Analysis of B-H and J-H Hysteresis Characteristics for Irreversible Demagnetization Modeling in NdFeB PMSM Applications

Md Imtiaz Kamrul[1], Shuhui Li[2], Sayem Ul Alam[3], Nathan Jeong[4], Yang-Ki Hong[5]
Department of Electrical and Computer Engineering
The University of Alabama, Tuscaloosa, Alabama 35487, USA
[1]mkamrul@crimson.ua.edu, [2]sli@eng.ua.edu, [3]salam7@crimson.ua.edu, [4]shjeong@ua.edu, [5]ykhong@eng.ua.edu

***Abstract*—Research on Permanent Magnet Synchronous Machines (PMSMs) inherently involves cross-disciplinary efforts spanning material science and electric machine design. Despite extensive study within each domain, existing literature provides limited systematic correlation study between the domain-specific intrinsic (J–H) and normal (B–H) magnetic representations, including in the context of irreversible demagnetization under combined thermal and magnetic stress. This paper presents a unified and detailed evaluation that explicitly bridges this gap between the two domains. Specifically, by integrating the material-centric intrinsic knee point (H_k) into a machine-design-oriented non-linear Norton equivalent magnetic circuit, this work systematically evaluates the parameters associated with demagnetization representation in each domain. Additionally, it establishes a rigorous basis for interpreting these parameters, explicitly defining how intrinsic stability limits should be translated into practical machine design constraints. By establishing clear connections between the two demagnetization frameworks, this study offers new insights that are not explicitly addressed in existing studies, enabling more effective cross-disciplinary collaboration and robust PMSM design.**

***Index Terms*—Magnetic hysteresis, coercivity, irreversible demagnetization, permanent magnet synchronous motor**

I. Introduction

Rare-earth permanent magnets, particularly compositions such as Neodymium–Iron–Boron (Nd–Fe–B) and Samarium–Cobalt (Sm–Co), are getting more and more attention nowadays due to the rising demand for higher power density in the advancement of electric machine technology [1], [2]. In order to meet this growing demand, the materials science domain focuses on enhancing intrinsic energy density. On the other hand, in the engineering domain, these materials are being pushed to their operational extremes in traction applications ranging from hybrid electric vehicles (HEVs) to wind turbine generators [1]. As operational demands drive these machines closer to their thermal and magnetic limits, the precise characterization in both domains of magnet stability becomes significant.

The magnetic hysteresis loop represents the key metric for the characterization of magnetic material [3], [4]; however, a significant methodological divergence currently limits the synergy between the material science and engineering field. To a material scientist, a permanent magnet is a complex physical system that is defined by its internal stability and the microscopic energy barriers required to resist domain reversal [1], [5]. Their analysis prioritizes the intrinsic ($J-H$) curve, which explicitly characterizes the physics that maintain the magnetized state against thermal and external agitation [6]. In contrast, electric machine designers predominantly treat the permanent magnet as a linear component which is a predictable energy source and can be characterized straightforwardly by the amount of flux it can deliver to the air gap [7]. Consequently, engineering approaches concentrate almost exclusively on the normal ($B-H$) curve, effectively ignoring the internal physics in favor of simplified linear models suitable for rapid electromagnetic design [2].

This gap creates a critical blind spot for both communities. For the machine designer, standard frameworks approximate safety using linear thresholds that work well for steady-state operation but fail to detect the non-linear internal collapse of the material under extreme stress [7]. This can lead to a calculated "safety margin" that is physically unrealistic. Conversely, the material science domain effectively characterizes this internal stability, but predominantly under static, uniform field conditions [5], [8]. This approach overlooks the complex, multi-constraint, and dynamic magnetic condition present in high-speed traction motor. Consequently, material-centric models often lack the context of dynamic machine operation, while machine-centric models lack the fidelity of intrinsic material physics. This paper aims to bridge this mutual gap, integrating the rigorous physics of the material scientist with the practical, dynamic constraints of the machine designer to reveal and prevent the latent vulnerabilities in modern PMSM design.

This paper is organized into five sections to methodically bridge these two fields. Section II describes the fundamental constitutive relationships linking normal and intrinsic hysteresis behaviors, which establishes the physical basis for the unified model. Section III investigates the demagnetization region, specifically evaluating the intrinsic knee point (H_k) as a temperature-dependent stability threshold. In Section IV-A, a non-linear Norton equivalent magnetic circuit is designed to explicitly model the interaction between air-gap permeance and the magnet's internal operating point to calculate flux linkage. Subsequently, Section IV-B explains the demagnetization Safe Operating Area, while Subsections IV-C and IV-D vali-

979-8-3315-8138-1

date the proposed framework through a comprehensive thermal sensitivity analysis, demonstrating the efficacy of the dynamic safety metric in predicting irreversible demagnetization under extreme operating conditions. Finally, Section V summarizes the key contributions and concludes the research.

II. Constitutive Analysis of Hysteresis and Thermal Instability in Permanent Magnets

The operational reliability of PMSMs is mainly determined by the correlation between the macroscopic flux required for torque production in electric machine design and the macroscopic domain stability of the magnet defined by material science to resist demagnetization. In soft magnetic materials, the distinction between intrinsic and normal behavior is often negligible; however, high-performance PMSMs using Nd–Fe–B magnets exhibit strong dipole alignment that necessitates a decoupled analysis of the normal (B–H) and intrinsic (J–H) hysteresis loops. This section discusses the core associations between these two characteristics and analyzes the physical factors that contribute to irreversible degradation [8].

A. The Divergence of Normal and Intrinsic Perspectives

In materials science, the magnetic state of a permanent magnet is strictly defined by its magnetic polarization, J, which represents the volume-averaged vector sum of atomic magnetic dipole moments. As shown in Fig. 1, the magnetic domain alignment evolves from a randomly oriented domain structure in the unmagnetized state (point 'a') to a highly aligned configuration under an applied external magnetizing field, H (point 'b').

This intrinsic polarization is the physical source of the magnet's strength. However, in electric machine design and finite element analysis (FEA), the magnetic behavior of the permanent magnet is primarily characterized by the magnetic flux density, B, which governs the air-gap flux and induces electromotive force (EMF) in the stator windings, with $B = \nabla \times A$. The scientific relationship between the material-centric intrinsic curve (J–H) and the engineering-focused normal curve (B–H) shown in Fig. 1 is determined by the fundamental constitutive equation [6]:

$$B = J + \mu_0 H \tag{1}$$

where μ_0 is the permeability of free space ($4\pi \times 10^{-7}$ H/m). Since B intrinsically couples the externally applied magnetic field with material magnetization, operating condition of the magnet and their associated thermal impacts affect normal and intrinsic demagnetization responses through distinct physical mechanisms, as illustrated in Fig. 1.

The intrinsic (J–H) curve determines the inner integrity of the magnetic domains. In modern sintered Nd–Fe–B grades, this curve exhibits high squareness, that is maintaining a near-constant polarization deep into the second quadrant. This behavior indicates that the magnetic domains remain stable and resistant to reversal under significant opposing fields. Conversely, the normal (B–H) curve represents the net flux density available to the external magnetic circuit. Due to the opposing element $\mu_0 H$ in the second quadrant, the B–H curve is sheared downwards [5] relative to the J–H curve, as shown in Fig. 1.

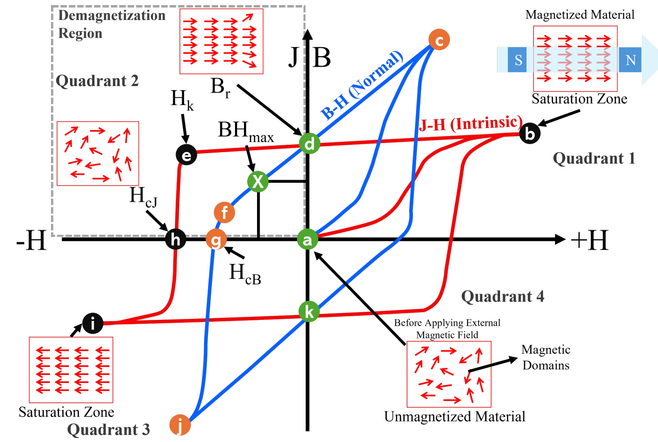

Fig. 1. Comparison of B–H and J–H hysteresis loops of a ferromagnetic material.

A major methodological issue in standard PMSM design is the dependency on the B–H curve for safety assessment. As illustrated in Fig. 1, at the point of normal coercivity (H_{cB}), the net flux density drops to zero, which can be misinterpreted as complete demagnetization. On the other hand, the intrinsic J–H curve reveals that at H_{cB}, the material retains a significant positive polarization ($J > 0$), signifying that the internal magnetic domains remain largely aligned and unreversed. True irreversible demagnetization occurs only at the intrinsic coercivity (H_{cJ}). At this point, the polarization, J, drops to zero, indicating permanent domain destabilization and reversal. Therefore, while the B–H curve is sufficient for determining steady-state performance (torque), the reliability of the machine against irreversible fault conditions is strictly influenced by the J–H curve and its proximity to the intrinsic knee point.

B. Thermal Degradation and Intrinsic Stability Threshold

The intrinsic demagnetization resistance of sintered Nd–Fe–B magnets is limited by thermal degradation. Particularly, the temperature dependence of the magnetocrystalline anisotropy field (H_A) [7] plays a central role in determining intrinsic coercivity (H_{cJ}) and, consequently, the stability margin against irreversible demagnetization. In contrast to soft magnetic materials, hard magnets exhibit a non-linear degradation of stability as thermal agitation reduces the energy threshold required for domain wall deformation. This degradation is commonly characterized through two reversible temperature coefficients: α for remanence (B_r) and β for intrinsic coercivity (H_{cJ}).

To quantify this behavior, the instantaneous magnetic properties at operating temperature T are calculated relative to a reference temperature T_0 (typically 20 °C) using the following linear approximations [9]:

$$B_r(T) = B_r(T_0) \cdot [1 + \alpha(T - T_0)] \tag{2}$$

$$H_{cJ}(T) = H_{cJ}(T_0) \cdot [1 + \beta(T - T_0)] \tag{3}$$

979-8-3315-8138-1

In commercial Nd–Fe–B grades, the coercivity coefficient β (approximately $-0.60\%/°\text{C}$ [9]) is significantly larger in magnitude than the remanence coefficient α (typically $-0.11\%/°\text{C}$ [9]). This parametric discrepancy creates a critical blind spot between material science and machine design. As rotor temperature rises, torque capability that is primarily governed by α degrades only marginally, while the intrinsic stability margin that is governed by β may collapse rapidly and remain undetected in conventional B–H based assessments.

The transition to irreversible demagnetization occurs when the externally applied magnetic field, acting in conjunction with thermal degradation, drives the magnet's operating point below the intrinsic knee point (H_k). Consequently, accurate failure prediction requires a unified modeling framework that explicitly couples the thermal state of the magnet with its magnetic operating trajectory.

III. Analytical Characterization of Demagnetization Thresholds

A robust PMSM design requires a precise stability limit. In PMSM design, normal coercivity often utilize for demagnetization safety assessment, but this is analytically insufficient, as it merely represents the zero-flux condition ($B = 0$) rather than irreversible domain reversal defined by material science. To address this, this section establishes the intrinsic knee point (H_k) as the definitive engineering constraint.

A. The Intrinsic Knee Point (H_k) as a Failure Criterion

Ideally, the polarization J would remain constant at B_r until the external field H reached the intrinsic coercivity H_{cJ}. In practice, microstructural defects cause the curve to deviate earlier at the knee point (H_k). This critical threshold is defined where polarization drops to 90% of remanence [10]

$$J(H_k) = 0.90 \cdot B_r \tag{4}$$

To integrate this into the unified model, H_k is linked to the temperature-dependent intrinsic coercivity (H_{cJ}) defined in Section II, through the squareness factor (SQ) [10]:

$$\text{SQ} = \frac{H_k}{H_{cJ}} \quad \Rightarrow \quad H_k(T) = \text{SQ} \cdot H_{cJ}(T) \tag{5}$$

Equation (5) provides the critical link between the material physics and the machine model, as it translates the thermal degradation of H_{cJ} defined in Equation (3) into a dynamic stability threshold $H_k(T)$. This threshold acts as the hard constraint in the Norton equivalent magnetic circuit detailed in Section IV, defining two distinct states:

1) *Safe Region* ($H < H_k$): Domain displacement is reversible. The demagnetizing field magnitude is below the threshold. Upon load removal, the magnet recoils along the major loop to restore full B_r.
2) *Failure Region* ($H > H_k$): The demagnetizing field magnitude exceeds the intrinsic knee H_k, overcoming domain pinning. This leads to plastic deformation of the magnetic state and permanent loss of flux density.

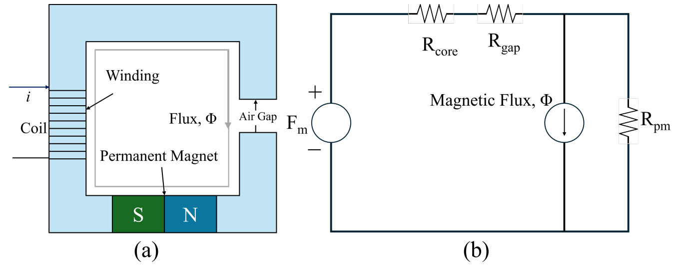

Fig. 2. (a) Physical magnetic circuit illustrating the permanent magnet, air gap, and winding. (b) Equivalent magnetic circuit model with MMF source (F_m), series reluctances of the core (R_{core}) and air gap (R_{gap}), and parallel reluctance of the magnet (R_{pm}).

B. Modeling Recoil Permeability and Irreversible Loss

When a magnet is driven beyond the intrinsic knee point (H_k), material science's definition of reversible domain wall motion is violated. Instead of returning along the original saturation curve,the operating point follows a minor recoil loop that is governed by a recoil permeability (μ_{rec}) [4]. This results in a permanent degradation of the magnetic state, quantified by a reduced remanence B'_r:

$$B'_r = B_r - \Delta B_{\text{irr}} \tag{6}$$

where ΔB_{irr} represents the irreversible loss of magnetic flux density.

In the context of machine design, this irreversible degradation dictates a significant adjustment to the magnetic circuit model. If the instantaneous operating point (H_m) evaluated in Section IV exceeds the intrinsic stability threshold (H_k), the Norton flux source (ϕ_r) should be irreversibly reduced to reflect the new B'_r. Therefore, accurate PMSM modeling requires treating the magnet not as a static component, but as a history-dependent source where the material limit (H_k) continuously constrains the machine performance (ϕ_r).

IV. Results, Analysis, and Practical Implications

To rigorously quantify PMSM operational boundaries, the physical distinction between flux generation in machine design and intrinsic coercivity governed by material science should be mathematically decoupled. Accordingly, this section integrates the thermal degradation mechanisms analyzed in Section II with the stability thresholds established in Section III into a unified Norton equivalent framework. This topology facilitates the simultaneous evaluation of the electromechanical torque capability and the safety margin against irreversible demagnetization during deep flux-weakening and fault operating conditions.

A. Analytical Derivation of the Non-Linear Norton Equivalent Circuit

To evaluate the risk of irreversible demagnetization, the physical PMSM structure in Fig. 2(a) is discretized into a lumped-parameter Equivalent Magnetic Circuit (EMC), as shown in Fig. 2(b). This study employs a Norton equivalent topology, where the permanent magnet is modeled as a constant flux source (ϕ_r) in parallel with an internal reluctance

979-8-3315-8138-1

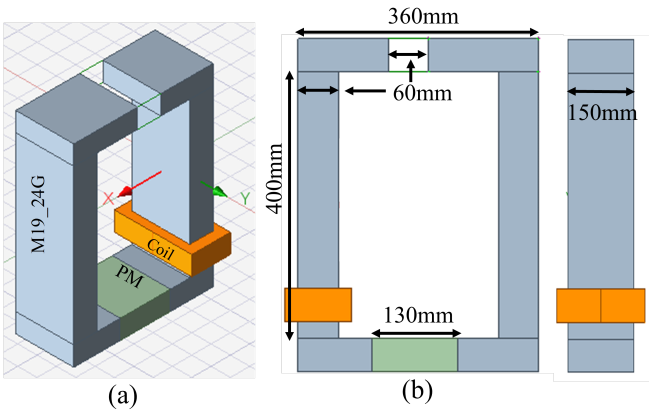

Fig. 3. FEA model configuration: (a) 3D isometric view of the magnetic test circuit, and (b) 2D cross-sectional dimensions (in mm).

(R_{pm}). This formulation is particularly well suited for demagnetization analysis as it explicitly preserves the parallel nature of rotor leakage paths and also facilitating a direct nodal solution for the magnet's operating point (B_m, H_m).

The magnetic behavior of the system is governed by the conservation of flux at the magnet poles. The Norton flux source ϕ_r represents machine design capability, as well the theoretical remanent flux available for torque production and has been modified based on the thermal coefficient α derived in Equation (2):

$$\phi_r = B_r(T)A_m \tag{7}$$

where A_m is the pole cross-sectional area. The magnet's internal resistance to demagnetizing fields is characterized by the internal reluctance R_{pm}, derived from the magnet geometry and its relative recoil permeability (μ_{rec}):

$$R_{pm} = \frac{l_m}{\mu_0 \mu_{\text{rec}} A_m} \tag{8}$$

The operating point of the magnet is determined by the magnetic potential difference ($\mathcal{F}_m$) across the source nodes. By applying Nodal Analysis to the circuit in Fig. 2(b) yields the governing flux balance relationship:

$$\phi_r = \phi_{pm} + \phi_{gap} + \phi_{leak} \tag{9}$$

$$B_r A_m = \mathcal{F}_m \left(\frac{1}{R_{pm}} + \frac{1}{R_{leak}} \right) + \frac{\mathcal{F}_m - F_{coil}}{R_{gap} + R_{core}} \tag{10}$$

where $R_{gap} = g'/(\mu_0 A_{gap})$ accounts for the effective air-gap reluctance including slotting effects, and F_{coil} represents the opposing demagnetizing MMF generated by the stator d-axis current. Solving this system for $\mathcal{F}_m$ yields the instantaneous magnetic field intensity

$$H_m = \frac{\mathcal{F}_m}{l_m}, \tag{11}$$

This calculated H_m serves as the bridge between the field of material science and machine operation. While Equation (10) determines the magnetic operating point associated with torque production, the demagnetization safety is verified by mapping H_m onto the intrinsic J–H curve to ensure it remains above the temperature-dependent intrinsic knee point ($H_k(T)$) defined in Equation (5).

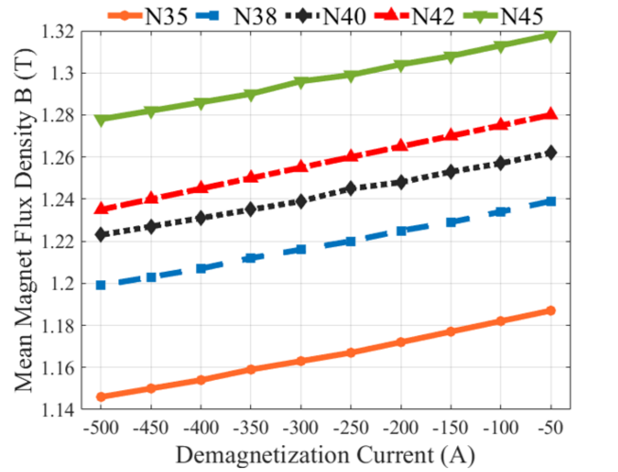

Fig. 4. Simulation results of the mean magnet flux density versus the applied demagnetizing current.

TABLE I
CHARACTERISTICS OF ND–FE–B MAGNET GRADES

PM Grade	Remanence B_r (T)	Coercivity H_c (MA/m)
N35H	1.21	0.911
N38H	1.26	0.943
N40H	1.27	0.951
N42H	1.30	0.979
N45H	1.35	1.01

B. Impact of Magnet Grade on the Demagnetization Safe Operating Area (SOA)

The selection of permanent magnet grade represents a direct coupling between machine design objectives and material science constraints. To quantify this sensitivity, five commercial Nd–Fe–B grades (N35–N45) were analyzed, with their key magnetic properties summarized in Table I. Within the Norton framework, upgrading the magnet grade directly enhances machine performance by increasing remanence (B_r). This linearly scales the Norton flux source ($\phi_r = B_r A_m$), resulting in higher air-gap flux density and proportional torque capability.

Beyond performance enhancement, the magnet grade also defines the material stability limit. Higher-grade magnets exhibit increased intrinsic coercivity (H_{cJ}), which shifts the intrinsic knee point (H_k) toward higher negative field intensities. This shift expands the Safe Operating Area (SOA), enabling the machine to sustain significantly higher negative d-axis currents (i_d) during flux-weakening without crossing the irreversible threshold. In contrast, lower-grade magnets such as N35 constrict the SOA, forcing the machine designer to limit high-speed power capability to ensure the instantaneous operating point (H_m) remains within safe material limits.

979-8-3315-8138-1

C. Simulation-Based Evaluation of Demagnetization Susceptibility

To validate the analytical framework, a 3D Finite Element Analysis (FEA) was conducted in Ansys Maxwell using the geometry in Fig. 3. The model utilized high-resolution meshing and open boundary conditions to accurately map the flux distribution near critical edges. The simulation results demonstrated in Fig. 4 clearly illustrate the divergence between machine-level performance metrics and material-level stability constraints.

While higher-grade magnets provide a linear increase in baseline air-gap flux density and thus torque capability, their primary advantage is the non-linear expansion of the stability region. As shown in Fig. 4, lower grades (N35) suffer a sharp flux collapse, indicating that the instantaneous operating point (H_m) has breached the intrinsic knee (H_k). These results confirm that robust PMSM design requires explicit consideration of material selection to decouple torque capability from the risk of irreversible demagnetization.

D. Impact of Temperature on Permanent Magnet Performance

Operating temperature serves as the dominant physical coupling between machine design objectives and material science constraints in PMSMs. To quantify this sensitivity, temperature-dependent magnetic characteristics of N45-grade Nd–Fe–B were synthesized from commercial specifications [9] across a range of $20°C$ to $120°C$, as shown in Fig. 5. This dataset establishes the temperature-dependent baselines required for evaluating demagnetization behavior within the Norton equivalent framework.

The analysis reveals a critical divergence in thermal sensitivity. While the remanence (B_r) degrades moderately with temperature, governed by the remanence coefficient α, the intrinsic magnet stability characterized by the intrinsic coercivity (H_{cJ}) collapses rapidly due to the significantly larger magnitude of coefficient β. Moreover, increasing temperature reduces the magnetocrystalline anisotropy energy, thereby lowering the energy barrier for domain reversal and allowing substantially smaller opposing magnetic fields to suppress the intrinsic magnetization (J).

For the electric machine designer, this discrepancy dictates a shrinking Safe Operating Area (SOA). As illustrated in Fig. 5, the intrinsic knee point (H_k) to shift aggressively toward lower field magnitudes with increasing temperature, even though the normal coercivity (H_{cB}) appears relatively stable. Consequently, a demagnetizing current (i_d) that is perfectly safe at room temperature may drastically overshoot the thermally contracted H_k at $120°C$, triggering immediate, irreversible flux loss.

V. Conclusion

This paper bridges the interdisciplinary gap between machine design and material science by decoupling the roles of normal ($B-H$) and intrinsic ($J-H$) hysteresis loops. While standard design practices correctly utilize the $B-H$ curve for torque production, this study demonstrates that such an

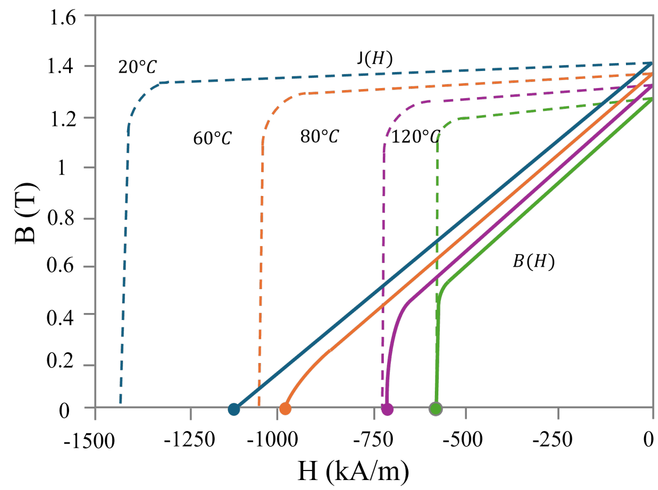

Fig. 5. Demagnetization curves (B–H and J–H) for an N45-grade Nd–Fe–B magnet at various operating temperatures.

approach alone obscures the true intrinsic stability limit of the magnetic material. Simulation results confirm that the intrinsic knee point (H_k) acts as the definitive failure threshold, particularly under thermal stress where the material constraint (H_{cJ}) degrades significantly faster than the performance-oriented metrics such as normal coercivity (H_{cB}). Consequently, ensuring robustness in high-performance traction systems requires a unified methodology that actively validates the machine's instantaneous operating point (H_m) against the thermally sensitive intrinsic Safe Operating Area (SOA).

Acknowledgment

The work is partially supported by NSF under grant number 2402634.

References

[1] J. M. D. Coey, "Permanent magnets: Plugging the gap," *Scripta Materialia*, vol. 67, no. 6, pp. 524–529, 2012.

[2] M. J. Melfi, S. D. Rogers, S. Evon, and B. Martin, "Permanent-magnet motors for energy savings in industrial applications," *IEEE Transactions on Industry Applications*, vol. 44, no. 5, pp. 1360–1366, 2008.

[3] M. Hodgdon, "Applications of a theory of ferromagnetic hysteresis," *IEEE Transactions on Magnetics*, vol. 24, no. 1, pp. 218–221, 1988.

[4] What is the magnetic hysteresis loop. Arnold Magnetic Technologies. Accessed: Nov. 3, 2025. [Online]. Available: https://www.arnoldmagnetics.com/wp-content/uploads/2017/10/FINAL_Tech-Library_Tech-Notes_Hysteresis-Loop.pdf

[5] S. Constantinides, "Manufacture of modern permanent magnet materials," Arnold Magnetic Technologies Corp., Rochester, NY, USA, Tech. Rep., 2014.

[6] D. Jiles, *Introduction to magnetism and magnetic materials*. CRC Press, 2015.

[7] J. R. Hendershot and T. J. E. Miller, *Design of brushless permanent-magnet motors*. Oxford university press, 1995.

[8] E. du Trémolet de Lacheisserie, D. Gignoux, and M. Schlenker, *Magnetism: Fundamentals*. Springer, 2003.

[9] Temperature effects on magnetic output. Arnold Magnetic Technologies. TechNote TN-0303, Accessed: Nov. 3, 2025. [Online]. Available: https://www.arnoldmagnetics.com/wp-content/uploads/2017/10/TN_0303_rev_150715.pdf

[10] Hk: A key magnetic figure of merit. Arnold Magnetic Technologies. Accessed: Nov. 3, 2025. [Online]. Available: https://www.arnoldmagnetics.com/wp-content/uploads/2017/10/Hk-A-Key-Magnetic-Figure-of-Merit-TN_0901_rev_160212.pdf