Time-Variant Problem of Distribution System Security Region: Phenomena, Reason and Solution

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***Abstract*—Distribution system security region (DSSR) has time-variant characteristics after including flexible resources (FRs). This paper conducts an in-depth study on the time-variance of DSSR, pointing out and addressing the problem it causes for the first time. Firstly, the reason for the time-variance is analyzed: FR regulations are reflected in the region varying. The problem caused by the time-variance of DSSR is analyzed, namely that the region-based method will lose its speed advantage in online security analysis. Secondly, a time-invariant DSSR model and its corresponding security analysis method are proposed. The key to addressing the time-variance of DSSR is to reflect FR regulations as the operating point movement. Finally, the proposed model and method are verified in a real case. The comparison shows that the proposed model maintains the speed advantage of the region-based method in security analysis. This work lays a key foundation for practical applications of DSSR in smart distribution networks.**

***Keywords*—distribution network, security region, flexible resources, time-variance, time-invariant model.**

I. INTRODUCTION

Distribution system security region (DSSR) is defined as the closed set of all the N-1 secure operating points in the state space of a distribution system [1]. As a kind of region-based method, the DSSR method has a significant speed advantage in online security analysis, compared with the point-by-point simulation method [2]. DSSR without flexible resource (FR) regulations is not affected by system operating state, so it can be determined as long as the network topology and component parameters are defined [2]. Hence, DSSR can be calculated offline beforehand [3]. The online security analysis only needs to determine the distances from operating points to the region boundary [4]. Then, the security conclusion can be obtained. Therefore, DSSR greatly improves security analysis speed [2].

The distribution system security is affected by the integration of FRs such as distributed generation (DG), energy storage system (ESS), and demand response (DR). The latest DSSR studies have accounted for FRs. Ref. [5] developed a static voltage security region with DG under the normal operation (N-0) security. Ref. [6] developed a steady-state security region with flexible load under the N-0 security. But these studies didn't consider real-time FR operating states. Ref. [7] proposed the DSSR model accounting for ESS. Ref. [8] proposed the DSSR model accounting for DG and DR. Ref. [9] proposes the DSSR model accounting for soft open point (SOP). These studies all mention that the DSSR with FRs is affected by FR operating states. Ref. [7] pointed out for the first time that the DSSR with FRs becomes time-variant, whose security boundaries will move with time. Besides, note that although loads and DGs also exhibit temporal correlation, they are reflected as operating point movements and don't cause DSSR time-variance [7].

The time-variance is a new phenomenon in DSSR research when FRs are involved. No literature has discussed the problem that the time-variance of DSSR may cause. This paper finds that the time-variant DSSR undermines the DSSR method's speed advantage in online security analysis. This is because the time-variance of DSSR implies that DSSR cannot be calculated offline beforehand, but must be calculated online in real time at each security analysis. Calculating the security region generally takes the longest time in the whole security analysis calculation. As a result, the time for online security analysis will increase significantly. Currently, no study clearly points out the problem or gives an effective solution.

This paper provides an in-depth study on the time-variance of DSSR and addresses the problem it causes. The main contributions of this paper are as follows:

1) The problem caused by the time-variance of DSSR is pointed out for the first time, namely that the region-based method will lose its speed advantage in security analysis.

2) The reason for the time-variance of DSSR after accounting for FRs is revealed. The reason is that the time-variant DSSR considers FR regulations in DSSR modeling, and thus FR regulations are reflected in the region varying.

3) The time-invariant model of DSSR and its security analysis method are proposed. The key is to reflect FR regulations as the operating point movement.

The proposed time-invariant DSSR model is verified by the urban distribution system part of the HPDS133-bus test system [10]. The proposed model can maintain the speed advantage of the region-based method with correct results.

II. TIME-VARIANT MODEL OF DSSR

The region-based security analysis method involves 2 steps. Step 1 is region modeling and calculation. Step 2 is using the region to analyze the operating point security. The existing studies consider FR regulations in Step 1 [7]. The FR regulation refers to FRs performing power regulations conducive to the system secure operation, following the distribution system operator's instructions or contracts [11].

A. Operating point only including customer demands

To analyze the reason for the time-variance of DSSR, this paper classifies nodes from the perspective of their functions in distribution network regulation, as follows.

1) Demand nodes are the nodes with pure customer demands, such as the fixed load and intermittent DG. They only require the distribution network to provide load power supply or DG accommodation services without participating in regulation. The set of demand nodes is denoted as $\boldsymbol{ND}$. The total number is denoted as n_D.

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2) Regulating nodes are the nodes with pure regulating resources, such as the public ESS whose property rights belong to the electric utility. They only participate in system regulation without customer demands. The set of regulating nodes is denoted as $\boldsymbol{NR}$. The total number is denoted as n_{R}.

3) Hybrid nodes are the nodes with both customer demands and regulating resources, such as the DR load and the ESS belonging to customers. The set of hybrid nodes is denoted as $\boldsymbol{NH}$. The total number is denoted as n_{H}.

The operating point of the time-variant DSSR (TV-DSSR) is defined as the net power vector of the demand nodes and hybrid nodes in the distribution system [7], as shown in Eq (1).

$$\boldsymbol{W}_{\mathrm{TV}}=[S_1,\ldots,S_i,\ldots,S_{n\mathrm{D}+n\mathrm{H}}],\ i\in \boldsymbol{ND}\cup\boldsymbol{NH} \tag{1}$$

where $\boldsymbol{W}_{\mathrm{TV}}$ is the operating point of TV-DSSR; i is node index; S_i is the net power of node i, and $S_i>0$ if node i consumes power and $S_i<0$ if node i injects power to the network.

As seen in Eq (1), $\boldsymbol{W}_{\mathrm{TV}}$ only includes customer demands, namely, demand nodes and hybrid nodes. The purpose is to characterize the security region of customer demands, while regulating resources are considered as a factor affecting the region. This is reasonable because the distribution system is to provide load power supply and DG accommodation services under secure conditions, that is, to meet customer demands. However, this operating point modeling leaves a hidden danger leading to the time-variance of DSSR.

B. Powers related to FR regulations

Hybrid nodes have FR regulations, such as reducing load by DR. The feasible power regulation of hybrid node i at time t is denoted as $\Delta S_i(t)$, where $\Delta S_i>0$ if S_i increases, and $\Delta S_i<0$ if S_i decreases. Note that ΔS_i can be time-variant. For example, the load that DR can reduce is time-variant.

Regulating nodes have FR regulations, such as the public ESS node. The power extremum of regulating node i at time t after the node i FR regulations is denoted as $S_i^{\mathrm{lim}}(t)$. Note that S_i^{lim} can be time-variant. For example, the ESS discharging power extremum varies with the state of charge varying.

This paper uses these two abstract concepts instead of specific FR types, ensuring the generalizability in analyzing the time-variance of DSSR.

C. DSSR model & reason for time-variance

The TV-DSSR $\boldsymbol{\Omega}_{\text{TV-DSSR}}$ model considers FR regulations in the DSSR modeling[7].The model is shown in Eq (2).

$$
\begin{aligned}
&\boldsymbol{\Omega}_{\text{TV-DSSR}}(t)=\{\boldsymbol{W}_{\mathrm{TV}}=[S_1,\cdots,S_i,\cdots,S_{n\mathrm{D}+n\mathrm{H}}], i\in\boldsymbol{ND}\cup\boldsymbol{NH}\ |\\
&s.t.A\quad S_i^{\min}\le S_i\le S_i^{\max}\\
&s.t.B\begin{cases}
|S_{\mathrm{F}j}|=|\sum\limits_{i\in\Lambda_{\mathrm{F}j}\cap(\boldsymbol{ND}\cup\boldsymbol{NH})}S_i+\sum\limits_{i\in\Lambda_{\mathrm{F}j}\cap\boldsymbol{NH}}\Delta S_i(t)+\sum\limits_{i\in\Lambda_{\mathrm{F}j}\cap\boldsymbol{NR}}S_i^{\mathrm{lim}}(t)|\\
\le c_{\mathrm{F}j}\quad\forall j\in\boldsymbol{B}\\
|S_{\mathrm{T}j}|=|\sum\limits_{i\in\Lambda_{\mathrm{T}j}\cap(\boldsymbol{ND}\cup\boldsymbol{NH})}S_i+\sum\limits_{i\in\Lambda_{\mathrm{T}j}\cap\boldsymbol{NH}}\Delta S_i(t)+\sum\limits_{i\in\Lambda_{\mathrm{T}j}\cap\boldsymbol{NR}}S_i^{\mathrm{lim}}(t)|\\
\le c_{\mathrm{T}j}\quad\forall j\in\boldsymbol{T}\\
U_i^{\min}\le U_i\le U_i^{\max}\qquad\forall i\in\boldsymbol{N}\\
U_i=U_0-\dfrac{1}{U_{\mathrm{N}}}\sum\limits_{j\in\Pi i}\{R_j[\sum\limits_{k\in\Lambda_{\mathrm{F}j}\cap(\boldsymbol{ND}\cup\boldsymbol{NH})}S_k\cos\varphi_k+\\
\sum\limits_{i\in\Lambda_{\mathrm{F}j}\cap\boldsymbol{NH}}\Delta S_i(t)\cos\varphi_k+\sum\limits_{i\in\Lambda_{\mathrm{F}j}\cap\boldsymbol{NR}}S_i^{\mathrm{lim}}(t)\cos\varphi_k]+\\
X_j[\sum\limits_{k\in\Lambda_{\mathrm{F}j}\cap(\boldsymbol{ND}\cup\boldsymbol{NH})}S_k\sin\varphi_k+\sum\limits_{i\in\Lambda_{\mathrm{F}j}\cap\boldsymbol{NH}}\Delta S_i(t)\sin\varphi_k+\\
\sum\limits_{i\in\Lambda_{\mathrm{F}j}\cap\boldsymbol{NR}}S_i^{\mathrm{lim}}(t)\sin\varphi_k]\}
\end{cases}\\
&s.t.C\begin{cases}
\forall\psi_z\in\boldsymbol{\Psi}\\
|S_{\mathrm{F}j}(z)|=|\sum\limits_{i\in\Lambda_{\mathrm{F}j}(z)\cap(\boldsymbol{ND}\cup\boldsymbol{NH})}S_i+\sum\limits_{i\in\Lambda_{\mathrm{F}j}(z)\cap\boldsymbol{NH}}\Delta S_i(t)+\\
\sum\limits_{i\in\Lambda_{\mathrm{F}j}(z)\cap\boldsymbol{NR}}S_i^{\mathrm{lim}}(t)|\le c_{\mathrm{F}j}\quad\forall j\in\boldsymbol{B}\setminus\{z\}\\
|S_{\mathrm{T}j}(z)|=|\sum\limits_{i\in\Lambda_{\mathrm{T}j}(z)\cap(\boldsymbol{ND}\cup\boldsymbol{NH})}S_i+\sum\limits_{i\in\Lambda_{\mathrm{T}j}(z)\cap\boldsymbol{NH}}\Delta S_i(t)+\\
\sum\limits_{i\in\Lambda_{\mathrm{T}j}(z)\cap\boldsymbol{NR}}S_i^{\mathrm{lim}}(t)|\le c_{\mathrm{T}j}\quad\forall j\in\boldsymbol{T}\setminus\{z\}\\
U_i^{\min}\le U_i(z)\le U_i^{\max}\qquad\forall i\in\boldsymbol{N}\\
U_i(z)=U_0-\dfrac{1}{U_{\mathrm{N}}}\sum\limits_{j\in\Pi i(z)}\{R_j[\sum\limits_{k\in\Lambda_{\mathrm{F}j}(z)\cap(\boldsymbol{ND}\cup\boldsymbol{NH})}S_k\cos\varphi_k+\\
\sum\limits_{i\in\Lambda_{\mathrm{F}j}(z)\cap\boldsymbol{NH}}\Delta S_i(t)\cos\varphi_k+\sum\limits_{i\in\Lambda_{\mathrm{F}j}(z)\cap\boldsymbol{NR}}S_i^{\mathrm{lim}}(t)\cos\varphi_k]+\\
X_j[\sum\limits_{k\in\Lambda_{\mathrm{F}j}(z)\cap(\boldsymbol{ND}\cup\boldsymbol{NH})}S_k\sin\varphi_k+\sum\limits_{i\in\Lambda_{\mathrm{F}j}(z)\cap\boldsymbol{NH}}\Delta S_i(t)\sin\varphi_k+\\
\sum\limits_{i\in\Lambda_{\mathrm{F}j}(z)\cap\boldsymbol{NR}}S_i^{\mathrm{lim}}(t)\sin\varphi_k]\}
\end{cases}\\
&\}
\end{aligned}
\tag{2}
$$

where $s.t.A$ is the state space constraint, which determines all the possible operating points; $s.t.B$ is the N-0 security constraint; $s.t.C$ is the N-1 security constraint; $S_{\mathrm{F}j}$ is the power on feeder branch j; $c_{\mathrm{F}j}$ is the capacity of branch j; $\boldsymbol{B}$ is the set of all the feeder branches; $S_{\mathrm{T}j}$ is the power on substation transformer j; $\Lambda_{\mathrm{T}j}$ is the set of nodes downstream of transformer j; $c_{\mathrm{T}j}$ is the capacity of transformer j; $\boldsymbol{T}$ is the set of all the substation transformers; U_i is the voltage magnitude of node i; $U_i^{\min}$ and $U_i^{\max}$ are the minimum and maximum of U_i, respectively; $\boldsymbol{N}$ is the set of all the nodes; U_0 is the feeder outlet voltage; U_{N} is the rated voltage; Π_i is the set of branches upstream node i; R_j and X_j are the resistance and reactance of branch j, respectively; φ_k is the power factor angle of node k; ψ_z is the contingency resulting in the isolation of component z, which leads to load transfers and topology changes; $\boldsymbol{\Psi}$ is the set of contingencies; $S_{\mathrm{F}j}(z)$ is the power on branch j after ψ_z; $\Lambda_{\mathrm{F}j}(z)$ is the set of nodes downstream of branch j after ψ_z; $\boldsymbol{B}\setminus\{z\}$ is the set of all the feeder branches except z; $S_{\mathrm{T}j}(z)$ is the power on transformer j after ψ_z; $\Lambda_{\mathrm{T}j}(z)$ is the set of nodes downstream of transformer j after ψ_z; $\boldsymbol{T}\setminus\{z\}$ is the set of all the substation transformers except z; $U_i(z)$ is the voltage magnitude of node i after ψ_z; $\Pi_i(z)$ is the set of branches upstream node i after ψ_z.

Note that the powers on feeder branches and substation transformers are the powers after FR regulations, which are determined by $\Delta S_i(t)$ and $S_i^{\mathrm{lim}}(t)$. Besides, power factors and approximations [5] are used to project voltage constraints onto S-space. The derivation process is given in Ref. [12].

As seen in Eq (2):

i) $\boldsymbol{\Omega}_{\text{TV-DSSR}}$ only describes the secure operation range of demand nodes and hybrid nodes. The operating point $\boldsymbol{W}_{\mathrm{TV}}$ only includes demand node powers and hybrid node powers.

ii) $\boldsymbol{\Omega}_{\text{TV-DSSR}}$ considers FR regulations in security constraints. $s.t.B$ and $s.t.C$ include the powers related to FR regulations $\Delta S_i(t)$ and $S_i^{\mathrm{lim}}(t)$. This means $\boldsymbol{W}_{\mathrm{TV}}$ needs to meet certain security constraints after considering FR regulations.

iii) $\boldsymbol{\Omega}_{\text{TV-DSSR}}$ will be time-variant if $\Delta S_i(t)$ or $S_i^{\mathrm{lim}}(t)$ is.

The reason for the time-variance of DSSR is that $\boldsymbol{\Omega}_{\text{TV-DSSR}}$ considers FR regulations in DSSR modeling (Step 1), and thus FR regulations are reflected in the region varying. Moreover, some FRs' power regulating ranges are time-variant, such as ESS and DR load. Hence, DSSR is time-variant.

III. TIME-INVARIANT MODEL OF DSSR & ANALYSIS METHOD

This section proposes the time-invariant DSSR (TI-DSSR) model. Different from TV-DSSR, FR regulations are considered in Step 2 of the region-based method, that is, in using the region to analyze the operating point security, rather than in the DSSR modeling and calculation (Step 1).

A. Operating point including customer demands and regulating resources

The operating point of TI-DSSR is defined as the net power vector of the demand nodes, hybrid nodes, and regulating nodes in the distribution system, as shown in Eq (3).

$$\boldsymbol{W}_{\text{TI}}=[S_1,\ldots, S_i,\ldots, S_{n\text{D}+n\text{H}},\ldots S_n],\ i\in \boldsymbol{ND}\cup \boldsymbol{NH}\cup \boldsymbol{NR} \quad (3)$$

where $\boldsymbol{W}_{\text{TI}}$ is the operating point of TI-DSSR; $n=n_{\text{D}}+n_{\text{H}}+n_{\text{R}}$.

Compared with $\boldsymbol{W}_{\text{TV}}$:

i) $\boldsymbol{W}_{\text{TI}}$ includes not only customer demands but also absolute regulating resources without customer demands.

ii) $\boldsymbol{W}_{\text{TI}}$ has more variables, and the difference is the total number of regulating nodes n_{R}. If there is no regulating node, that is, $\boldsymbol{NR}$ is an empty set, then $\boldsymbol{W}_{\text{TV}}$ is the same as $\boldsymbol{W}_{\text{TI}}$.

TI-DSSR builds the operating point including customer demands and regulating resources. The purpose is to reflect FR regulations as the operating point movement instead of the DSSR boundary movement. Thus, the DSSR boundary is not affected by FR states and is no longer time-variant.

B. Time-invariant DSSR model

The TI-DSSR $\boldsymbol{\Omega}_{\text{TI-DSSR}}$ model is shown in Eq (4).

$$
\begin{aligned}
&\boldsymbol{\Omega}_{\text{TI-DSSR}} = \{\boldsymbol{W}_{\text{TI}} = [S_1,...,S_i,...,S_{n\text{D}+n\text{H}},...,S_n],\\
&i \in \boldsymbol{ND} \cup \boldsymbol{NH} \cup \boldsymbol{NR} \,|\\
&s.t.A \quad S_i^{\min} \le S_i \le S_i^{\max}\\
&s.t.B \begin{cases}
|S_{\text{F}j}| = |\sum_{i\in\Lambda_{\text{F}j}} S_i| \le c_{\text{F}j} \quad \forall j \in \boldsymbol{B}\\
|S_{\text{T}j}| = |\sum_{i\in\Lambda_{\text{T}j}} S_i| \le c_{\text{T}j} \quad \forall j \in \boldsymbol{T}\\
U_i^{\min} \le U_i \le U_i^{\max} \quad \forall i \in \boldsymbol{N}\\
U_i = U_0 - \sum_{j\in\Pi i} \dfrac{R_j \sum_{k\in\Lambda_{\text{F}j}} S_k \cos\varphi_k + X_j \sum_{k\in\Lambda_{\text{F}j}} S_k \sin\varphi_k}{U_{\text{N}}}
\end{cases}\\
&s.t.C \begin{cases}
\forall \psi_z \in \boldsymbol{\Psi}\\
|S_{\text{F}j}(z)| = |\sum_{i\in\Lambda_{\text{F}j}(z)} S_i| \le c_{\text{F}j} \quad \forall j \in \boldsymbol{B}\setminus\{z\}\\
|S_{\text{T}j}(z)| = |\sum_{i\in\Lambda_{\text{T}j}(z)} S_i| \le c_{\text{T}j} \quad \forall j \in \boldsymbol{T}\setminus\{z\}\\
U_i^{\min} \le U_i(z) \le U_i^{\max} \quad \forall i \in \boldsymbol{N}\\
U_i(z) = U_0 - \sum_{j\in\Pi i(z)} \dfrac{R_j \sum_{k\in\Lambda_{\text{F}j}(z)} S_k \cos\varphi_k + X_j \sum_{k\in\Lambda_{\text{F}j}(z)} S_k \sin\varphi_k}{U_{\text{N}}}
\end{cases}\\
&\}
\end{aligned} \tag{4}
$$

As seen in Eq (4):

i) $\boldsymbol{\Omega}_{\text{TI-DSSR}}$ describes the secure operation range of demand nodes, hybrid nodes, and regulating nodes. The operating point $\boldsymbol{W}_{\text{TI}}$ includes demand node powers, hybrid node powers, and demand node powers.

ii) $\boldsymbol{\Omega}_{\text{TI-DSSR}}$ doesn't consider FR regulations. The security constraints $s.t.B$ and $s.t.C$ only include $\boldsymbol{W}_{\text{TI}}$ variables and don't include the powers related to FR regulations.

iii) $\boldsymbol{\Omega}_{\text{TI-DSSR}}$ is time-invariant.

The $\boldsymbol{\Omega}_{\text{TI-DSSR}}$ modeling doesn't account for FR regulations. Even though some FRs' power regulating ranges are time-variant, it doesn't affect $\boldsymbol{\Omega}_{\text{TI-DSSR}}$.

Besides, the analytical method [7] is used to solve DSSR model to determine security boundaries, which are described by hyperplanes [1]. A security boundary analytical expression can be obtained by taking one security constraint inequality to be equal. The No. p security boundary is denoted as $\text{B}_{\text{TI},p}$.

C. Treatment of FR regulations

FR regulations are not reflected in DSSR varying in the TI-DSSR model. If using TI-DSSR in security analysis, the treatment of FR regulations is to analyze the operating point's potential for position regulation according to the FR security regulating capability (*SRC*). *SRC* uniformly quantifies the security regulating capability of different FRs [11], ensuring the generalizability in security analysis. The security analysis object is the operating point regulated according to *SRC*.

The node *SRC* is the feasible maximum FR power regulation amount within the node that meets the system security demand direction and can last longer than the security demand time [11], and can be calculated according to the Ref. [11] method. The security demand direction is the power regulation direction required by the distribution network [11]. The security demand time is the minimum time for which the power regulation should be maintained, required by the distribution network [11].

The node *SRC* can be expressed by the powers related to FR regulations, as shown in Eq (5).

$$SRC_i(t) = \begin{cases} |\Delta S_i(t)|,\ i \in \boldsymbol{NH} \\ |S_i^{\text{lim}}(t) - S_i(t)|,\ i \in \boldsymbol{NR} \end{cases} \tag{5}$$

The operating point regulated according to *SRC* is in (6).

$$\boldsymbol{W}_{\text{TI},SRC} = \begin{cases} \boldsymbol{W}_{\text{TI}} - \boldsymbol{SRC} \\ = [S_1 - SRC_1,\ldots,S_i - SRC_i,...,S_n - SRC_n], \\ \text{in supply scenario} \\ \boldsymbol{W}_{\text{TI}} + \boldsymbol{SRC} \\ = [S_1 + SRC_1,\ldots,S_i + SRC_i,...,S_n + SRC_n], \\ \text{in reverse scenario} \end{cases} \tag{6}$$

where $\boldsymbol{W}_{\text{TI},SRC}$ is the operating point regulated according to *SRC*; $\boldsymbol{SRC}$ is the vector of all the node *SRC*s; SRC_i is the node i *SRC*. A feeder is in the supply scenario if its outlet power flow is positive, approaching positive overload or low voltage [11]. At that time, the security demand direction is reducing outflow and increasing injection, and the FR regulations include reducing load by DR and increasing ESS discharging power [11]. Conversely, the feeder is in the reverse scenario.

D. Security analysis method using time-invariant DSSR

A security analysis method using TI-DSSR is proposed, as shown in Fig. 1.

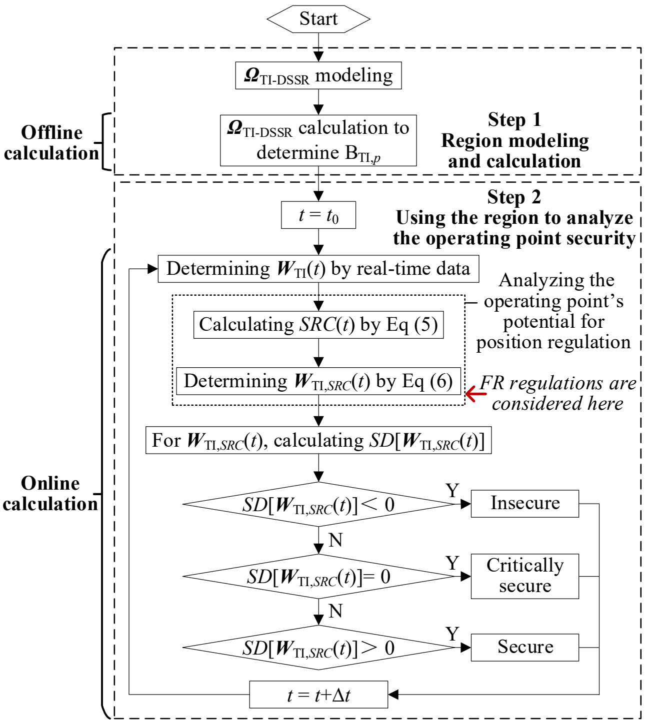

Fig. 1. Security analysis method using TI-DSSR.

There is a $W_{\text{TI}}(t)$ at every moment t. The t_0 is the initial time. The Δt is the sampling time interval of the distributed supervisory control and data acquisition. The SD is the security distance, defined as the distance from $W_{\text{TI}}(t)$ to the security boundary [4], and calculated by the Ref. [11] method.

For each $W_{\text{TI}}(t)$, $SRC(t)$ is calculated, and $W_{\text{TI},SRC}(t)$ is determined by Eq (6). Then, for $W_{\text{TI},SRC}(t)$, $SD[W_{\text{TI},SRC}(t)]$ is calculated, and thus assesses the system security at time t.

As seen in Fig. 1:

i) FR regulations are considered in Step 2, in analyzing the operating point's potential for position regulation.

ii) TI-DSSR is calculated offline instead of online.

IV. Case Study

A. Case information

The urban distribution system part of the HPDS133-bus test system [10] is used, which is based on the real 10 kV urban distribution network in Jiaxing, China. ESS and DR are further considered. The system structure is shown in Fig. 2.

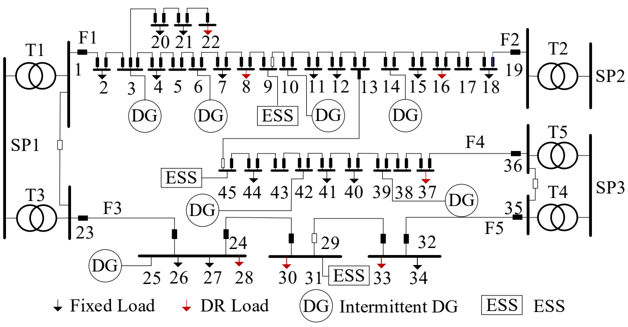

Fig. 2. Case study system structure.

In this case, demand nodes include all the fixed load nodes and intermittent DG nodes, regulating nodes include all the ESS nodes, and hybrid nodes include all the DR load nodes.

The data are given in Ref. [12].

B. Solution of time-invariant DSSR

Solve the TI-DSSR model. The security boundaries are shown in Table I.

TABLE I. Security Boundaries of TI-DSSR

Time	Security boundary analytical expressions
9:00~13:00	$B_{\text{TI},1}$: $\sum_{i1} S_{\text{L}i1}+\sum_{i2} S_{\text{DG}i2}+\sum_{i3} S_{\text{ESS}i3}$=**6000**, $i_1\in\{2,4,7,8,11,12,15,16,18,20,21,22\}$, $i_2\in\{3,6,10,14\}$, $i_3\in\{9\}$
	$B_{\text{TI},2}$: $\sum_{i1} S_{\text{L}i1}+\sum_{i2} S_{\text{DG}i2}+\sum_{i3} S_{\text{ESS}i3}$=**6000**, $i_1\in\{11,12,15,16,18,37,40,41,44\}$, $i_2\in\{10,14,39,42\}$, $i_3\in\{45\}$
	$B_{\text{TI},3}$: $\sum_{i1} S_{\text{L}i1}+\sum_{i2} S_{\text{DG}i2}+\sum_{i3} S_{\text{ESS}i3}$=**6000**, $i_1\in\{26,27,28,30,33,34\}$, $i_2\in\{25\}$, $i_3\in\{31\}$

As seen in Table I, there are 3 security boundaries: $B_{\text{TI},1}$, $B_{\text{TI},2}$, and $B_{\text{TI},3}$, which remain unchanged with time.

The 3D visualization of TI-DSSR from 9:00 to 13:00 is shown in Fig. 3. S_{DG25}, S_{L33}, and S_{ESS31} are selected as the observation variables, and the other node powers are fixed to the 11:00 operating point powers.

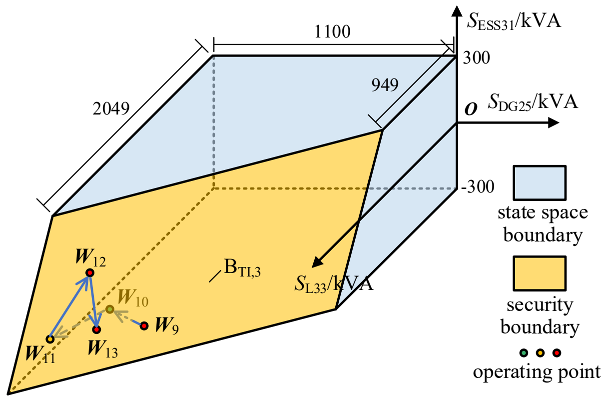

Fig. 3. Visualization of TI-DSSR from 9:00 to 13:00.

As seen in Fig. 3, TI-DSSR is time-invariant. The size and shape remain unchanged. The security boundary doesn't move.

C. Comparison with existing security analysis method

1) Comparison of security analysis results

If using TI-DSSR, the online security analysis needs to analyze the operating point's potential for position regulation based on FR SRC. Taking the 11:00 operating point W_{11} as an example, as shown in Fig. 4.

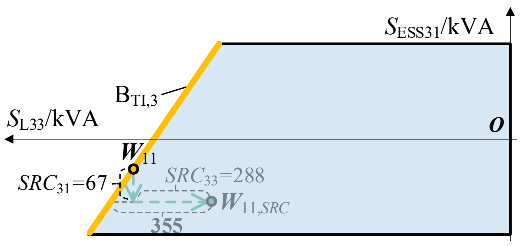

Fig. 4. Analyzing the operating point's potential for position regulation based on FR SRC.

As seen in Fig. 4, after the FR regulation according to SRC_{31} and SRC_{33}, W_{11} moves inside DSSR from the security boundary $B_{\text{TI},3}$ along the negative direction of the S_{ESS31} and S_{L31} axes. The security distance changes from 0 to 355kVA. Hence, the online security analysis conclusion is that W_{11} is critically secure before FR regulations, and is secure after the ESS31 and DR load 33 regulations.

The online security analysis results from 9:00 to 13:00 based on TV-DSSR and TI-DSSR are shown in Table II.

TABLE II. COMPARISON OF ONLINE SECURITY ANALYSIS RESULTS BASED ON TV-DSSR AND TI-DSSR

Time	$\boldsymbol{W}_{\mathrm{TI}}$ (S_{L33}, S_{DG25}, S_{ESS31}) /kVA	TV-DSSR [7]		TI-DSSR (This paper)				
				Before FR regulations		After FR regulations according to *SRC*		
		Security	Security distance/kVA	Security	Security distance/kVA	$\boldsymbol{W}_{\mathrm{TI},SRC}$	Security	Security distance/kVA
9:00	(2552, -830, -182)	Secure	372	**Insecure**	-114	(2320, -830, -300)	Secure	372
10:00	(2030, -870, -255)	Secure	822	Secure	258	(1691, -870, -300)	Secure	822
11:00	(2509, -980, -100)	Secure	462	**Critically secure**	0	(2222, -980, -167)	Secure	462
12:00	(2794, -890, 168)	Insecure	-676	Insecure	-1329	(2527, -890, -67)	Insecure	-676
13:00	(2687, -900, -126)	Critically secure	0	**Insecure**	-526	(2446, -900, -219)	Critically secure	0

As seen in Table II, using the operating point regulated according to *SRC* $\boldsymbol{W}_{\mathrm{TI},SRC}$, the proposed TI-DSSR can get exactly the same analysis results as TV-DSSR. Note that the proposed TI-DSSR cannot get the correct results if directly using $\boldsymbol{W}_{\mathrm{TI}}$. Because $\boldsymbol{W}_{\mathrm{TI}}$ doesn't include FR regulations. The proposed TI-DSSR must be used in conjunction with FR *SRC*.

2) Comparison of security analysis time

The online security analysis based on TV-DSSR and TI-DSSR is separately implemented by the MATLAB R2023a program. The program is carried out on a PC with Intel(R) Core (TM) i5-13500H @ 2.60GHz CPU, 16GB RAM. The comparison of the calculation time is shown in Table III.

TABLE III. COMPARISON OF ONLINE SECURITY ANALYSIS CALCULATION TIME BASED ON TV-DSSR AND TI-DSSR

Items	TV-DSSR [7]	TI-DSSR (This paper)	Difference
DSSR calculation/ms	6624.09	7173.83[a]	549.74
SRC calculation/ms	0[b]	255.61	255.61
Security distance calculation/ms	6.25	10.29	4.04
Online total/ms	6630.34	265.90	-6364.44
Reducing online total by	95.99%		

[a] TI-DSSR is calculated offline only once, and the elapsed time is not included in the online calculation.
[b] There is no need to calculate *SRC* if using TV-DSSR.

As seen in Table III:

i) The proposed TI-DSSR model has the benefit of being calculated offline only once. Hence, TI-DSSR can be directly used in each subsequent online security analysis, saving the online DSSR calculation time (6624.09ms).

ii) The benefit also comes at a certain cost. On the one hand, the DSSR calculation time increases by about 8%, and the security distance calculation time increases by 4.04ms. The reason is that the number of $\boldsymbol{W}_{\mathrm{TI}}$ variables is n_{R} more than $\boldsymbol{W}_{\mathrm{TV}}$. On the other hand, the *SRC* needs to be calculated additionally, which costs 255.61ms.

iii) The benefit (6624.09ms) far outweighs the cost (255.61+4.04=259.65ms in the online calculation). The benefit is one order of magnitude greater than the cost. Hence, the total online calculation time is significantly reduced (by about 96%). Note that although the DSSR calculation time increases by about 8%, it refers to the offline calculation, which has a lower speed requirement than the online calculation, and is only once.

V. CONCLUSION

Distribution system security region (DSSR) has time-variant characteristics after considering flexible resources (FRs). If continuing to use the existing DSSR model, the DSSR must be calculated in real time, which loses the speed advantage of the region-based method for online security analysis. This paper points out this problem for the first time, reveals the reason for the time-variance of DSSR, and proposes the time-invariant DSSR (TI-DSSR) model and its security analysis method. TI-DSSR reflects FR regulations as the operating point possessing the potential for position regulation rather than DSSR varying, and can be calculated offline beforehand and thus maintains the speed advantage.

This work lays a theoretical foundation for practical applications of DSSR in online security analysis of smart distribution networks. Economic factors and the coordination of various FRs will be further considered in the future.

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