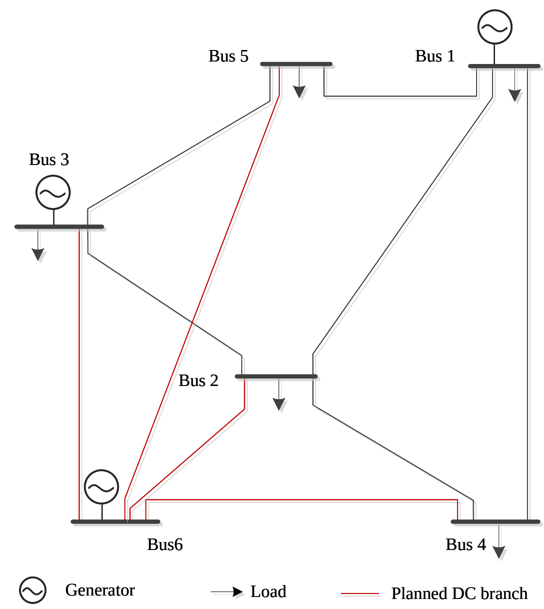

Fig. 2. Cumulative expansion outcome corresponding to the end-of-horizon network topology in Case 2.

IV. Case Study

To evaluate the proposed multi-period hybrid AC/DC SCT-NEP model, numerical experiments are conducted on a modified IEEE 6-bus AC/DC transmission system, where candidate corridors are synthetically generated according to electrical distance and geographical proximity [16]. Each candidate line is characterized by its reactance, thermal limit, and investment cost proportional to line length, the detailed toplogy of the system is shown as Fig. 1. Both AC and DC reinforcement options are included, enabling the optimization to determine hybrid network topologies.

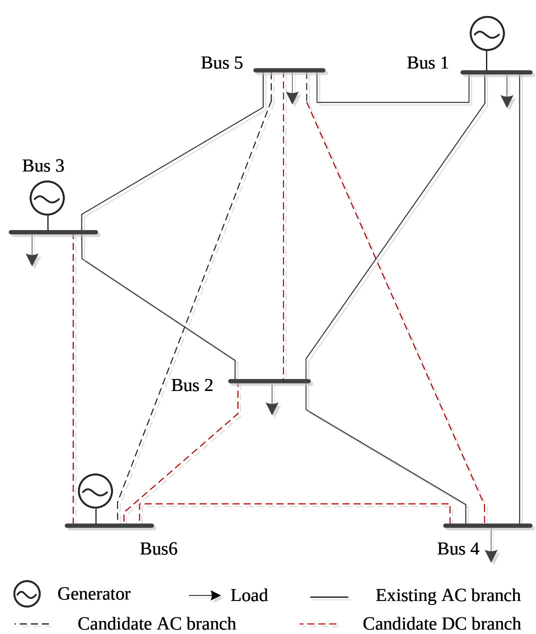

Fig. 1. Topology of the modified IEEE 6-bus AC/DC test system used for planning studies. In addition to the existing AC network, each bus except Bus 1 is modeled as a potential DC node with an associated candidate converter. For each AC/DC bus pair, multiple DC transmission options are included in the candidate set (e.g., 200 MW and 400 MW variants) with different ratings and investment costs.

The planning horizon covers four representative stages from 2025 to 2035, representing a 10-year development pathway. Within each stage, system operation is evaluated over a typical 24-hour period to capture diurnal variations in load and renewable generation. Hourly load profiles are scaled from the IEEE Reliability Test System data and normalized such that peak demand increases gradually by 2 % per year.

Two planning configurations are compared:

- Case 1: Investment decisions are determined without embedding contingency constraints.
- Case 2: The proposed formulation enforces N-1 security for all single-line outages within each planning stage.

All optimization models are implemented in the Julia environment using the JuMP.jl [17] modeling framework together with PowerModelsACDC.jl [18]. The problems are solved with the Gurobi optimizer on a MacBook Pro M3 system with 36 GB RAM.

Fig. 2 shows the cumulative network configuration obtained at the end of the 10-year horizon for Case 2. The selected reinforcements form a compact hybrid structure that strengthens the main transfer path of the system, with DC branches placed where controllable power transfer contributes most effectively to maintaining feasibility under contingency

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TABLE I
PLANNED INVESTMENTS COMPARISON BETWEEN CASES

Year	Case 1 investments	Cost[1]	Case 2 investments	Cost
2026	Converter2, Converter4, Converter5, Converter6, DC-Branch8,[2] DC-Branch11	23.70	Converter2, Converter4, Converter5, Converter6, DC-Branch10, DC-Branch3, DC-Branch8	26.10
2028	—[3]	0.00	Converter3, DC-Branch4	4.54
2029	DC-Branch2	2.01	—	0.00
2032	Converter3, DC-Branch4	3.60	—	0.00
Total		**29.31**		**30.64**

[1] All investment costs are expressed in normalized monetary units and are used for comparative analysis between planning cases.
[2] Indices DC branches correspond to their ordering in the candidate dataset.
[3] "—" denotes no new investments in that year.

conditions. The resulting topology indicates that embedding N-1 constraints at the planning stage naturally guides the solution toward a consolidated AC/DC backbone rather than dispersed upgrades. Table I summarizes the multi-period investment schedule and the corresponding annual investment costs based on the specified escalation factors. In Case 1, upgrades are delayed until they become strictly necessary for meeting the growing load. After the substantial build in 2026, no further additions occur until 2029 and 2032, leading to a relatively back-loaded investment pattern. In contrast, Case 2 exhibits an earlier concentration of capital. Although both cases share similar initial actions in 2026, Case 2 introduces an additional converter–DC pair in 2028 to ensure feasibility under single-line outages across the planning stages. No further reinforcements are required afterwards, suggesting that this mid-horizon upgrade effectively removes emerging security bottlenecks that would otherwise necessitate later corrective expansions.

Using the applied escalation factors, the total investment cost over the horizon is 29.31 in Case 1 and 30.64 in Case 2. The difference is modest relative to the overall scale of investment and reflects the additional cost associated with maintaining N-1 feasibility in the planning formulation.

V. CONCLUSION

This paper presented a multi-period hybrid AC/DC transmission expansion planning framework that incorporates N-1 feasibility directly into the investment formulation. The model captures the interaction between AC/DC reinforcement options and the evolving demand across planning stages while remaining computationally tractable.

Results on a modified IEEE 6-bus system show that enforcing N-1 security influences both the schedule and placement of grid upgrades. Compared with the non-secure case, the secure plan introduces an earlier, targeted reinforcement that reduces contingency-driven limitations later in the horizon and produces a more coherent hybrid topology. The associated increase in investment cost is modest, indicating that incorporating security criteria at the planning stage can be achieved with limited additional expenditure.

The framework provides a practical basis for secure hybrid transmission development and can be extended to large-scale systems, such as synthetic models of the Australian NEM, as well as to richer candidate sets and uncertainty-aware planning settings involving renewable variability and long-term demand uncertainty.

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