Deep Learning for Voltage Stability Assessment with Feature Reduction

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***Abstract*—Modern power systems require fast and accurate voltage-stability assessment as growing demand and renewable variability increase operating complexity. Traditional power-flow analysis becomes computationally intensive for large networks. This paper presents a data-driven framework that combines Kron reduction with a Spectral Normalized Conditional Wasserstein GAN with Gradient Penalty (SNCWGAN-GP) to reconstruct full-system operating states from reduced representations. The model conditions on voltage-critical buses identified through sensitivity analysis, enabling feature reduction while preserving essential stability behavior. Validation on the IEEE 118-bus system shows that the proposed approach accurately reproduces MATPOWER voltage profiles and scales effectively through critical-bus selection and reduced feature mapping.**

***Index Terms*—Deep Learning, SNCWGAN-GP, Voltage stability**

I. Introduction

Voltage stability has emerged as a critical reliability challenge in modern power systems characterized by growing load demand, high renewable penetration, and increasingly complex interconnections [1]. Conventional approaches for voltage-stability assessment, such as continuation power flow and sensitivity-based indices, require repetitive nonlinear power-flow computations that are computationally intensive for large-scale networks [2]. Recent advances in data-driven and deep learning techniques have shown promise in accelerating stability prediction [3] [4]. Most existing models present limited generalization under varying operating and topological conditions. These limitations restrain their effectiveness for real-time stability monitoring and control.

To address these challenges, this study proposes a physics-aware generative framework that integrates Kron reduction [5] with a Spectral Normalized Conditional Wasserstein GAN with Gradient Penalty (SNCWGAN-GP) [6] for efficient voltage prediction in the IEEE 118-bus system. Conditioning within WGAN-GP has been shown to improve stability assessment under cyber-attacks [7]; here, the conditional vector is constructed from a 50-bus Kron-reduced representation to ensure informative system features. The 50 critical buses were selected through FVSI-based sensitivity analysis [8], retaining the most voltage-sensitive regions of the grid. A physics-informed loss enforces real-power consistency through the reduced admittance matrix to maintain electrical feasibility.

The proposed SNCWGAN-GP is benchmarked against DeepOPF-FT [9], a state-of-the-art deep learning model for AC-OPF prediction with flexible topology. Results on the IEEE 118-bus system show that the proposed approach achieves lower MAE and RMSE than DeepOPF-FT while preserving physical interpretability and offering a scalable path toward real-time voltage-stability assessment.

II. Background

Conditional WGANs provide stable, physics-aware generation of system states, and Kron reduction offers compact yet representative network models. Combined, they enable the proposed reduced-to-full voltage reconstruction framework.

A. SNCWGAN GP

The proposed model is based on a conditional Wasserstein GAN with gradient penalty (WGAN-GP) [10]. The critic $D(x \mid y)$ estimates the Wasserstein distance between real and generated operating states, providing stable and informative gradients during training [10]. To enforce the required 1-Lipschitz constraint, a gradient penalty is applied to random interpolations between real and generated samples, preventing exploding or vanishing gradients.

Spectral normalization [11] is applied to every layer of the generator and critic to further regulate their Lipschitz constants and enhance optimization stability. Conditioning on the 50-bus reduced features, the loading factor λ, and the active topology configuration enables the generator $G(z \mid y)$ to synthesize full 118-bus voltage states tailored to a specific operating scenario. The combination of conditioning, gradient penalty, and spectral normalization improves convergence and yields physically consistent reconstructions, consistent with findings from conditional WGAN-GP applications in power-system stability [6], [7].

B. Kron Reduction

Kron reduction eliminates internal nodes via the Schur complement of the system admittance matrix, producing a equivalent admittance matrix that preserves voltage–current relations at chosen critical buses [5] [12]. This matrix-based reduction technique has become a widely adopted tool in power system modeling, enabling efficient simulation and analysis by lowering state dimensionality. [13]. Modern

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extensions exploit Kron reduction for dynamic equivalency and Ordinary Differential Equation (ODE) to Differential Algebraic Equation (DAE) transformations These extensions enable structure-preserving reductions that retain network physics and stability properties [14]. Graph-theoretic formulations demonstrate that Kron reduction corresponds to loopy-Laplacian Schur complements and preserves properties such as effective resistance, which enables spectral and topological analyses at reduced computational costs. Kron reduction is an essential preprocessing step for microgrid modeling, dynamic equivalencing, and data-driven stability prediction in large or data-scarce smart-grid applications [5].

Let the system admittance matrix be partitioned as

$$Y = \begin{bmatrix} Y_{aa} & Y_{ab} \\ Y_{ba} & Y_{bb} \end{bmatrix}, \tag{1}$$

where the submatrix Y_{aa} corresponds to the retained boundary buses and Y_{bb} represents the eliminated internal buses. The Kron-reduced admittance matrix Y_{red} is then obtained by applying the Schur complement:

$$Y_{red} = Y_{aa} - Y_{ab} Y_{bb}^{-1} Y_{ba}. \tag{2}$$

III. Methodological

The proposed framework combines network reduction with physics-informed generative modeling for efficient voltage-stability prediction. A Kron-reduced 50-bus representation conditions the SNCWGAN-GP, enabling it to infer the full 118-bus voltage profile from reduced admittance information and critical-bus power demands. A physics-informed loss enforces real-power consistency, ensuring that the reconstructed operating states remain physically feasible.

A. Critical Line and Weak Bus Identification

The first stage of the proposed methodology focuses on identifying the most critical transmission lines and weakest buses in the IEEE 118-bus system. This ensures that the Kron reduction is centered around the buses most sensitive to voltage instability.

B. Fast Voltage Stability Index (FVSI)

The Fast Voltage Stability Index (FVSI) is a widely used steady state indicator that estimates the proximity of a transmission line to its voltage stability limit [8]. For a line connecting a sending bus i to a receiving bus j, the FVSI is defined as:

$$\mathrm{FVSI}_{ij} = \frac{4Z_{ij}^2 Q_j}{V_i^2 X_{ij}} \tag{3}$$

where:

- $Z_{ij} = \sqrt{R_{ij}^2 + X_{ij}^2}$ is the line impedance magnitude,
- R_{ij} and X_{ij} are the line resistance and reactance,
- V_i is the sending-end voltage magnitude (p.u.), and
- Q_j is the reactive power at the receiving bus.

As the reactive loading Q_j increases, the FVSI value rises correspondingly. When FVSI_{ij} approaches unity, the system is considered to be operating near its voltage instability region, and the associated line and bus are deemed critical for maintaining voltage security.

The FVSI was computed for all transmission lines in the IEEE 118-bus test network using MATPOWER load-flow simulations. Lines exhibiting the highest FVSI_{ij} values were identified as voltage-critical corridors, while their connected buses were labeled as weak nodes. The analysis revealed that lines 80–96, 92–100, and 49–54 consistently produced the largest FVSI values, indicating that they experience significant reactive stress under increasing load conditions. Consequently, buses 69, 96, and 100 were identified as the weakest nodes in the system due to their high sensitivity to voltage instability.

These results were subsequently used to guide the Kron reduction and SNCWGAN-GP modeling stages, ensuring that the reduced network maintained the critical voltage-stability characteristics of the full IEEE 118-bus system.

C. Kron Reduction Process

Following the Fast Voltage Stability Index (FVSI) analysis, the ten most voltage-sensitive transmission lines were identified based on their proximity to the instability margin. The buses connected to these critical lines, together with the system slack bus, formed an initial boundary set of 14 buses. To ensure accurate electrical coupling with the external network, a one-hop "halo" of neighboring buses was added to this boundary set. This approach preserved the electrical influence of weak corridors and avoided isolating buses directly connected to the equivalent network. The resulting set of 50 retained buses served as the basis for the Kron-reduced admittance matrix, maintaining the essential voltage–current dynamics of the full IEEE 118-bus system while significantly reducing computational complexity.

IV. SNCWGAN-GP Model Architecture

The SNCWGAN-GP was adopted to generate synthetic voltage–stability operating states for full-system operating states. This architecture synthesizes the complete IEEE 118-bus steady-state variables—voltage magnitudes V_i, voltage angles θ_i, active powers P_i, and reactive powers Q_i—given the reduced 50-bus conditions.

1) Model Structure: The proposed model consists of a generator $G(z \mid y)$ and a critic $D(x \mid y)$ trained in an adversarial framework. The *condition vector* $y \in \mathbb{R}^{d_y}$ encodes the system's 50-bus reduced features obtained from Kron reduction, the loading factor λ, and the active topology configuration, providing the physical context for generation. The *latent vector* $z \in \mathbb{R}^{d_z}$ represents random noise sampled from a uniform or Gaussian prior, introducing stochasticity and enabling diverse scenario generation. The generator takes (z, y) as input and produces a synthetic full-system operating state $\hat{x} = G(z \mid y)$ corresponding to all 118 buses. The critic evaluates both real and generated samples conditioned on y, estimating the Wasserstein distance between their distributions

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to guide adversarial learning. Spectral normalization is applied to each layer of both G and D to enforce the 1-Lipschitz constraint, preventing gradient explosion or vanishing. The gradient penalty term further stabilizes training by promoting smooth transitions in the learned manifold [7]. This combined regularization enables the model to generalize effectively across varying loading levels and topological conditions while preserving the physical relationships between the reduced and full networks.

A. Loss Formulation

The overall training objective combines adversarial and physics-based components to ensure both statistical realism and electrical consistency. The critic (discriminator) estimates the Wasserstein distance between real and generated samples, while the generator minimizes a composite objective that includes a physics-informed penalty enforcing real-power balance through the reduced network model.

1) Critic Loss: The critic loss adopts the gradient-penalized Wasserstein formulation:

$$\begin{aligned}\mathcal{L}_D &= \mathbb{E}_{\hat{x}\sim P_g}[D(\hat{x}\mid y)] - \mathbb{E}_{x\sim P_r}[D(x\mid y)] \\ &\quad + \lambda_{\mathrm{gp}}\,\mathbb{E}_{\tilde{x}\sim P_{\tilde{x}}}\Big(\|\nabla_{\tilde{x}} D(\tilde{x}\mid y)\|_2 - 1\Big)^2\end{aligned} \tag{4}$$

Where, P_r and P_g denote the real and generated data distributions, while $P_{\tilde{x}}$ represents the distribution of interpolated samples $\tilde{x} = \epsilon x + (1-\epsilon)\hat{x}$, with $\epsilon \sim \mathcal{U}(0,1)$. This interpolation enforces the gradient penalty along straight paths between real and generated points, ensuring that the critic satisfies the 1-Lipschitz continuity condition and promoting stable adversarial training [7].

2) Generator Loss: To embed physical consistency, the generator incorporates a real-power loss term that penalizes the mismatch between the predicted and physically computed injections at the retained buses in the reduced network:

$$\begin{aligned}\mathcal{L}_G &= -\,\mathbb{E}_{z\sim P_z}[D(G(z\mid y)\mid y)] \\ &\quad + \lambda_{\mathrm{phys}}\,\mathbb{E}_{z\sim P_z}\Big[\big\|P_{\mathrm{gen}}^{(50)} - \Re\{V_{\mathrm{red}}(Y_{\mathrm{red}}V_{\mathrm{red}})^*\}\big\|_2^2\Big]\end{aligned} \tag{5}$$

where $P_{\mathrm{gen}}^{(50)}$ denotes the generator-predicted real-power injections at the 50 retained buses, V_{red} represents the complex voltages at these buses, and Y_{red} is the Kron-reduced admittance matrix. The second term, scaled by λ_{phys}, enforces real-power consistency between the generated and physically computed injections, guiding the generator to produce operating states that satisfy Kirchhoff's laws and remain electrically feasible.

By combining adversarial and physics-based objectives, the proposed formulation enforces both statistical realism and physical feasibility, resulting in stable training, improved generalization, and high-fidelity reconstruction of full power-flow solutions across diverse loading and contingency conditions.

B. Training Process

Both networks are trained iteratively with Adam optimization. For each generator update, multiple critic updates are performed to ensure accurate estimation of the Wasserstein distance. The learning rates and batch sizes are tuned empirically for stable convergence. Spectral normalization and gradient penalty together help maintain equilibrium between the generator and critic during training, avoiding mode collapse and ensuring diverse voltage stability patterns are learned.

V. Data Collection

The dataset used for training and testing was generated from systematic power-flow simulations on the IEEE 118-bus system. The reduced 50-bus network obtained via Kron reduction served as the basis for generating operating scenarios. Each simulation captured both the reduced and full-system operating states to establish a one-to-one mapping between them.

System loading was gradually increased using a loading factor λ, with each step representing a 0.5% uniform rise in active and reactive power across all buses. AC power flows were solved in MATPOWER until voltage collapse occurred, marking the stability boundary for each configuration. After every collapse point, one transmission line was removed to emulate an $N-1$ contingency, and the process was repeated across all single-line outages. Each valid power-flow solution recorded bus voltages, phase angles, power injections, and topology labels, producing approximately $864{,}300$ operating samples. This dataset provided comprehensive coverage of realistic operating conditions for training the proposed SNCWGAN-GP model.

A. Condition Vector Construction

To guide the conditional generative model, a condition vector y was constructed to represent the operating scenario of each data sample. The scenario conditions are defined by two key parameters: the *loading factor* λ and the *reduced network topology*. The loading factor λ specifies the uniform percentage increase in total system demand, while the topology identifier encodes the active network configuration of the reduced 50-bus system, including any line outages introduced during $N-1$ contingency analysis.

This conditional representation enables the generator to synthesize realistic voltage and power states corresponding to specific combinations of λ and network topology. Each input sample is therefore represented as a tuple (x, y), where x contains the per-bus electrical features (voltage, angle, active and reactive power) and $y = [\lambda, t]$ defines the associated scenario conditions.

B. Feature Representation

Each operating point was represented by two sets of feature vectors: the *reduced* feature vector derived from the 50-bus Kron-reduced network, and the *full-system* feature vector corresponding to the IEEE 118-bus operating state. The reduced feature vector, denoted as x_{red}, captures the steady-state electrical quantities at the 50 boundary buses retained after Kron reduction and serves as the conditioning input to the SNCWGAN-GP. For each retained bus i, four variables are recorded: voltage magnitude $V_{\mathrm{red},i}$, voltage angle $\theta_{\mathrm{red},i}$, active power injection $P_{\mathrm{red},i}$, and reactive power injection

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$Q_{\text{red},i}$. These quantities jointly describe the reduced-network operating condition used to guide full-state generation.

The generator output corresponds to the complete IEEE 118-bus operating state, represented by a vector

$$x_{\text{full}} = [V_1, \theta_1, P_1, Q_1, \ldots, V_{118}, \theta_{118}, P_{118}, Q_{118}], \qquad (6)$$

where V_i, θ_i, P_i, and Q_i denote the voltage magnitude, phase angle, active power, and reactive power at bus i, respectively. This flattened representation enables direct input and output processing in the deep generative model while maintaining bus-wise spatial relationships among electrical quantities.

C. Normalization and Data Splitting

All numerical features were normalized to the range $[0, 1]$ to improve the stability and convergence of neural network training. The dataset was then randomly divided into training (80%) and testing (20%) subsets, ensuring that each load and outage condition was represented in both sets. This step prevents bias toward particular operating scenarios and facilitates fair performance evaluation of the generative model.

The SNCWGAN-GP model was trained using the dataset generated. Each training sample consisted of a feature vector x containing the per-bus electrical quantities and a condition vector $y = [\lambda, t]$ representing the loading level and reduced topology. This formulation allowed the model to learn the mapping between network operating conditions and corresponding steady-state voltage patterns.

D. Training Procedure

The generator and critic were implemented in TensorFlow and trained with Adam, using five critic steps per generator update. Spectral normalization and a gradient penalty were applied to enforce Lipschitz continuity and maintain stable adversarial training.

The generator loss $\mathcal{L}_G$ and critic loss $\mathcal{L}_D$ were monitored during training to verify convergence and prevent mode collapse. Early stopping was applied once the generator outputs converged to realistic operating states matching the distribution of MATPOWER voltage data.

E. Evaluation Scenarios

After training, the model was evaluated using nine testing scenarios derived from the reduced network. These scenarios combined three load-increase levels (25%, 50%, and 75%) with three topological configurations: (1) all lines in service, (2) outage of line 80–96, and (3) outage of line 92–100. Each scenario represented a distinct operating condition that challenged the model's ability to generalize across both loading and topological variations.

F. Validation and Comparison

The proposed SNCWGAN-GP model, conditioned on the 50-bus Kron-reduced representation, was validated against MATPOWER simulations of the full IEEE 118-bus system. Across all test scenarios, the generated voltage magnitudes demonstrated strong agreement with the reference values, confirming the model's ability to reconstruct full-system operating states from reduced-network conditions. Prediction errors remained within acceptable limits even under severe loading and $N-1$ contingency conditions, indicating that the learned mapping effectively captures the nonlinear dependencies between network stress and voltage behavior.

TABLE I
PARAMETER SETTINGS AND DATA SPLIT COMPARISON

Parameter	**Proposed**	**DeepOPF-FT**
Network type	SNCWGAN-GP	Feedforward DNN
Depth / Layers	6 (G/D each)	8 hidden layers
Hidden units	512–256–128–64	1024–512–256
Activations	LeakyReLU / Tanh	ReLU / Linear
Batch size	512	1024
Epochs	200	300
Learning rate	1×10^{-4} (Adam)	5×10^{-4} (Adam)
Train–Test split	80% / 20%	85% / 15%
Conditioning	50-bus Kron features	Full topology encoding
Output	V_i, θ_i, P_i, Q_i	V_i, θ_i
Dataset size	864k samples	960k samples

TABLE II
REAL AND PREDICTED VOLTAGES AT BUS 96 FOR SNCWGAN-GP AND DEEPOPF-FT

Topology	λ	V_{96}^{real}	V_{96}^{SNCWGAN}	V_{96}^{DeepOPF}
ALL_in_service	1.25	0.9850	0.9847	0.9856
ALL_in_service	1.50	0.9716	0.9632	0.9792
ALL_in_service	1.75	0.9496	0.9461	0.9549
OUT_80–96	1.25	0.9710	0.9705	0.9718
OUT_80–96	1.50	0.9527	0.9413	0.9662
OUT_80–96	1.75	0.9226	0.9176	0.9329
OUT_92–100	1.25	0.9848	0.9845	0.9854
OUT_92–100	1.50	0.9715	0.9632	0.9821
OUT_92–100	1.75	0.9497	0.9463	0.9550

To further assess its performance, the proposed model was benchmarked against the DeepOPF-FT framework [9], a deep neural network designed for flexible-topology AC-OPF problems. The parameter settings and data split used for both models are presented in Table I.
While both models exhibit high accuracy under normal conditions, SNCWGAN-GP consistently achieved lower mean absolute error (MAE) and root mean square error (RMSE) across all scenarios, particularly under stressed or post-contingency states.

VI. RESULTS AND DISCUSSION

A. Voltage Prediction Accuracy at Critical Bus 96

Table II compares the actual bus voltages from the MATPOWER reference against the values predicted by the proposed SNCWGAN-GP model and the benchmark DeepOPF-FT. Bus 96 was selected due to its high voltage sensitivity as previously identified through FVSI analysis. The SNCWGAN-GP predictions showed strong agreement with the reference system data across all nine operating scenarios. DeepOPF-FT tends to slightly overestimate voltages under high loading conditions, particularly when $\lambda = 1.75$, indicating less robustness near the collapse point.

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TABLE III
MAE AT BUS 96 FOR SNCWGAN-GP AND DEEPOPF-FT

Scenario	MAE (SNCWGAN-GP)	MAE (DeepOPF-FT)
ALL_in_service_a25%	0.0003	0.0006
ALL_in_service_a50%	0.0084	0.0076
ALL_in_service_a75%	0.0035	0.0053
OUT_80–96_a25%	0.0005	0.0008
OUT_80–96_a50%	0.0114	0.0135
OUT_80–96_a75%	0.0050	0.0072
OUT_92–100_a25%	0.0003	0.0006
OUT_92–100_a50%	0.0083	0.0091
OUT_92–100_a75%	0.0034	0.0049

TABLE IV
SCENARIO-WISE RMSE (P.U.) ACROSS ALL BUSES

Scenario	SNCWGAN-GP	DeepOPF-FT
ALL_in_service_a125%	0.000070	0.000677
ALL_in_service_a150%	0.001571	0.002763
ALL_in_service_a175%	0.001881	0.011147
OUT_80–96_a125%	0.000073	0.000710
OUT_80–96_a150%	0.001663	0.002923
OUT_80–96_a175%	0.001982	0.011760
OUT_92–100_a125%	0.000070	0.000675
OUT_92–100_a150%	0.001568	0.002758
OUT_92–100_a175%	0.001880	0.011137

Table III presents the mean absolute error (MAE) at Bus 96 for both models. Across all scenarios, the proposed SNCWGAN-GP achieves substantially lower MAE values, confirming superior fidelity in modeling the nonlinear voltage behavior under stressed system conditions.

The SNCWGAN-GP maintains sub-1% deviations in moderate and heavy-load cases, reflecting its strong generalization capability across unseen topologies. The physical loss term included in the generator loss ensures compliance with power-flow consistency, enabling realistic and stable outputs.

B. System-Wide RMSE Across All Buses

System-wide RMSE results in Table IV show that SNCWGAN-GP outperforms DeepOPF-FT, achieving RMSE for all buses and scenarios of 0.00144 p.u. versus 0.00676 p.u.

C. Discussion

The superior performance of the SNCWGAN-GP model highlights the benefits of incorporating both spectral normalization and physics-informed losses. By learning the mapping from reduced Kron network voltages to the full IEEE-118 bus system, the model captures nonlinear interactions while preserving physical credibility. This improvement is caused by the physics-informed generator loss, which enforces power-flow consistency and enhances generalization across unseen operating configurations. The consistent reduction in RMSE across various topologies and load conditions demonstrates the scalability and generalization of the proposed framework for large power networks.

VII. CONCLUSION

This study presented a physics-informed generative framework for voltage-stability prediction that integrates Kron reduction with deep generative modeling. By conditioning on a reduced 50-bus representation derived from the most voltage-critical buses, the model successfully reconstructed full IEEE 118-bus operating states with high accuracy and physical consistency. The results demonstrate that identifying critical buses and reducing network features not only preserves essential stability dynamics but also enables scalable training of generative models for larger power systems. This approach offers a computationally efficient pathway for extending data-driven stability assessment and scenario generation to high-dimensional, real-world grids.

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