

LMP-Based Transmission and Battery Storage Co-Planning for Reducing Price Disparities

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Abstract—Rising regional energy price disparities, worsened by unequally distributed energy resource integration and frequent extreme weather, place significant burdens on disadvantaged communities. This paper introduces a co-planning approach for transmission capacity and battery energy storage systems (BESS) to address these disparities by incorporating locational marginal pricing (LMP) and price disparity metrics, including equality indices and the Gini coefficient, directly into transmission planning. Our primal-dual model optimizes LMP for equitable energy distribution and stabilizes prices using BESS to mitigate peak demand and congestion. Case studies on the IEEE 118-bus system show the model's effectiveness in promoting fair, accessible electricity pricing and aligning transmission planning with energy equity goals.

Index Terms—Transmission expansion planning, battery energy storage systems, locational marginal price, mathematical program with equilibrium constraints.

NOMENCLATURE

A. Sets

$\mathcal{N}$	Set of all buses
$\mathcal{N}_i$	Set of adjacent buses connected to bus i via a transmission line
$\mathcal{G}$	Set of all online generators
$\mathcal{G}_i$	Set of online generators at bus i
$\mathcal{L}$	Set of all lines
$\mathcal{T}$	Set of time intervals

B. Parameters

B_{ij}	Susceptance of line (i, j)
C_g	Marginal production cost of generator g
C_{ij}	Amortized investment cost of line (i, j)
C_i^P, C_i^E	Amortized investment for BESS power and energy capacity at bus i
D_{it}	Power demand at bus i in interval t
$F_{ij}^{\max}$	Thermal flow limit for line (i, j)
$P_g^{\max}$	Maximum generation capacity of generator g
X_i	Community characteristics at bus i for energy justice metrics
Δ	Allowable disparity threshold for energy justice metrics
α	Weighting factor for energy justice objective
η	Charging efficiency for BESS

C. Decision Variables

$f_{ij}^{\max}$	Transmission capacity for line (i, j)
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$p_i^{\max}$	BESS power capacity at bus i
$e_i^{\max}$	BESS energy capacity at bus i
e_{it}	State of charge (SoC) for BESS at bus i in interval t
p_{it}^+/p_{it}^-	Charge/Discharge power for BESS at bus i in interval t
p_{gt}	Active power output of generator g in interval t
θ_{it}	Voltage phase angle at bus i in interval t
λ_{it}	LMP at bus i in interval t
λ_i	Average LMP at bus i over all intervals
π_{ijt}^+/π_{ijt}^-	Dual variable for transmission upper/lower bound of line (i, j) in interval t
μ_{gt}	Dual variable for generation capacity limit of generator g in interval t

I. INTRODUCTION

Recently, rising disparities in energy prices across regions have placed additional financial strain on many families, leading to significant social concerns [1]. Addressing these price differences through transmission planning allows energy generated from more affordable resources to reach disadvantaged communities. Additionally, advancements in battery energy storage systems (BESS) have enhanced their accessibility and capabilities, offering the potential to stabilize energy prices and reduce peak-hour price surges. Consequently, the co-planning of transmission lines and BESS presents a promising approach to alleviate spatial and temporal market price disparities.

A. Energy Price and Regional Disparity

With the increasing yet uneven penetration of distributed energy resources and the more frequent occurrence of extreme weather events, electricity price disparities across power grids have become a major concern, as highlighted in the *National Transmission Needs Study* conducted by the US Department of Energy [2]. As shown in Fig. 1, many electricity hubs experienced an average hourly energy price difference exceeding \$20 within and across regions in 2022. For a 1,000 MW transmission link, such price disparities can amount to \$175 million annually.

These substantial price gaps impose significant financial burdens on households and deviate from the objective of energy justice. Addressing this issue is crucial for societal

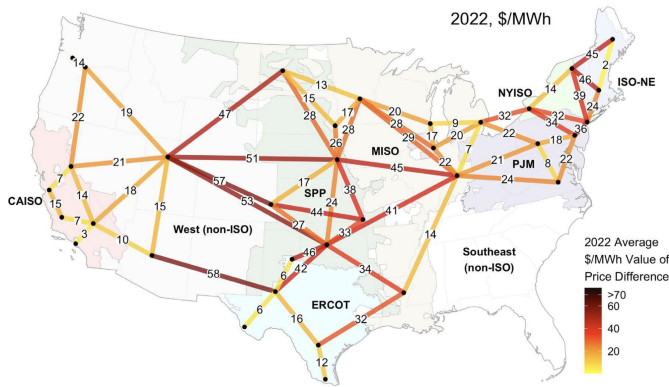


Fig. 1. Average absolute hourly energy price differences across selected links in 2022 [3]. The number on each line denotes the price difference between two hubs at either end of the line. Darker red lines represent higher price differences, while light yellow lines indicate relatively lower differences.

well-being and economic stability. The United Nations has set a goal of energy justice to provide affordable and modern energy to all people by 2030 [4]. Additionally, the Biden Administration's *Justice40 Initiative* seeks to direct 40% of the overall benefits in climate and clean energy to disadvantaged communities [5].

Energy prices in the power system are influenced by multiple factors, including generation costs, load demand, and transmission capacity. For example, when a transmission line reaches its capacity, congestion occurs, raising energy prices at the receiving end. While it is impractical to relocate low-cost power plants to urban centers or move customers out of their communities, it is possible to make energy more affordable and equitable through transmission capacity planning [6]. Such planning holds the potential to advance the long-term goal of energy justice.

B. Related Literature

The transmission planning problem addresses optimal strategies for expanding and reinforcing transmission networks to ensure reliable, efficient, and cost-effective power delivery over time [7]. This process begins when utilities and independent system operators (ISOs) identify new transmission needs within their regions [2]; federal agencies, such as Federal Energy Regulatory Commission (FERC), provide oversight and policy guidance [8]; research groups and laboratories study and propose planning models; while utility and transmission companies ultimately build the new transmission lines [9]. These additional lines alter network topology and increase transmission capacity.

Although network topology and transmission capacity influence energy prices, transmission planning processes typically do not prioritize energy price distribution. Instead, their primary focuses are reliability requirements, overall total cost effectiveness, load growth, and renewable integration [10], [11], [12].

A few attempts have been made to integrate energy prices into the transmission planning process. Researchers have ex-

amined interactions between ISOs and transmission companies by calculating locational marginal prices (LMPs) based on simulated investment plans and iteratively revising planning decisions until all parties are satisfied [6], [13]. Another study focuses on planning the transmission network under major congested scenarios, filtered by an LMP threshold, as higher congestion levels can result in elevated LMPs, highlighting the need for transmission expansion [14]. Additionally, a separate approach incorporates price-responsive demand, where load demand is a decision that depends on the LMP [15]. Here, LMPs are determined using the Karush–Kuhn–Tucker (KKT) conditions of an energy market equilibrium model. Since the KKT conditions are nonlinear, the researchers employed a successive linear program to solve the problem on a 33-country (node) network, though convergence and global optimality were not guaranteed.

To further address price disparities and congestion identified through transmission planning, incorporating BESS has emerged as a complementary solution. BESS co-planning can stabilize energy prices and manage congestion by leveraging benefits like fast response times, high ramp rates, and geographical independence [16]. By charging BESS when prices are low and discharging when prices are high, BESS planning enables temporal demand shifts [17]. Another approach uses BESS by transporting charged BESS units from low-LMP regions to high-LMP regions [18]. Integrating both stationary and mobile BESS into transmission planning can enhance renewable energy utilization and alleviate transmission congestion. Moreover, issues such as the relatively short life cycle of BESS [19] and revenue competition between BESS and transmission companies [13] have also been explored in the context of co-planning challenges.

Furthermore, recent research on energy justice in transmission planning has primarily focused on policy-level analysis. These studies seek to assess energy justice issues—including energy price disparities—within current transmission infrastructures in various regions, such as in the United States [20] and Colombia [21]. They provide analytical frameworks and qualitative policy recommendations to guide future transmission planning efforts in addressing these issues [22].

C. Challenge and Our Contributions

While existing studies highlight energy justice concerns related to price differentials and recognize the link between energy prices and transmission planning, reaching energy justice through transmission planning faces two main challenges. First, because LMP is derived as the dual value in the economic dispatch problem, optimizing LMP simultaneously with the transmission planning problem is complex. Second, there has been limited exploration of quantitative energy justice metrics, such as the Gini coefficient and general entropy indices, within the whole optimization framework [23].

To address these gaps in the literature on transmission planning through the lens of energy justice, we propose a primal-dual formulation for the co-planning of transmission

capacity and BESS, integrating both LMP and energy justice metrics. The main contributions are summarized as:

1. Develop a primal-dual model to directly optimize LMP within the transmission planning process.
2. Incorporate various quantitative energy justice metrics into the proposed model to foster more equitable LMP distributions.
3. Integrate BESS planning to stabilize LMP and enhance energy justice for disadvantaged communities.
4. Conduct case studies on co-planning with the energy justice model using the IEEE 118-bus system, demonstrating significant improvements in energy justice outcomes.

The remaining part of the paper is organized as follows. Section II illustrates the effects of transmission planning on LMP through illustrative examples and proposes a primal-dual formulation to calculate LMP directly within the transmission planning process. This section also integrates energy justice metrics to quantify and optimize the LMP for different communities in the planning process. Section III presents the additional constraints for co-planning BESS with transmission expansion while upholding energy justice requirements. Section IV presents case study results that validate the effectiveness of the proposed models. Finally, Section V concludes our study.

II. LMP-BASED TRANSMISSION PLANNING

LMPs, derived from dual variables for an economic dispatch problem as shown below, reflect the most cost-effective generator output schedule to satisfy load demands within transmission and generation capacities:

$$\begin{aligned}
\min \quad & \sum_{g \in \mathcal{G}} \sum_{t \in \mathcal{T}} C_g p_{gt} \quad (1a) \\
\text{s.t. } \lambda_{it} : \quad & \sum_{g \in \mathcal{G}_i} p_{gt} - \sum_{j \in \mathcal{N}_i} B_{ij}(\theta_{it} - \theta_{jt}) = D_{it}, \\
& \forall i \in \mathcal{N}, t \in \mathcal{T}, \quad (1b) \\
\pi_{ijt}^-, \pi_{ijt}^+ : \quad & -F_{ij}^{\max} \leq B_{ij}(\theta_{it} - \theta_{jt}) \leq F_{ij}^{\max}, \\
& \forall (i, j) \in \mathcal{L}, t \in \mathcal{T}, \quad (1c) \\
\mu_{gt} : \quad & 0 \leq p_{gt} \leq P_g^{\max}, \forall g \in \mathcal{G}, t \in \mathcal{T}, \quad (1d) \\
& \theta_{\text{ref}, t} = 0, \forall t \in \mathcal{T}. \quad (1e)
\end{aligned}$$

The objective function (1a) minimizes total generation costs. For clarity, the dual variables are shown next to each constraint: constraint (1b) enforces power balance for each bus where the power flow is calculated using phase angle formulation, constraint (1c) limits line flows to their respective capacities, constraint (1d) sets maximum generation output, and constraint (1e) set phase angle of the reference bus to 0.

Upon solving (1), the LMP at bus i in interval t is given by the value of λ_{it} . When a transmission constraint binds (i.e., the flow reaches capacity), the congestion component of the LMP becomes non-zero, raising the LMP. Conversely, with sufficient transmission capacity, the congestion component reduces to zero, thus lowering LMP.

A. Transmission Planning Model

Different from the economic dispatch problem, transmission planning considers generation schedules over the long term, factoring in the construction costs of transmission lines while meeting stability and reliability requirements.

Since line capacities significantly impact LMPs, particularly when the locations of power plants and consumer communities are hard to move places, we optimize thermal flow limit $f_{ij}^{\max}$ in existing transmission corridors to minimize both construction costs and long-term operational costs, as expressed in the following transmission planning model:

$$\begin{aligned}
\min \quad & \sum_{(i,j) \in \mathcal{L}} C_{ij} f_{ij}^{\max} + \sum_{g \in \mathcal{G}} \sum_{t \in \mathcal{T}} C_g p_{gt} \quad (2a) \\
\text{s.t. } \quad & (1b), (1d) - (1e), \quad (2b) \\
& -f_{ij}^{\max} \leq B_{ij}(\theta_{it} - \theta_{jt}) \leq f_{ij}^{\max}, \\
& \forall (i, j) \in \mathcal{L}, t \in \mathcal{T}. \quad (2c)
\end{aligned}$$

Here, the objective function (2a) minimizes total system cost over the representative operational horizon. The first term represents the amortized investment cost for new transmission capacity, while the second term captures the actual generation costs incurred during that same time frame. Constraint (2c) specifies variable capacity for each transmission line to determine the optimal planning strategy. By applying a high-level DC planning method, we assume fixed line impedance while changing thermal limits [24].

B. Primal-Dual Formulation

Since the operational problem (1) is a linear program, strong duality holds [25], implying that the optimal primal and dual objective values are equal. This allows us to replace the lower-level optimization with its optimality conditions, i.e., a strong duality equality constraint. This transforms the bilevel planning problem into a mathematical program with equilibrium constraints (MPEC) [26], [27].

Through this formulation, the dual variables (LMPs) become decision variables accessible to the upper-level problem. This allows us to explicitly restrict or optimize LMPs to reduce price disparities. The unified primal-dual transmission planning formulation is as follows:

$$\begin{aligned}
\min \quad & (2a) \quad (3a) \\
\text{s.t. } \quad & (2b) - (2c), \quad (3b) \\
p_{gt} : \quad & \lambda_{i(g), t} - \mu_{gt} \leq C_g, \forall g \in \mathcal{G}, t \in \mathcal{T}, \quad (3c) \\
\theta_{it} : \quad & \sum_{j \in \mathcal{N}_i} B_{ij}(\lambda_{it} - \lambda_{jt}) + \sum_{j: (i,j) \in \mathcal{L}} B_{ij}(\pi_{ijt}^+ - \pi_{ijt}^-) - \\
& \sum_{k: (k,i) \in \mathcal{L}} B_{ki}(\pi_{kit}^+ - \pi_{kit}^-) = 0, \forall i \neq \text{ref}, t \in \mathcal{T}, \quad (3d) \\
\mu_{gt}, \pi_{ijt}^+, \pi_{ijt}^- : \quad & \mu_{gt}, \pi_{ijt}^+, \pi_{ijt}^- \geq 0, \lambda_{it} \text{ free}, \quad (3e) \\
& \sum_{t \in \mathcal{T}} \sum_{g \in \mathcal{G}} C_g p_{gt} = \sum_{t \in \mathcal{T}} \left(\sum_{i \in \mathcal{N}} D_{it} \lambda_{it} - \right. \\
& \left. \sum_{(i,j) \in \mathcal{L}} f_{ij}^{\max} (\pi_{ijt}^+ + \pi_{ijt}^-) - \sum_{g \in \mathcal{G}} P_g^{\max} \mu_{gt} \right). \quad (3f)
\end{aligned}$$

Here, the objective (3a) remains the same as in the planning problem. Constraints (3b) enforce primal feasibility, while (3c)-(3e) enforce dual feasibility. Constraint (3f) imposes strong duality by equating the primal and dual objectives. This MPEC framework not only integrates LMPs into transmission planning but also facilitates the direct optimization of energy justice metrics.

C. Energy Justice Metrics

With LMPs exposed as decision variables, we can apply various energy justice metrics based on price and community characteristics. In this subsection, we present four widely-used formulas for quantifying energy justice, utilizing the following generalized framework derived from model (3):

$$\min \sum_{(i,j) \in \mathcal{L}} C_{ij} f_{ij}^{\max} + \sum_{t \in \mathcal{T}} \sum_{g \in \mathcal{G}} C_g p_{gt} + \alpha G(\lambda) \quad (4a)$$

$$\text{s.t.} \quad (3b) - (3f), \quad (4b)$$

$$H(\lambda) \leq 0, \quad (4c)$$

where α adjusts the weight of the energy justice term in the objective, and $G(\lambda)$ and $H(\lambda)$ represent optional functions of the LMP variable that capture equity goals. Since energy justice requirements can be modeled as either soft penalties or as hard constraints, we include both forms for flexibility. Below, we demonstrate four example applications.

1) *Equality*: restricts price spreads to a specified bandwidth Δ via $H_{ij}(\lambda) = |\lambda_i - \lambda_j| - \Delta$, where time-averaged LMP at location i is represented by $\lambda_i = 1/|\mathcal{T}| \sum_{t \in \mathcal{T}} \lambda_{it}$.

2) *X-Equality*: scales prices by community attributes X_i (e.g., income level, race composition, gid accessibility), defined as $H_{ij}(\lambda) = |\lambda_i/X_i - \lambda_j/X_j| - \Delta$, where X_i represents an energy justice feature for the community at bus i .

3) *Gini coefficient*: A widely used metric for assessing energy justice is the Gini coefficient [23], which ranges from 0 (perfect equality) to 1 (maximum inequality). To incorporate the Gini coefficient, we set:

$$G(\lambda) = \frac{\sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{N}} |\lambda_i - \lambda_j|}{2|\mathcal{N}| \sum_{k \in \mathcal{N}} \lambda_k}. \quad (5)$$

This nonlinear term in the objective function can be linearized if needed.

4) *Minimax*: caps the highest LMP by setting $H_{it}(\lambda) = \lambda_{it} - d$ and minimizing d .

In addition to these LMP-based metrics, other energy justice measures, such as *energy burden* (defined as the LMP divided by the average household income for a community), can also be adopted into this framework (4).

III. CO-PLANNING WITH BESS

To incorporate BESS into the transmission planning process, we introduce additional variables and constraints to model (2). The problem for this co-planning problem is as follows:

$$\min \sum_{(i,j) \in \mathcal{L}} C_{ij} f_{ij}^{\max} + \sum_{i \in \mathcal{N}} (C_i^P p_i^{\max} + C_i^E e_i^{\max}) +$$

$$\sum_{g \in \mathcal{G}} \sum_{t \in \mathcal{T}} C_g p_{gt} \quad (6a)$$

$$\text{s.t.} \quad (1d) - (1e), (2c), \quad (6b)$$

$$\sum_{g \in \mathcal{G}_i} p_{gt} + p_{it}^- - p_{it}^+ - \sum_{j \in \mathcal{N}_i} B_{ij}(\theta_{it} - \theta_{jt}) = D_{it}, \quad \forall i \in \mathcal{N}, t \in \mathcal{T}, \quad (6c)$$

$$e_{it} = e_{i,t-1} + \eta p_{it}^+ - p_{it}^- / \eta, \forall i \in \mathcal{N}, t \in \mathcal{T}, \quad (6d)$$

$$e_{i,0} = e_{i,|\mathcal{T}|} = 0, \forall i \in \mathcal{N}, \quad (6e)$$

$$0 \leq p_{it}^+, p_{it}^- \leq P_i^B, 0 \leq e_{it} \leq E_i^B, \forall i \in \mathcal{N}, t \in \mathcal{T}. \quad (6f)$$

In this model, the amortized construction costs for BESS are added into the objective function (6a). Constraint (6c) ensures power balance; constraints (6d)-(6f) model the inventory level (SoC) dynamics of the BESS.

This co-planning model integrates BESS into the transmission system in a manner consistent with the LMP-based framework described in Section II, enabling a unified approach to optimize both transmission and battery storage resources based on LMPs.

IV. CASE STUDY

We validate the proposed framework on the IEEE 24-bus reliability test system from PG-Lib library (v23.07) [28]. Optimization models were solved using Gurobi 12.0.3 on macOS 26.2 with an Apple M1 Pro chip and 16GB RAM.

The system consists of 24 buses, 33 generators, and 38 transmission lines. Transmission expansion costs are estimated based on line reactance (proxy for distance) at \$1.5M/mile over a 40-year lifespan. BESS capital costs are set to \$300k/MW for power and \$250k/MWh for energy capacity with a 15-year lifespan. The load profile consists of four representative hours weighted to model a 24-hour horizon, capturing the temporal arbitrage value of storage.

We evaluate three planning scenarios to demonstrate the trade-offs between cost and price disparity:

- S1: Cost-minimization transmission planning based on model (2). No equity constraints are applied.
- S2: Transmission planning with a equality constraint based on model (4), set to 50% of the spread observed in S1.
- S3: Joint transmission and BESS planning with the same LMP spread constraint as S2 based on model (6).

Table I summarizes the economic and equity outcomes. In S1, the model builds a minimal transmission capacity required to serve the load, resulting in significant congestion and a high LMP spread of \$13.19/MWh. In S2, enforcing the equity constraint requires \$25,853 more in investment in transmission lines, increasing the total cost to \$1.37M. In S3, BESS provides a more cost-effective mechanism to equalize prices. It creates temporal arbitrage that smooths congestion without requiring as many expensive line builds, as shown in Fig. 2. Consequently, S3 achieves the lowest Gini coefficient while reducing the total system cost to \$1.26M, demonstrating that storage investment effectively offsets operational inefficiencies.

TABLE I
COMPARISON OF COSTS AND EQUITY METRICS

Metric	S1	S2	S3
Total Cost (M\$)	1.35	1.37	1.26
Line Investment (\$)	698624.53	724477.90	636261.68
BESS Investment (\$)	0.00	0.00	53984.84
Operational Cost (\$)	656152.15	647591.84	572752.41
Gini Coefficient	0.06	0.04	0.01
Max LMP Diff (\$/MWh)	13.19	6.60	3.05
Solution time (s)	0.10	2.75	0.20

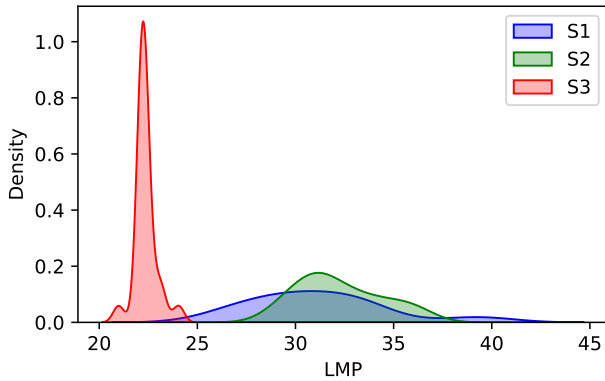


Fig. 2. Distribution of time-averaged LMPs across 24 buses for each scenario.

V. CONCLUSION

This paper developed an LMP-based transmission planning model with integrated energy justice metrics and co-planning for battery storage. Our framework promotes efficient and equitable energy access, with validation showing reduced price disparities and enhanced fairness. Future research could apply this model to larger grids and varying policy needs.

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