An Ultra-fast Analytical Approach for Voltage Drop Estimation in Low-Voltage Feeders

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***Abstract*—Low-voltage (LV) networks are the largest electrical network, which are often the least observed network due to the size and the cost of installing monitoring devices. Thus, there are significant data gap to make detailed power-flow analysis at scale. This paper presents a sampled analytical algorithm for ultra-fast power flow analyses, that would estimate voltage drop along LV feeders at fraction of time. The method derives voltage–power relationship, made the broad assumption of load uniformly distributed along the feeders. The approach is based on estimated LV network profiles, LV substation voltage magnitudes, and group power factors, enabling ultra-fast voltage estimation under limited monitoring conditions. Validation on the IEEE European LV Test Feeder demonstrates that the proposed method produced reasonable overall voltage patterns, particularly under balanced loading and moderate feeder lengths, when compared with OpenDSS simulations. Its applicability and limitations are also discussed. The proposed approach offers a practical and computationally efficient solution for feeder-level voltage assessment under limited observability.**

***Index Terms*—Low-voltage networks, voltage estimation, analytical model, observability.**

I. Introduction

In the UK, the operational capacity of LV distribution networks has traditionally been defined by thermal, voltage and fault level constraints [1]. With the rapid growth of low carbon technologies such as electric vehicles (EVs), heat pumps (HPs), and rooftop photovoltaics (PV), distribution system operators (DSOs) are placing increasing attention on voltage limit violations [2]. These technologies introduce highly variable and unbalanced power flows, causing significant voltage fluctuations and potential phase asymmetry along feeders. Consequently, the ability to accurately estimate voltage profiles at scale has become essential for assessing network performance, identifying possible constraint violations, and supporting future LV reinforcement and control strategies. However, due to the limited observability of LV networks, accurately estimating their voltage states remains a significant challenge in the UK and internationally.

Several methods have been proposed for voltage estimation in UK LV networks. For example, deep-learning approaches have been developed to predict feeder voltage distributions under partial observability [3]. Data-driven capacity and loss estimation methods further reveal the challenges caused by limited smart-meter coverage and sparse network monitoring [4]. In addition, weighted least squares (WLS)-based state estimation techniques have also been applied to LV networks with significant penetration of LCTs [5].

However, these existing methods generally require extensive measurements or data availability. Due to the historical "fit-and-forget" design principle [1], UK LV feeders were originally built for unidirectional power flow with limited network monitoring; only transformer-level voltage and current measurements are sometimes available [6] [7]. Although smart meters are the most widely deployed monitoring devices in the UK, their data are primarily held by individual energy suppliers and are not fully accessible to DSOs for feeder-level analysis [8]. In addition, LV networks typically involve a large number of customers and long overall feeder distances, making it difficult to perform voltage estimation using computationally intensive methods to assess possible voltage violations. Moreover, data-driven algorithms often lack physical interpretability, preventing DSO engineers from understanding the underlying mechanisms and thus limiting their direct applicability in practical operations.

Despite the lack of direct monitoring, LV networks can still be modeled through appropriate assumptions and power-flow formulations. Previous studies, such as Baran and Wu's analytical formulation of distribution flow equations [9], have demonstrated that voltage variations along feeders can be effectively represented using ultra-fast analytical models. Building upon this foundation, voltage estimation can be achieved by exploiting transformer-level measurements and feeder parameters, enabling rapid and scalable computation suitable for large-scale LV network analysis.

This paper proposes an ultra-fast analytical approach for voltage drop estimation in LV feeders. The voltage-drop formulation is derived from the simplified power-flow equations commonly used in distribution systems. In this paper, we made further simplification to assume load is uniformly distributed along the feeders to enable analytical solutions. The method is specifically designed for LV networks with very limited network nor customer visibility. Owing to its analytical nature and low computational complexity, the proposed model enables ultra-fast voltage estimation suitable for large-scale LV network analysis. Rather than competing with detailed power-flow or data-driven voltage estimation methods, the proposed approach targets an unresolved engineering problem under limited observability. It provides a physically interpretable and computationally efficient framework that enables DSOs

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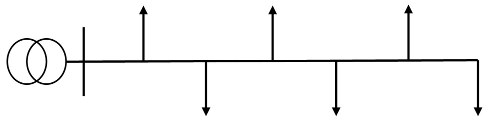

Fig. 1. Diagram of LV feeder layout originating from the distribution substation.

to identify feeders and operating conditions for subsequent detailed analysis. The approach is validated using the IEEE European LV Test Feeder, where its results are compared with those from conventional power-flow calculations, with further discussion on its applicability and limitations.

This paper is organized as follows. Section II introduces the proposed ultra-fast analytical formulation for voltage estimation in LV networks. Section III presents a case study based on the IEEE European LV Test Feeder, compares the results with OpenDSS simulations and discusses the limitations of the proposed approach. Finally, Section IV concludes the paper and outlines directions for future work.

II. Ultra-fast Analytical Model for LV Voltage Estimation

A. Modeling Load Distribution along LV Feeders

In many local-planning frameworks in the UK, residential or community developments are guided by target housing-density ranges, such as 20 to 40 dwellings per hectare in Nottingham [10]. Given these normative density objectives and the relatively regular spatial dispersion of households, particularly in suburban and rural areas, the aggregated load along the feeder can be approximated as continuously distributed. Although this assumption involves a practical approximation, it provides a reasonable balance between computational efficiency and data requirements. It enables analytical voltage estimation when monitoring is limited, while comparison with full power-flow simulations indicates the model's fidelity and informs DSOs of the associated uncertainty.

In typical UK LV networks, such spatially dispersed residential units are connected to the distribution system through radial feeders extending from local transformers, as illustrated in Fig. 1. Although individual user load profiles can be measured, they are rarely collected due to the high cost and scale of monitoring. For analytical purposes, the feeder load is therefore assumed to be continuously and uniformly distributed along its physical length L_0.

This assumption of uniformly distributed load does not imply homogeneous customer demand, but represents a spatially averaged demand density along the feeder. This assumption is primarily intended for planning-oriented analysis and large-scale voltage screening, where detailed customer-level information is unavailable, rather than for capturing detailed temporal or spatial load variations.

Assuming a linear decrease of power flow from the substation to the remote end, the apparent power transmitted along the feeder can be expressed as:

$$s(l) = s_0 - \frac{s_0}{L_0} l, \tag{1}$$

where s_0 represents the total apparent power injected at the transformer, and L_0 is the total feeder length. Under this assumption, the apparent power flow gradually decreases with distance, reflecting the aggregated contribution of multiple downstream loads that are evenly distributed along the feeder.

This assumption of uniformly distributed load does not imply uniform customer demand, but rather represents a spatially averaged demand density along the feeder. It is primarily intended for planning-oriented analysis and large-scale voltage estimation, where customer-level information is unavailable, rather than for capturing detailed temporal or spatial load variations.

B. Formulation of Feeder Voltage Drop

The voltage drop, $d(\Delta V)$, along a small feeder segment of length dl, can be approximated from the basic power–flow relationship as [11]:

$$d(\Delta V) = P \cdot R\, dl + Q \cdot X\, dl, \tag{2}$$

where P and Q are the active and reactive power flows through the segment, and R and X are the line resistance and reactance per unit length. By expressing the power components in terms of the apparent power S and power factor angle φ, the incremental voltage drop becomes:

$$d(\Delta V) = (R\cos\varphi + X\sin\varphi) S\, dl. \tag{3}$$

Since the apparent power flow gradually decreases along the feeder, substituting the linear load-flow function $s(l)$ gives:

$$d(\Delta V) = (R\cos\varphi + X\sin\varphi)\left(s_0 - \frac{s_0}{L_0} l\right) dl. \tag{4}$$

Integrating along the entire feeder length from 0 to L_0 yields the total end-to-end voltage drop:

$$\Delta V = (R\cos\varphi + X\sin\varphi)\int_0^{L_0}\left(s_0 - \frac{s_0}{L_0} l\right) dl. \tag{5}$$

By evaluating the integral, the closed-form solution of the feeder voltage drop in physical units can be expressed as:

$$\Delta V = \frac{(R'\cos\varphi + X'\sin\varphi)}{V_0}\left(SL_0 - \tfrac{1}{2}\alpha L_0^2\right) \tag{6}$$

where V_0 denotes the sending-end voltage, S is the apparent power at the sending end, and $\alpha = S/L_0$ represents the linear load-flow gradient along the feeder. The term $R'\cos\varphi + X'\sin\varphi$ defines an equivalent impedance coefficient that combines the resistive and reactive effects under a given power-factor angle φ.

For a general non-uniform load distribution $s(l)$, the voltage profile along the feeder can be written as:

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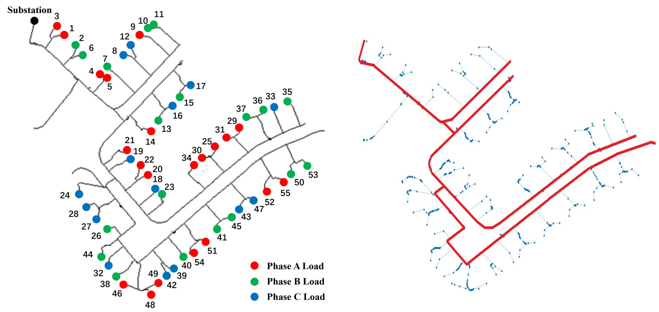

(a) (b)

Fig. 2. IEEE European Low Voltage Test Feeder and connections used for the ultra-fast Analytical Approach.

$$V(l) = V_0 - \frac{(R' \cos\varphi + X' \sin\varphi)}{V_0} \int_0^l s(\xi)\, d\xi. \quad (7)$$

This analytical formulation provides a physically interpretable relationship between feeder loading and voltage profile, relying solely on measurable substation quantities.

III. Case Study and Comparative Analysis

A. Test System Description

In this case study, the proposed modeling approach is tested via the IEEE European LV Test Feeder, shown in Fig. 2 (a), which represents a typical 0.4 kV radial distribution network. For validation, the feeder was simulated in OpenDSS, and voltage and power flow results were exported for comparison with the analytical calculations implemented in MATLAB. The load profiles used in the OpenDSS and MATLAB simulations are taken directly from the IEEE European LV Test Feeder dataset. The main objective is to assess how accurately the proposed model can reproduce the voltage profile along each feeder phase using only substation-level measurements and basic network parameters.

B. Implementation of the Ultra-fast Analytical Model

In this study, the ultra-fast analytical flow model focuses on calculating voltage variations along the main LV feeder. Only the principal line section connecting the distribution transformer to downstream nodes is considered, while secondary service cables are neglected. Each user's voltage is therefore assumed to be equal to the voltage at the corresponding connection point on the main feeder.

The known measurable parameters include the transformer's voltage magnitude, power factor, and active and reactive power at the secondary side. The distance and impedance values from each service connection point to the nearest node on the main feeder are taken directly from the IEEE European LV Test Feeder dataset. The analysed section in the network diagram, as shown in Fig. 2 (b), corresponds to the red-highlighted feeder, where the connected loads are uniformly distributed along the line. The total length of this feeder section is 516.87 m.

C. Results Comparison

1) A Snapshot of Estimation Results: Fig. 3 presents the spatial configuration of the LV network, where each colored line denotes a primary feeder segment under a given operating condition. The substation voltage is set to 230 V, with a power factor of 0.95 and an apparent power of 30 kVA.

Using the proposed ultra-fast analytical model, the voltage variation $V(l)$ is estimated along the main feeders, which are decomposed into multiple segments for clarity. Each color corresponds to a specific feeder section, and the embedded subplots illustrate the voltage drop from the upstream to the downstream end of each segment as a function of feeder length.

In Fig. 3, the blue, orange, and yellow segments exhibit nearly linear voltage declines, as these branches mainly transfer power toward downstream loads. In contrast, the remaining segments display nonlinear voltage behavior, reflecting concentrated load conditions. The purple segment shows slight irregularities in its voltage curve due to the presence of two different cable types along that section, which introduce variations in impedance.

2) Voltage Profile Comparison: Fig. 4 compares the voltage profiles obtained from OpenDSS simulations (blue solid lines) and the ultra-fast analytical model (red dashed lines) for three representative users: Load 1 (Fig. 4(a)), Load 32 (Fig. 4(b)), and Load 53 (Fig. 4(c)). Each load is located at a different distance from the secondary substation, representing typical operating positions along the feeder. These three nodes are selected to maintain consistency with the IEEE European LV Test Feeder documentation [12], where the same points are used to demonstrate the results difference between GridLAB-D and OpenDSS.

As shown, Load 1, located near the transformer, exhibits the smallest voltage fluctuation due to its short cable length and low impedance. Load 32, positioned at the mid-feeder, shows stronger voltage variations, reflecting the impact of unbalanced and diverse phase loading. Load 53, located at the far end, experiences a slightly larger voltage drop but maintains a consistent overall trend with the benchmark.

3) Error Distribution Analysis: Fig. 5 illustrates the time-varying apparent power S (blue solid lines) and the corresponding average voltage error $|\Delta V|_{\text{avg}}$ (red dashed lines) for Phases A, B, and C, respectively. The results show a clear correlation between the magnitude of feeder loading and the deviation between the ultra-fast analytical model and OpenDSS simulation. Periods of high load demand, particularly between 400 and 1000 minutes, tend to coincide with larger estimation errors, indicating that the analytical approximation becomes less accurate when the feeder operates under heavier and more unbalanced conditions. The maximum instantaneous deviation was found to be 8.56 V at Load 35 around 568 minutes, occurring during peak feeder loading.

Among the three phases, Phase B exhibits the largest error fluctuations, with peak deviations reaching above 4 V. This phase is also characterized by the highest apparent power and strongest temporal variability, consistent with the observed

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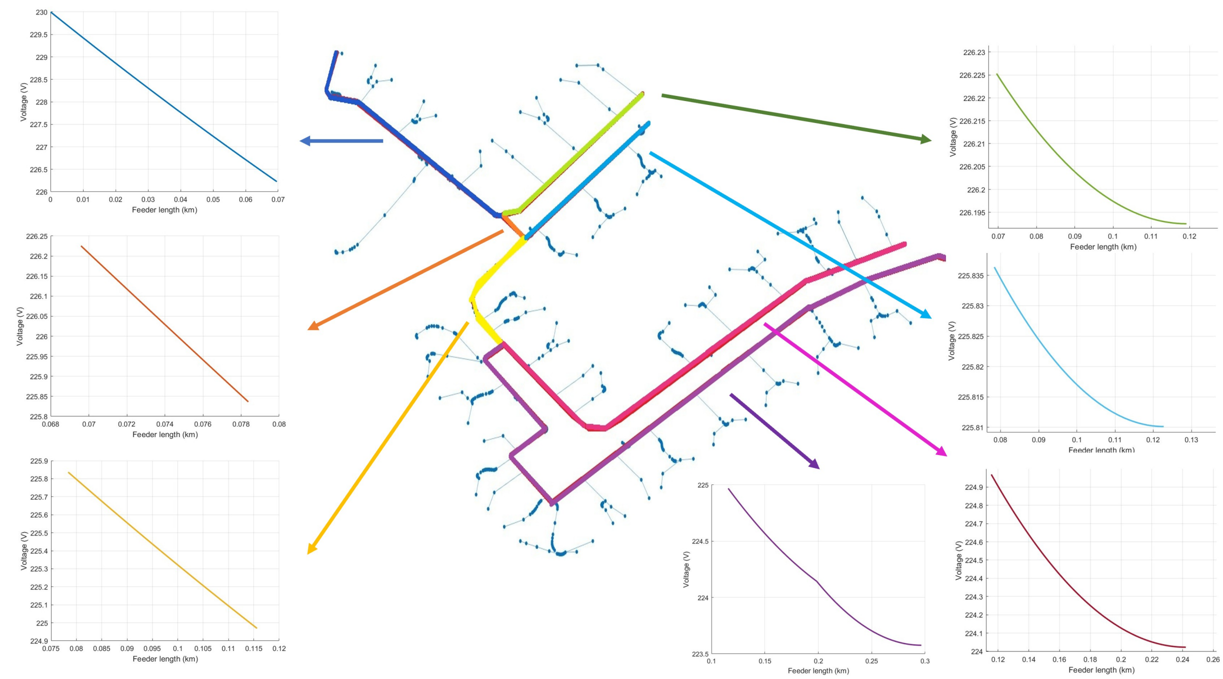

Fig. 3. Voltage profiles along different main feeder sections in the IEEE European LV Test Feeder.

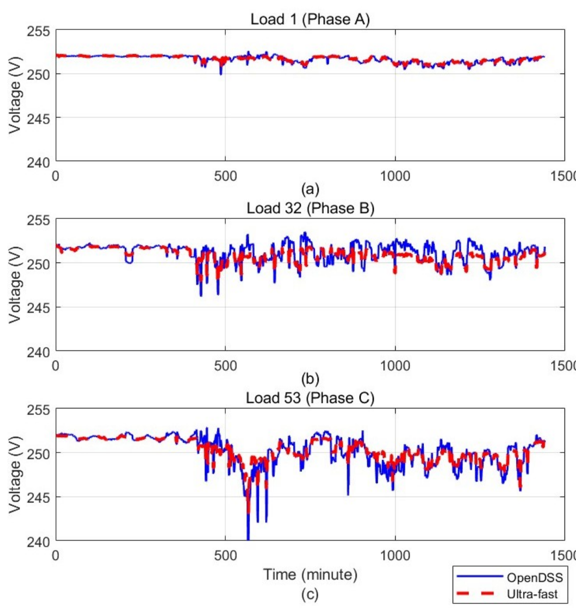

Fig. 4. Voltage comparison between OpenDSS simulation and ultra-fast analytical model.

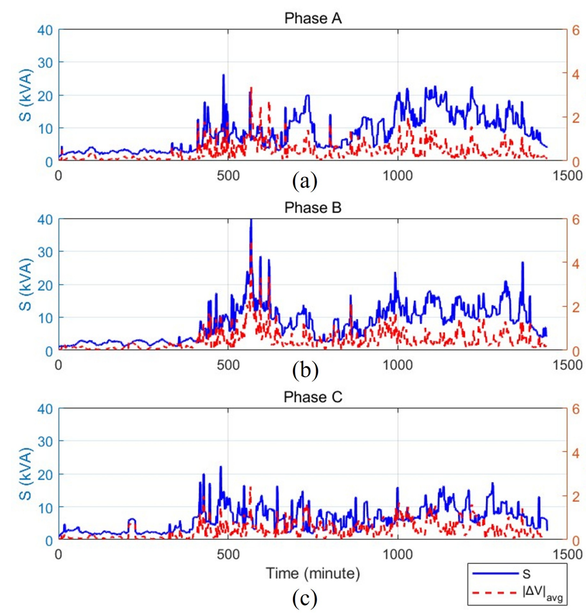

Fig. 5. Three-phase voltage estimation error and apparent power comparison.

voltage mismatch in Fig. 4 Phases A and C show smaller and more stable errors, typically remaining below 2 V. These observations suggest that the ultra-fast model captures the steady-state voltage behaviour well under moderate loading but tends to over-simplify the effects of phase imbalance and localized voltage drops during peak demand.

Fig. 5 further visualizes the error magnitude distribution across all loads and time periods. Most of the voltage deviations are concentrated below 1 V (shown in blue), while the red regions ($\geq$ 2 V) indicate larger discrepancies. These high-error areas are mainly observed toward the end of the feeder, such as near Load 35, where cumulative line impedance and longer service distances amplify the voltage estimation error. The spatial pattern indicates that the modeling error is not uniformly distributed—it increases with feeder length and load variability.

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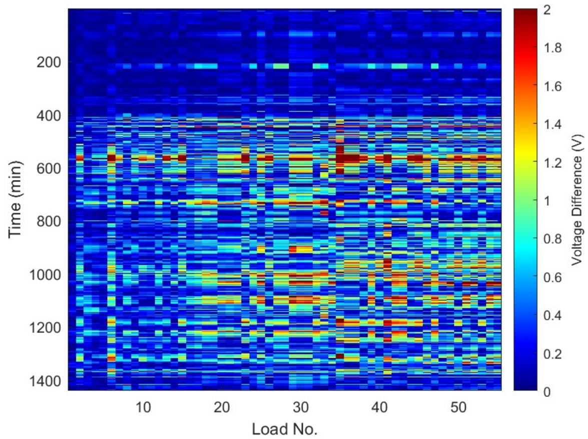

Fig. 6. Error magnitude distribution across all loads and time periods.

D. Discussion on Model Applicability and Limitations

The results presented in Figs. 3–5 demonstrate that the proposed ultra-fast analytical model can effectively reproduce the general voltage behaviour along LV feeders while significantly reducing computational effort compared to full power-flow simulations.

Its analytical closed-form formulation enables fast evaluation of voltage drops and feeder constraints using only a few measurable quantities—namely the substation voltage V_0, apparent power S, and power factor φ.

This simplicity makes the method particularly suitable for applications where detailed network data or extensive monitoring are unavailable, such as distribution planning, large-scale sensitivity assessment, or rapid voltage envelope estimation under limited observability.

However, the comparative analysis also highlights several important limitations. As shown in Fig. 3, while the ultra-fast model aligns well with OpenDSS results under normal loading, deviations appear during periods of high demand or strong phase unbalance, especially in Phase B. Fig. 4 further confirms that the voltage estimation error correlates strongly with the instantaneous apparent power, implying that the model assumes a quasi-linear load–voltage relationship that becomes less accurate under nonlinear or unbalanced conditions. In Fig. 5, larger errors ($\geq$ 2 V) are concentrated near the end-of-feeder region (e.g., around Load 35), where the cumulative impedance and local variations in service-cable characteristics amplify voltage drops not captured by the uniform distribution assumption.

These findings indicate that the model is most applicable for feeders with relatively balanced loading and moderate length, where the voltage variation is dominated by distributed demand rather than by localized peaks.

IV. Conclusion

This paper developed an ultra-fast analytical model to estimate voltage profiles along LV feeders using only transformer-level measurements. The model provides a relationship between power flow and voltage drop, enabling fast evaluation under limited network observability. Comparison with OpenDSS simulations on the IEEE European LV Test Feeder shows good agreement in overall voltage trends, with small deviations mainly occurring at feeder ends and during highly unbalanced loading. The method offers a computationally efficient alternative for feeder-level voltage assessment.

Future work will focus on extending the proposed approach by incorporating non-uniform load distribution and phase-specific correction factors to improve its accuracy under diverse operating conditions. In addition, the influence of different load coincidence factors on voltage estimation accuracy will be further investigated. The model will also be validated on real LV networks to quantify its performance and deviation under practical measurement constraints.

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