Optimal Valley-Filling Utilizing Water-Filling Algorithm

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***Abstract*—To accommodate the growing integration of energy storage systems, such as Battery Energy Storage Systems (BESS), into power distribution grids, efficient control coordination algorithms are essential. We propose an innovative distributed control scheme designed for valley-filling through optimized charging in BESS. To minimize the communication bandwidth required, local BESS controllers compute the optimal charging power, followed by a simplified coordination command broadcast via a central controller. The water-filling concept in convex optimization is employed to develop the theoretical framework for the proposed coordination algorithm. A real-time simulation involving a 3-BESS system, along with comparative studies involving up to 50 BESS systems, demonstrates the performance and scalability of the proposed approach.**

***Index Terms*—BESS, Optimization, Valley-Filling, Optimal Charging, Primal-Dual Dynamics**

I. Introduction

Recent advancements in battery energy storage systems (BESS) including electrical vehicles (EVs) contribute towards regulating electricity costs. As such, peak shaving and valley filling concepts have been introduced to manage the electricity cost of the facility considering hourly energy price [1]. When the energy price is high, the peak shaving method is introduced, defined as utilizing BESS dispatch cutting off the peak demand in the facility. On the contrary, during off-peak hours the valley filling is introduced, BESS charging without creating a non-coincidental peak (NCP). Facilities with higher electricity demands such as high-rise residential buildings, office buildings and factories with limited shiftable loads impel to install BTM BESSs for peak shaving [1], [2]. Scheduling BESS charging during off-peak hours can help smooth the electric load profile, potentially reducing the need for new power plants [3].

Integrating BESSs and EVs to the smart grid requires a coordinated charging and discharging protocol to ensure grid stability while minimising the customers' charging costs [4]. Charging a large number of BESSs simultaneously during off-peak hours creates NCP in the grid causing thermal generators to operate. In contrast, the allocation of reduced power for charging causes BESSs to be partially charged creating a limited margin for the next peak-shaving event. Several control algorithms have been proposed in the literature to optimize valley filling. Integrating particle swarm optimization and genetic algorithm, a centralized charging strategy for EVs is proposed in [5]. The proposed method assesses EV SOC and charging location while incorporating battery swapping. A decentralized optimal control strategy for EV charging is proposed in [6] to minimize the charging cost considering the temperature effect on Li-ion batteries. The nonlinear optimization uses daily prices, EV data, customer needs, and temperature inputs.

Based on the urgency of charging and allocated charge power, a two-level optimal schedule for EV charging is proposed in [7]. The first echelon of the introduced two-tiered model directs urgent EVs to immediate charging without a schedule, while the second echelon of EVs is subject to scheduled charging. However, there is a risk of grid instability and NCP if the majority of EVs are in the first echelon. Allowing real-time updates to charging profiles based on control signals broadcast by the utility company, a decentralized optimal scheduling algorithm for EV charging is proposed in [3]. Moreover, the proposed algorithm converges to the optimal charging solution irrespective of the uniqueness of the EV characteristics which was necessary for the decentralized algorithm presented in [8]. Considering the discrete charge levels of EVs an optimal scheduling algorithm is proposed in [9] based on binary programming. The method's two levels achieve valley filling and reduce EV switching. Due to its non-convex constraints, a heuristic algorithm is proposed to solve this NP-hard problem. [5] employs heuristic methods such as particle swarm optimization while [6] employs sequential quadratic programming algorithm in non-linear programming.

BESS charging and valley-filling proposals can be centralized or decentralized, using contemporary or heuristic methods. Centralized algorithms yield globally optimal solutions but tend to converge slowly. Conversely, decentralized methods achieve local optima with minimal coordination. Hence, a fast-converging algorithm that can compute a global valley-filling solution is essential. In this paper, we propose a fast-converging algorithm to achieve a globally optimal solution, where the considered BESS systems determine their charging power with minimal communication between local BESS control agents and a centralized control unit. The total valley-filling margin is determined by the utility at each time interval and broadcast to local controllers, which compute individual charging power based on remaining local constraints. Consequently, the proposed method does not create NCP, and grid stability is maintained through controlled power extraction for BESS charging.

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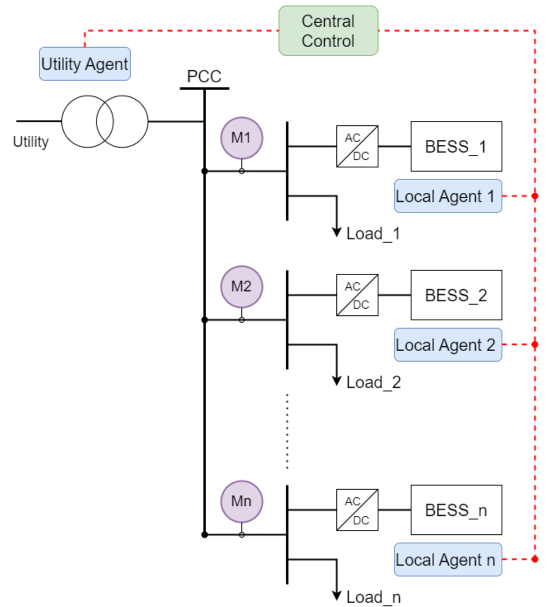

Fig. 1: BTM BESS architecture

The remainder of this paper is organized as follows: Section II introduces the Behind-The-Meter (BTM) BESS architecture considered in this paper. Section III presents the proposed control algorithm, and Section IV evaluates it via practical simulation studies. Concluded in Section V.

II. System Architecture

This section presents the considered BTM BESS architecture. The utility supplies electricity to each facility, while BESS units are installed behind the gross load billing meters (M1…Mn) as shown in Fig.1. Each BESS follows charge or discharge commands from local agents connected to the central coordination unit.

Each BESS is modelled by considering the characteristics of lithium-ion batteries commonly used in the industry [10], [11]. The BESS is connected to the AC side via a DC-AC converter and a series R-L circuit. The converter control is modelled utilizing the PI controller in the DQ frame of reference, as stated in [12]. This way, each BESS ($BESS_i$) is capable of functioning according to the external reference active power ($P_{ref,i}$) and reactive power ($Q_{ref,i}$) commands. Moreover, a PI controller is implemented in the central coordination unit to facilitate valley-filling. Accordingly, the power required for valley-filling is adjusted by allocating excess energy to charge the BESS. A limiter is introduced in the controller to prevent the system from exceeding its maximum charge capacity for the BESS.

III. Proposed Optimization Algorithm

In this section, the optimal valley-filling problem is first formally formulated as a convex optimization problem. Nonlinear programming solvers, such as MATLAB's 'fmincon', which is widely employed for solving the minimum of constrained nonlinear multivariable functions, can be utilized to address this optimization problem [6]. Next, the formal optimization problem is converted into a dual optimization problem that can be solved in a distributed manner. Consequently, all BESS systems in the network iteratively update their local charging power based on a simple signal broadcasted by the central coordination unit. A similar approach is discussed in [13], focusing on demand curtailment within the context of a demand response program. Upon reaching optimality, the aggregate power consumed by all BESS in the network will equal the total valley-filling power that the utility's central coordination unit planned to achieve.

A. Valley-Filling Problem Formulation

Valley-filling allocates excess power in the network for charging BESS during off-peak hours. The amount of power that each participating BESS control agent should consume to achieve this aggregate allocation, while also preserving local BESS constraints such as maximum charging power, is formulated in Eq. 1.

$$\mathcal{P}_C: \quad \min_{P_B(t)} \frac{1}{2} P_B(t)^T A(t) P_B(t) \quad st. \tag{1}$$

$$\sum_{i=1}^{n} P_{b,i}(t) = P_U(t) - \sum_{i=1}^{n} P_{l,i}(t) \tag{2}$$

$$0 \le P_{b,i}(t) \le P_{MaxCh,i}(t) \tag{3}$$

$$\forall i \in 1 \dots n$$

$P_B(t) \in \mathbb{R}^{n\times 1}$ is a vector representing the charging power of each $BESS_i$ at time t, where n is the number of BESS in the participating network. As such $P_B(t) = [P_{b,1}(t), \dots, P_{b,n}(t)]^T$. $P_U(t)$ is the power supplied by the utility at time t whereas $P_{l,i}(t)$ is load at subsystem i. When the utility aims to maintain the power supply $P_U(t)$, the remaining power is allocated to BESS charging, as specified in the equality constraint Eq.2. However, if needed, this equality constraint can be relaxed to an inequality constraint with an offset applied. $P_{MaxCh,i}(t)$ is the maximum power that $BESS_i$ can be charged at time t. This time-varying parameter depends on the BESS's internal conditions, such as internal temperature, state of charge (SOC), and mode of operation. Additionally, the $P_{MaxCh,i}$ becomes 0 when the SOC of $BESS_i$ reach 100% [14].

$A(t) \in \mathbb{R}^{n\times n}$ is a diagonal matrix consisting of penalty factors. Each diagonal element $a_{i,i}(t)$ in $A(t)$ is computed based on the $SOC_i(t)$ and $Load_i$ at each subsystem. Accordingly, the higher the SOC_i the higher the $a_{i,i}(t)$, which limits the charging power allocation to the BESS with a high SOC level. Similarly, the higher the $Load_i$ the lower the $a_{i,i}(t)$, which increases the charging power allocation for subsystems with high loads. As such the penalty factors can be computed as stated in Eq. 4. Here, W_{SOC} and W_{Load} are weight factors for SOC and Load, respectively, which can be adjusted according to system requirements. Moreover, other

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factors, such as BESS internal temperature and load priorities, can be integrated into the calculation of $a_{i,i}$ as appropriate.

$$a_{i,i} = \frac{W_{SOC} \times SOC_i(t)}{W_{Load} \times Load_i(t)} \tag{4}$$

B. Primal and Dual Problem

The formulated convex optimization problem can be written in matrix form, as stated in Eq.5. The time variable is omitted for brevity.

$$\mathcal{P}_{CM}: \quad \min_{P_B} \frac{1}{2} P_B^T A P_B \quad st. \tag{5}$$

$$\mathbf{1}^T P_B = P_U - \mathbf{1}^T P_L \tag{6}$$

$$0 \leq P_B \leq P_{MaxCh} \tag{7}$$

Here, $P_L \in \mathbb{R}^{n\times 1}$ is a vector representing the loads at each subsystem (i.e $P_L = [P_{L,1}, \ldots, P_{L,n}]^T$). $P_{MaxCh} \in \mathbb{R}^{n\times 1}$ is a vector representing the maximum charge power of each BESS. The notation $\mathbf{1}$ referred to an all-ones vector with dimensions $(n \times 1)$. The Lagrangian of the primal problem $\mathcal{P}_{CM}$ is stated in 8. In this context, all constraints are incorporated into the Lagrangian.

$$\mathcal{L}(P_B, \lambda_1, \lambda_2, \nu) = \frac{1}{2} P_B^T A P_B - \lambda_1^T P_B + \lambda_2^T [P_B - P_{MaxCh}] + \nu[\mathbf{1}^T P_B - P_U - \mathbf{1}^T P_L] \tag{8}$$

$\lambda_1 \in \mathbb{R}^{n\times 1}$ and $\lambda_2 \in \mathbb{R}^{n\times 1}$ are vectors of Lagrangian multipliers associated with the inequality constraints, while $\nu \in \mathbb{R}$ is the Lagrangian multiplier associated with the equality constraint. This equality constraint can be identified as the coupling constraint, which requires global information (i.e., connected loads in each subsystem). The primal problem $\mathcal{P}_{CM}$ is then converted into a dual problem by employing the Lagrangian multipliers $\lambda_1, \lambda_2, \nu$ as stated in Eq. 9

$$\mathcal{P}_{DM}: \quad \max_{\lambda_1,\lambda_2,\nu} \min_{P_B} \; \mathcal{L}(P_B, \lambda_1, \lambda_2, \nu) \quad st. \tag{9}$$

$$\lambda_1 > 0 \quad , \quad \lambda_2 > 0 \tag{10}$$

Since $\min_{P_B} \; \mathcal{L}(\mathcal{P_B}, \lambda_\infty, \lambda_\in, \nu)$ is a concave function with respect to P_B, for all feasible P_B values $L(P_B, \lambda_1, \lambda_2, \nu)$ will be the lower bound for the primal problem. Hence the optimal value of the dual variables $\lambda_1^*, \lambda_2^*, \nu^*)$ is the lower bound for the optimal primal variable P_B^*. Strong duality holds when no gap exists between the primal and dual variables [15].

Assumption 1: The allocated valley filling target at each time step can be achieved by the BESS in the network.

According to Assumption 1, strong duality holds, indicating that the problem satisfies Slater's condition. As such the four optimality criteria derived from the Karush-Kuhn-Tucker (KKT) conditions will ensure the optimality of $\mathcal{P}_{DM}$ and hence $\mathcal{P}_{CM}$ [13], [15]. Consequently, the complementary slackness conditions $\lambda_1^{*T} P_B^* = 0$ and $\lambda_2^{*T}[P_B^* - P_{MaxCh}] = 0$ imply four possible scenarios for P_B^* as listed in Eq. 11.

Algorithm 1 Central Coordination

Set: $P_{VF,Target}$, $min(a_{ii})$; $a_{ii} \in A$
Initialization:
$k \leftarrow 0$
$\nu_{max}^k \leftarrow 0$, $\nu_{min}^k \leftarrow -P_{VF,Target}/min(a_{ii})$
$\nu^k \leftarrow (\nu_{max}^k - \nu_{min}^k)/2$
Algorithm:
while $\|(\nu_{max}^k - \nu_{min}^k)\| < \epsilon$ **do**

- Broadcast ν^k in distribution network
- Compute $P_{VF,feedback}$
- Update ν^k
  If $(P_{VF,feedback} - P_{VF,Target}) > 0$ **Then**
  $\nu_{min}^k \leftarrow \nu^k$
  Else
  $\nu_{max}^k \leftarrow \nu^k$
  End If
  $\nu^k = (\nu_{max}^k + \nu_{min}^k)/2$
- Update iteration $k \leftarrow k + 1$

end while

Algorithm 2 Local BESS Charge Control : $BESS_i$

Set: $P_{MaxCh,i}$, $a_{ii} \in A$
Initialization: Read ν from the broadcast
Algorithm:

- Compute $P_{b,i}$
  If $\nu \geq 0$ **Then**
  $P_{b,i} \leftarrow 0$
  Else If $-P_{MaxCh,i}.a_{ii} < \nu < 0$ **Then**
  $P_{b,i} \leftarrow -\nu/a_{ii}$
  Else
  $P_{b,i} \leftarrow P_{MaxCh,i}$
  End If

$$\begin{cases} 0 < P_B^* < P_{MaxCh}, & \text{if } \lambda_1^{*T} = 0, \lambda_2^{*T} = 0 \\ P_B^* = 0, & \text{if } \lambda_1^{*T} > 0, \lambda_2^{*T} = 0 \\ P_B^* = P_{MaxCh}, & \text{if } \lambda_1^{*T} = 0, \lambda_2^{*T} > 0 \\ P_B^* \;\; undefined, & \text{if } \lambda_1^{*T} > 0, \lambda_2^{*T} > 0 \end{cases} \tag{11}$$

The stationary KKT condition is derived considering the gradient of the Lagrangian Eq. 8 whereas $\nabla_{P_B}\mathcal{L}(P_B^*, \lambda_1^*, \lambda_2^*, \nu^*) = 0$. Therefore substituting Eq. 11 in stationary condition $AP_B^* - \lambda_1^{*T} + \lambda_2^{*T} + \nu^* \mathbf{1}^T = 0$ will allow computing P_B based on ν as stated in Eq.12.

$$P_B^* = \begin{cases} 0, & \text{if } \nu \geq 0 \\ -A^{-1}\mathbf{1}^T \nu^*, & \text{if } -A.P_{MaxCh} < \mathbf{1}^T \nu < 0 \\ P_{MaxCh}, & \text{if } \nu \leq -A.P_{MaxCh} \end{cases} \tag{12}$$

IV. Results

This section demonstrates the proposed distributed valley-filling algorithm via MATLAB/Simulink simulations on a realistic BESS-connected grid. The network model is first presented, followed by results of the strategy and a comparison of processing times with existing methods.

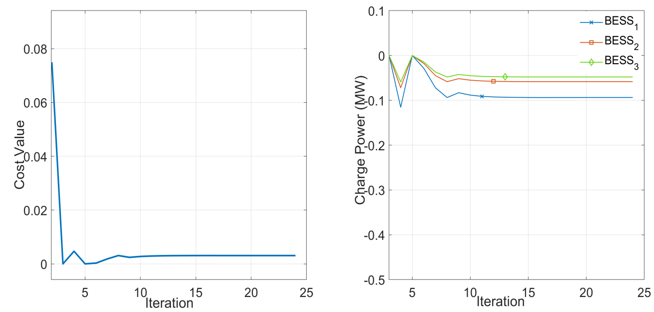
A. BESS-connected distribution grid model

The architecture of the considered BESS-connected distribution grid model is illustrated in Fig. 1. Each subsystem includes a dynamic load and a fixed resistive load, with the BESS connected through a DC–AC converter and series filter. The distribution network operates at a nominal voltage of $600V$ and a nominal frequency of $60Hz$. In simulation a distribution network with 3 BESS is modelled in MATLAB Simulink. Each BESS is connected to the utility grid via a three-phase series R-L circuit with 0.01Ω resistance and $0.001H$ inductance. BESS nominal DC voltage is $782V$ while the distribution grid nominal voltage $(V_{LL,rms})$ is $480V$. The DC-AC converter is modelled using IGBT/diode pairs, controlled by firing pulses generated by a PWM generator. A PI controller is set up with a proportional gain K_P of 5 and an integral gain K_I of 100. Additionally, a clamping anti-windup method is enabled to avoid controller saturation. The initial SOC of each BESS is set to 30%, 70%, and 90% respectively while the maximum charge limit is predefined to 95%. When a BESS reaches its maximum SOC, its maximum charging power constraint is set to 0 to prevent overcharging and maintain safe operation.The PI controller uses a proportional gain $K_P = 10$ and an integral gain $K_I = 100$ to track the setpoint of the central coordination unit.

B. Simulation of the Valley Filling Algorithm

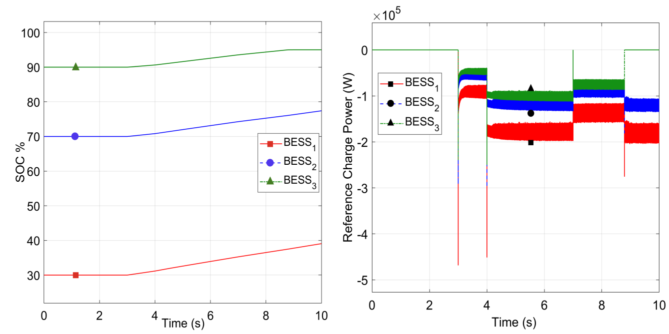
The proposed algorithm is simulated in MATLAB for one complete cycle, during which the set point for the utility supply is fixed at $1MW$. The initial loads in each subsystem are set at $0.4MW$, $0.25MW$, and $0.15MW$, respectively, while the BESS SOC levels are as previously stated. This configuration allows for a thorough evaluation of the algorithm's effectiveness in coordinating the distribution of charge power among the BESS throughout the system. Fig.2 (a) illustrates the trajectory of the cost function. Initially, the cost value is high due to the assignment of maximum charge power, without considering the available SOC and local loads for each BESS. When the optimal valley-filling algorithm is executed, the charge power is distributed accordingly, causing a rapid decrease in cost until it reaches steady state. The algorithm stops when the threshold $(\nu_{max} - \nu_{min})$ falls below 10^{-8}. Fig.2 (b) illustrates the reference value for BESS charge power.

The algorithm is simulated over a $10s$ horizon in MATLAB/Simulink. During the simulation, key variables including BESS dispatch power, utility supply, load levels, and BESS SOC are recorded. The SOC tracking ensures that each BESS operates within safe limits and maintains optimal performance. Fig. 3 (a) illustrates the trajectory of the SOC levels for each BESS. The SOC increase rate is notably higher when the subsystem load is elevated and the initial SOC level is low. Fig. 3 (b) illustrates the set point computed by the local BESS controller based on the broadcast signal ν from the central coordination. The set point for BESS 3 reaches 0 when the SOC reaches the maximum allowable limit of 95%.



(a) Cost of Charge (b) BESS Charge Power

Fig. 2: Simulation of the algorithm - one complete cycle



(a) SOC Increase (b) BESS Ref Charge Power

Fig. 3: SOC change and Reference Charge Power behaviour

The real-time simulation horizon is scaled down from the realistic load variation pattern in the distribution network to evaluate the applicability of the proposed algorithm. Initially, the loads in each subsystem are equal until $t = 2s$, and therefore the utility supply is equal to the sum of all loads. Uneven step load changes are initiated at $t = 2s$, which increases the utility supply from $0.3MW$ to $0.8MW$. BESS charging is enabled at $t = 3s$, allowing the proposed valley-filling algorithm to operate. Consequently, the utility supply increases to $1MW$, which is the target supply. Additional step load changes are initiated at $t = 4s$ and $t = 7s$, respectively. Fig. 4 illustrates the change in the charging power of each BESS in response to these variations, maintaining the target utility supply at a constant value. When $t \approx 8.75s$, BESS 3 reaches its maximum SOC and stops charging, reallocating the remaining power to charge BESS 1 and 2 accordingly.

C. Scalability

The valley-filling algorithm must remain independent of a fixed grid configuration due to the uncertain contribution of EVs. The proposed algorithm adapts to grid changes without additional reconfiguration. When a BESS or EV connects to the grid, the central coordination system updates the penalty parameter weight based on its SOC and local loads, adjusting the broadcast variable accordingly. Fig. 5(a) and (b) illustrate

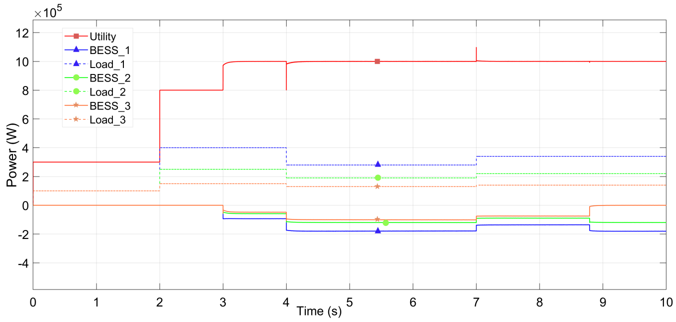
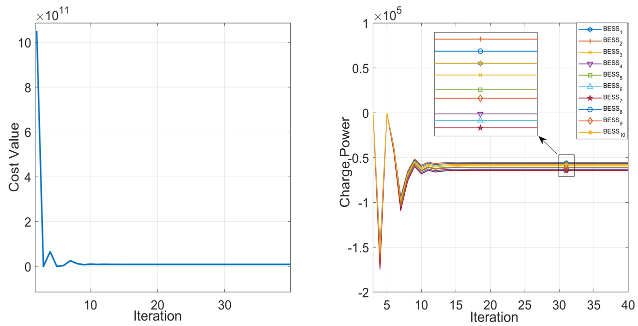


Fig. 4: Real-time simulation of a network with 3 BESS



(a) Cost of Charge (b) BESS Charge Power

Fig. 5: Scalability

the cost function trajectory and reference BESS charge power for a distribution system with 10 BESS, respectively.

D. Comparison

Fast convergence is crucial for efficient BESS charge coordination. The proposed algorithm is compared with MATLAB's nonlinear solver 'fmincon' [9] under identical conditions, recording the time to find the optimal solution for one cycle. As illustrated in Fig. 6, 'fmincon' is nearly 100 times slower than the proposed algorithm, and processing time increases with the number of BESS. For a system with 100 BESS, 'fmincon' requires $0.6s$, whereas the proposed algorithm takes only $0.003s$. These results demonstrate that the proposed algorithm is fast enough to handle typical latencies in the broadcasting network.

V. Conclusion

This paper presents an innovative valley-filling algorithm that optimizes BESS charge coordination in distribution networks, with emphasis on peak shaving. A detailed theoretical analysis shows that the algorithm yields optimal charging power for each BESS when sufficient absorption capacity exists, and this is validated through simulation results. The algorithm enhances system efficiency by reducing processing time, ensuring adequate time for broadcasting the centralized coordination setpoint during events. Future work will integrate peak-shaving and valley-filling into a unified distributed optimization framework.

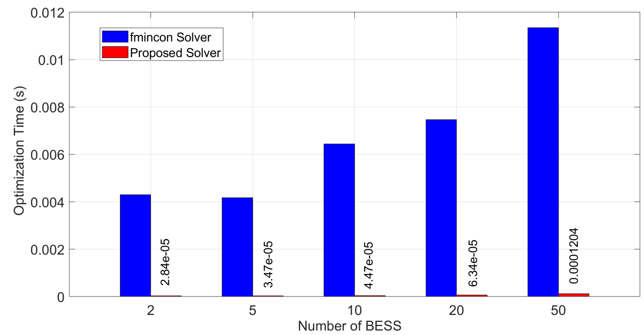


Fig. 6: Comparison of processing time

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