N-1 Security Analysis of the Snow-Shaped Distribution Network Based on Analytical Expression of Transfer Capability

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***Abstract*—To analytically evaluate the load transfer capability and security margin of the snow-shaped distribution network (SDN) under high-reliability supply scenarios, this paper proposes an analytical expression method based on a topological correlation matrix. First, a feeder correlation matrix is established to describe the inter-station and intra-station connections. Then, by introducing margin parameters of main transformers and feeders, the maximum transfer capability matrix is derived, achieving the analytical representation of load transfer capability. Based on the analytical results, multidimensional security assessment indices are defined to systematically evaluate the security level of SDNs under N-1 contingencies. Finally, case studies verify the correctness and applicability of the proposed model. The results show that, compared with the traditional enumeration method, the proposed approach can efficiently characterize the transfer characteristics among feeders and achieve N-1 analysis and weak-point identification of SDNs without the need for fault enumeration.**

***Keywords*—snow-shaped distribution network; analytical expression; transfer capability; N-1 security**

I. Introduction

With the evolution of distribution networks toward high-penetration distributed energy integration and multi-point interconnection [1],[2], the impact of network topological characteristics on system reliability and fault response capability has become increasingly significant. In the SDN, multiple intra-station and inter-station connections among feeders form complex coupling relationships. Under a single contingency, the load transfer capability is no longer determined by an individual feeder or a single transfer path, but by a system-level quantity jointly constrained by the overall connection pattern, the margins of main transformers and feeders, and the associated interconnection limits. Based on this, developing a mathematical formulation capable of analytically characterizing the inter-feeder transfer relationships within the snow-shaped topology [3] becomes a core requirement for determining whether the system can maintain supply continuity under N-1 conditions.

Existing studies on load transfer capability in distribution networks mainly fall into two categories: simulation- based or path search–based approaches that enumerate fault scenarios and obtain transfer results through iterative power flow calculations [4]; and heuristic or optimization-based approaches that perform numerical solving for specific network reconfiguration strategies or for scheduling of energy storage and flexible resources [5]. The former requires intensive computation and is unsuitable for rapid structural assessment, while the latter focuses on operational optimization and usually depends on case-specific parameters and extensive simulations for validation, lacking an interpretable analytical description of the network topology itself. For the SDN with multilayer interconnection characteristics, both methods fail to directly reveal the analytical relationship among the connection pattern, margin distribution, and maximum transfer capability.

In the direction of analytical expression modeling, some studies have introduced graph-based or matrix-based adjacency/correlation representations to describe the connectivity of power-supply paths. However, these are mostly limited to connectivity or reachability analysis and fail to systematically incorporate engineering parameters such as transformer and feeder capacities or loading rates into the analytical expression of transfer capability [6],[7]. Other studies have proposed analytical formulations of local transfer capability based on simplified parallel-capacity assumptions. Nevertheless, these formulations usually neglect the coupling effects of intra-station and inter-station connections within multi-station clusters and lack a unified matrix-based representation to support vector/matrix operations and index derivation [8],[9]. Therefore, existing studies exhibit two major limitations: the lack of a scalable and matrix-form feeder correlation model that can simultaneously characterize intra-station and inter-station transfer paths; and the absence of an analytical derivation of maximum transfer capability and a corresponding system security index framework based on such a correlation model, which restricts reproducible quantitative analysis in structural design and rapid security assessment.

To fill the above research gaps, this paper develops an analytical expression model for transfer capability tailored to the SDN topology and conducts a comprehensive N−1 security analysis based on this framework. The main contributions of this paper are summarized as follows:

(1) A unified feeder correlation matrix modeling method is proposed to structurally characterize multilayer intra-station and inter-station connections, providing a fundamental formulation for the analytical modeling of the SDN.

(2) Engineering parameters such as transformer and feeder capacities and loading rates are systematically embedded into the matrix-based analytical model, from which the maximum transfer capability matrix considering transformer and feeder margins is derived.

(3) A multidimensional N−1 security assessment index system is established, enabling rapid and accurate calculation of transfer margins, balance degree, and security margin distribution, and realizing quantitative identification of weak points in the system.

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II. Snow-Shaped Distribution Network

The snow-shaped distribution network refers to a multi-substation, multi-feeder distribution topology in which feeder clusters are symmetrically interconnected through inter-station and intra-station tie lines, forming a structure resembling a snowflake. The snow-shaped structure is constructed based on 10 kV ring main units (RMUs) as the fundamental elements, forming a feeder cluster composed of three substations with six feeders or four substations with eight feeders. Each substation is equipped with two 10 kV feeders, where each feeder includes one inter-station and one intra-station tie line. The overall network operates in an open-ring configuration. An electrical topology of the three-substation six-feeder SDN is shown in Fig. 1. Compared with conventional ring networks, the SDN topology significantly enhances the selectivity of load transfer paths and the reconfigurability of power supply chains at the spatial structural level. This topology is particularly suitable for urban distribution scenarios characterized by high load density, heterogeneous resources, and imbalanced supply–demand conditions [10].

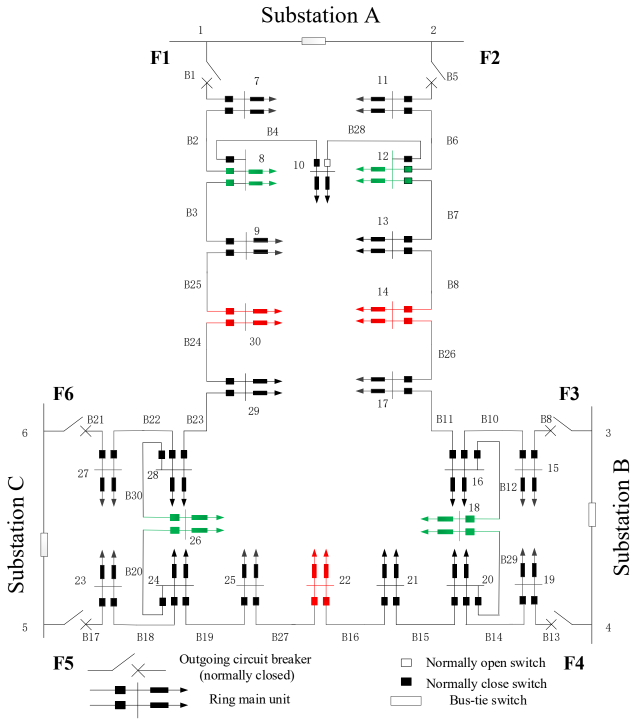

Fig. 1. Electrical topology of the three-substation six-feeder SDN

III. Load Transfer Capability Modeling of SDNs

A. Feeder Correlation Matrix

In the SDN, RMUs and branch lines jointly form the feeders. Each feeder typically contains multiple RMUs and branch sections, operating in a radial configuration, while feeders are interconnected through intra-station and inter-station tie lines. When a feeder becomes heavily loaded or experiences a fault, part of its RMU loads can be transferred to other feeders through these tie lines. This section establishes a feeder correlation matrix model to describe the interconnection relationships among feeders within the SDN.

Assume that the SDN consists of N feeders, numbered 1, 2, …, N. The feeder correlation matrix $\boldsymbol{F}$ is defined as follows:

$$\boldsymbol{F}=\begin{bmatrix} F_{1,1} & \cdots & F_{1,j} & \cdots & F_{1,N} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ F_{i,1} & \cdots & F_{i,j} & \cdots & F_{i,N} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ F_{N,1} & \cdots & F_{N,j} & \cdots & F_{N,N} \end{bmatrix} \tag{1}$$

where, $F_{i,j}$ represents the correlation relationship between feeder F_i and feeder F_j, taking binary values of 0 or 1. When $F_{i,j}$=1, feeders F_i and F_j are correlated, indicating that a tie line exists between them. When $F_{i,j}$=0, the feeders are uncorrelated, and no tie line exists between them. The diagonal elements of matrix $\boldsymbol{F}$ are defined as zero, i.e., $F_{i,i}$=0. Matrix $\boldsymbol{F}$ is a symmetric matrix, satisfying $F_{i,j}= F_{j,i}$, both representing the interconnection relationship between feeders F_i and F_j. The dimension of matrix $\boldsymbol{F}$ is consistent with the total number of feeders.

Assume that the distribution network consists of n substations, numbered 1, 2, …, n. Each substation contains N_1, N_2, …,N_n feeders, and the total number of feeders is N=N_1+ N_2+…+N_n. In an SDN topology, n=3 or 4, N_1=N_2=…=N_n=2, resulting in N=6 or 8.

$$\boldsymbol{F}=\left[\begin{array}{ccc:ccc:c:ccc} F_{1,1} & \cdots & F_{1,N1} & F_{1,N1+1} & \cdots & F_{1,N1+N2} & \cdots & \cdots & \cdots & F_{1,N} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & & \vdots & \ddots & \vdots \\ F_{N1,1} & \cdots & F_{N1,N1} & F_{N1,N1+1} & \cdots & F_{N1,N1+N2} & & \cdots & \cdots & F_{N1,N} \\ \hdashline F_{N1+1,1} & \cdots & F_{N1+1,N1} & F_{N1+1,N1+1} & \cdots & F_{N1+1,N1+N2} & & \cdots & \cdots & F_{N1+1,N} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \cdots & \vdots & \ddots & \vdots \\ F_{N1+N2,1} & \cdots & F_{N1+N2,N1} & F_{N1+N2,N1+1} & \cdots & F_{N1+N2,N1+N2} & & \cdots & \cdots & F_{N1+N2,N} \\ \hdashline & \vdots & & & \vdots & & \ddots & & \vdots & \\ \hdashline \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & & \cdots & \cdots & \cdots \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \cdots & \vdots & \ddots & \vdots \\ F_{N,1} & \cdots & F_{N,N1} & F_{N,N1+1} & \cdots & F_{N,N1+N2} & & \cdots & \cdots & F_{N,N} \end{array}\right] \tag{2}$$

In the SDN, load transfer can occur either between different feeders within the same substation or between feeders belonging to different substations. To facilitate the analytical expression of transfer capability based on the correlation matrix, the transfer modes are classified into intra-station transfer and inter-station transfer according to the type of tie line, and the corresponding load transfer matrices are constructed, respectively.

In the partitioned feeder correlation matrix $\boldsymbol{F}$, the submatrices on the main diagonal represent the correlation relationships between feeders within the same substation. By extracting these n diagonal submatrices, the intra-station transfer matrix $\boldsymbol{A}$ can be defined as follows:

$$\boldsymbol{A}=[a_{i,j}]_{N\times N}=\left[\begin{array}{c:c:c:c} \begin{matrix} F_{1,1} & \cdots & F_{1,N1} \\ \vdots & \ddots & \vdots \\ F_{N1,1} & \cdots & F_{N1,N1} \end{matrix} & 0 & \cdots & 0 \\ \hdashline 0 & \begin{matrix} F_{N1+1,N1+1} & \cdots & F_{N1+1,N1+N2} \\ \vdots & \ddots & \vdots \\ F_{N1+N2,N1+1} & \cdots & F_{N1+N2,N1+N2} \end{matrix} & \cdots & 0 \\ \hdashline \vdots & \vdots & \ddots & \vdots \\ \hdashline 0 & 0 & \cdots & \begin{matrix} \cdots & \cdots & \cdots \\ \vdots & \ddots & \vdots \\ \cdots & \cdots & F_{N,N} \end{matrix} \end{array}\right] \tag{3}$$

where, $a_{i,j}$ in matrix $\boldsymbol{A}$ indicates whether feeder F_j can transfer load to feeder F_i through an intra-station tie line. If $a_{i,j}$=1 load transfer is possible; if $a_{i,j}$=0, it is not.

Similarly, the off-diagonal submatrices of $\boldsymbol{F}$ — a total of $n(n-1)$ — represent the inter-station transfer matrix $\boldsymbol{B}$:

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$$
\boldsymbol{B}=[b_{i,j}]_{N\times N}=\left[\begin{array}{ccc|ccc|c|ccc}
 & & & F_{1,N1+1} & \cdots & F_{1,N1+N2} & & \cdots & \cdots & F_{1,N} \\
 & 0 & & \vdots & \ddots & \vdots & \cdots & \vdots & \ddots & \vdots \\
 & & & F_{N1,N1+1} & \cdots & F_{N1,N1+N2} & & \cdots & \cdots & F_{N1,N} \\
\hline
F_{N1+1,1} & \cdots & F_{N1+1,N1} & & & & & \cdots & \cdots & F_{N1+1,N} \\
\vdots & \ddots & \vdots & & 0 & & \cdots & \vdots & \ddots & \vdots \\
F_{N1+N2,1} & \cdots & F_{N1+N2,N1} & & & & & \cdots & \cdots & F_{N1+N2,N} \\
\hline
 & \vdots & & & \vdots & & \ddots & & \vdots & \\
\hline
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & & & & \\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \cdots & & 0 & \\
F_{N1+N2,1} & \cdots & F_{N1+N2,N1} & F_{N,N1+1} & \cdots & F_{N,N1+N2} & & & &
\end{array}\right] \tag{4}
$$

where, $b_{i,j}$ in matrix $\boldsymbol{B}$ indicates whether feeder F_j can transfer load to feeder F_i through an inter-station tie line. If $b_{i,j}$=1 load transfer is possible; if $b_{i,j}$=0, it is not.

The relationship between the $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{F}$ can thus be expressed as:

$$\boldsymbol{F}=\boldsymbol{A}+\boldsymbol{B} \tag{5}$$

B. Transfer Capability Matrix

Building upon the correlation matrix model established above, this section further derives the maximum transfer capability matrix of feeders under N−1 conditions, thereby enabling an analytical verification of power supply security. When a specific feeder experiences a fault, the maximum capacity of the remaining feeders in the SDN to undertake the transferred load is defined as the maximum transfer capability of the SDN centered on that feeder.

Let the maximum transfer capability matrix $\boldsymbol{TM}$ be expressed as:

$$\boldsymbol{TM}=\left[TM_1\cdots TM_i\cdots TM_N\right]^{\mathrm{T}} \tag{6}$$

where, TM_i represents the maximum transfer capability of the distribution network when feeder F_i fails.

Let $\Psi(i)\in\{1,\ldots,n\}$ denote the substation index that feeder i belongs to. For the studied SDN, $\Psi(1)=\Psi(2)=1$, $\Psi(3)=\Psi(4)=2$, $\Psi(5)=\Psi(6)=3$.

Let the main transformer capacity vector $\boldsymbol{R}$ be:

$$\boldsymbol{R}=\left[R_1\cdots R_{\Psi(i)}\cdots R_{\Psi(N)}\right]^{\mathrm{T}} \tag{7}$$

where, $R_{\Psi(i)}$ represents the load capacity that main transformer $R_{\Psi(i)}$ can accommodate.

Let the feeder capacity vector $\boldsymbol{L}$ be:

$$\boldsymbol{L}=\left[L_1\cdots L_i\cdots L_N\right]^{\mathrm{T}} \tag{8}$$

where, L_i represents the maximum allowable load capacity of feeder F_i.

Let the main transformer loading rate vector $\boldsymbol{RM}$ be:

$$\boldsymbol{RM}=\left[RM_1\cdots RM_{\Psi(i)}\cdots RM_{\Psi(N)}\right]^{\mathrm{T}} \tag{9}$$

where, $RM_{\Psi(i)}$ represents the loading rate of main transformer $R_{\Psi(i)}$.

Let the feeder loading rate vector $\boldsymbol{LM}$ be:

$$\boldsymbol{LM}=\left[LM_1\cdots LM_i\cdots LM_N\right]^{\mathrm{T}} \tag{10}$$

where, LM_i represents the loading rate of feeder F_i.

Then, the main transformer margin matrix $\boldsymbol{R'}$ can be expressed as:

$$\boldsymbol{R'}=\boldsymbol{R}\circ\left(1-\boldsymbol{RM}\right)=\left[R_1'\cdots R_{\Psi(i)}'\cdots R_{\Psi(N)}'\right]^{\mathrm{T}} \tag{11}$$

where, $R'_{\Psi(i)}$ represents the available capacity margin of main transformer $R_{\Psi(i)}$, that is, the remaining load it can accommodate.

Similarly, the feeder margin matrix $\boldsymbol{L'}$ is defined as:

$$\boldsymbol{L'}=\boldsymbol{L}\circ\left(1-\boldsymbol{LM}\right)=\left[L_1'\cdots L_i'\cdots L_N'\right]^{\mathrm{T}} \tag{12}$$

where, L_i' represents the available capacity margin of feeder F_i.

According to the feeder N−1 verification method presented in [11], the transfer capability of the SDN can be analytically derived based on the intra-station and inter-station transfer matrices $\boldsymbol{A}$ and $\boldsymbol{B}$, together with the margin matrices $\boldsymbol{R'}$ and $\boldsymbol{L'}$

The intra-station transfer capability matrix $\boldsymbol{TAM}$ is calculated as:

$$
\begin{aligned}
\boldsymbol{TAM} &= \boldsymbol{A}\cdot\min\left\{diag\left(\boldsymbol{R'}\right),diag\left(\boldsymbol{L'}\right)\right\}\cdot[1\cdots1\cdots1]^{\mathrm{T}} \\
&= \begin{bmatrix}
\min\{R_1',L_2'\} \\
\min\{R_1',L_1'\} \\
\min\{R_2',L_4'\} \\
\min\{R_2',L_3'\} \\
\min\{R_3',L_6'\} \\
\min\{R_3',L_5'\}
\end{bmatrix}
\end{aligned} \tag{13}
$$

The inter-station transfer capability matrix $\boldsymbol{TBM}$ is calculated as:

$$
\begin{aligned}
\boldsymbol{TBM} &= \boldsymbol{B}\cdot\min\left\{diag\left(\boldsymbol{R'}\right),diag\left(\boldsymbol{L'}\right)\right\}\cdot[1\cdots1\cdots1]^{\mathrm{T}} \\
&= \begin{bmatrix}
\min\{R_3',L_6'\} \\
\min\{R_2',L_3'\} \\
\min\{R_1',L_2'\} \\
\min\{R_3',L_5'\} \\
\min\{R_2',L_4'\} \\
\min\{R_1',L_1'\}
\end{bmatrix}
\end{aligned} \tag{14}
$$

Accordingly, the maximum transfer capability matrix $\boldsymbol{TM}$ can be expressed as:

$$
\begin{aligned}
\boldsymbol{TM} &= \boldsymbol{TAM}+\boldsymbol{TBM} \\
&= \begin{bmatrix}
\min\{R_1',L_2'\}+\min\{R_3',L_6'\} \\
\min\{R_1',L_1'\}+\min\{R_2',L_3'\} \\
\min\{R_2',L_4'\}+\min\{R_1',L_2'\} \\
\min\{R_2',L_3'\}+\min\{R_3',L_5'\} \\
\min\{R_3',L_6'\}+\min\{R_2',L_4'\} \\
\min\{R_3',L_5'\}+\min\{R_1',L_1'\}
\end{bmatrix}
\end{aligned} \tag{15}
$$

The matrix shown in (15) provides the analytical foundation for subsequent N−1 security index computation and margin assessment in SDN.

IV. N−1 SECURITY INDICES FOR FEEDERS IN SDN

In distribution network planning, reconstruction, operation, dispatch, and maintenance, the N−1 security criterion is commonly applied to assess the supply reliability of network reconfiguration strategies [12]. Building upon the analytical load transfer capability model developed in Section III, this section defines a set of feeder-level N−1 security indices, including the transfer margin, load balance degree, and minimum security margin, to quantitatively evaluate the post-contingency security level of the snow-shaped distribution network. These indices enable fast, non-iterative N−1 assessment and explicitly reveal the relationship between network topology, feeder capacity, and transfer capability. Moreover, they facilitate the identification of critical feeders and bottleneck connections, thereby providing a quantitative basis for reliability enhancement and topology optimization.

The transfer margin matrix $\boldsymbol{TM'}$ is defined as follows:

$$\boldsymbol{TM'} = \boldsymbol{TM} - \boldsymbol{L} \circ \boldsymbol{LM} \tag{16}$$

where, TM_i' in the transfer margin matrix $\boldsymbol{TM'}$ represents the remaining transfer margin of feeder F_i after load transfer. If $TM_i' \geq 0$, it indicates that the maximum transfer capability of feeder F_i is greater than or equal to its own load under fault conditions, meaning that feeder F_i satisfies the N−1 security criterion. In this case, the transfer margin value is positive, and a larger absolute value implies higher system security. Conversely, if $TM_i' < 0$, feeder F_i does not meet the N−1 security criterion; the transfer margin value is negative, and the larger its absolute value, the lower the system security.

The average transfer margin ATM' is defined as:

$$ATM' = \frac{1}{N}\sum_{i=1}^{N} TM_i' \tag{17}$$

where, ATM' represents the average transfer capability margin of all feeders in the system, serving as an indicator of the overall security margin. A larger ATM' indicates better system-wide security.

The transfer margin balance degree BTM' is defined as:

$$BTM' = \frac{1}{N}\sum_{i=1}^{N} (TM_i' - ATM')^2 \tag{18}$$

where, BTM' represents the variance of transfer capability margins among feeders, characterizing the dispersion level of transfer capability within the system. From an operational perspective, a low BTM' suggests that localized measures should be applied to specific bottleneck feeders, while a high BTM' implies that system-wide strategies, such as coordinated load redistribution or substation dispatch optimization, are more appropriate.

The transfer margin balance ratio RTM' is defined as:

$$RTM' = \frac{BTM'}{ATM'} \tag{19}$$

where, RTM' is the ratio of the balance degree BTM' to the average margin ATM'. A small RTM' indicates that the overall transfer capacity is constrained by a single critical feeder or its upstream substation.

Finally, the minimum transfer margin MTM' is defined as:

$$MTM' = \min\{TM_1', TM_2', \cdots, TM_N'\} \tag{20}$$

where, MTM' represents the minimum transfer margin among all feeders. If $MTM' > 0$, all feeders in the system satisfy the N−1 security criterion. A larger MTM' indicates a higher overall system security level.

V. Case Study

To verify the effectiveness and applicability of the proposed analytical expression-based N−1 security analysis method, case studies are carried out on a representative three-station and six-feeder SDN using the MATLAB R2022a platform. The system topology is illustrated in Fig. 1, where the three substations (A, B, and C) are interconnected through both intra-station and inter-station tie switches. The parameters of the main transformers are listed in Table I, and the feeder capacity parameters are provided in Table II. The load ratios of six feeders on a typical day are shown in Fig. 2, which are used as the input data for the N−1 security analysis.

TABLE I. Parameters of Main Transformers

Substation	Transformer Node	Transformation Ratio (kV/kV)	Capacity (MVA)
A	1	35/10	20
	2	35/10	20
B	3	110/10	50
	4	110/10	50
C	5	35/10	20
	6	35/10	20

TABLE II. Feeder Capacity Parameters

Feeder	Allowable Capacity (MVA)
F1、F2	5.83
F3、F4、F5、F6	6.91

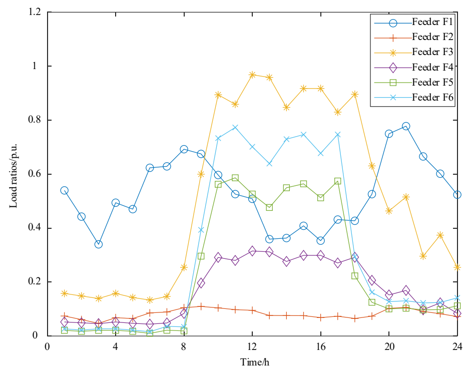

Fig. 2. Load ratios of six feeders on a typical day

During the typical day, feeders F1, F3, and F6 experience relatively high load ratios. Specifically, feeder F1 reaches its peak load of 77.78% during the peak hours between 18:00 and 23:00, with the maximum occurring at 21:00. Feeder F3 shows a high load ratio between 9:00 and 19:00, peaking at 96.81% at 12:00, while feeder F6 maintains a high load ratio from 10:00 to 17:00, with a peak of 77.29% at 11:00. In contrast, feeder F2 remains lightly loaded, with a maximum load ratio of only 10.96%. These observations indicate a clear coexistence of heavily and lightly loaded feeders in urban SDNs. By leveraging the multi-interconnection advantages of intra- and inter-station links within the SDN feeder cluster, loads can be flexibly transferred from heavily loaded feeders to lightly loaded ones, thereby mitigating branch overloads and achieving system-wide load balance.

The proposed analytical method is then applied to calculate the feeder transfer capability of the SDN. The minimum transfer margin of feeder F4 is −0.5925 MVA, indicating six time periods where the N−1 security criterion is not satisfied. Therefore, feeder load transfer in the spatial dimension is required to enhance system security. Taking 9:00–18:00 as an example, both the proposed method and the traditional enumeration method are employed to compute the transfer margin balance degree and transfer margin balance ratio of the SDN, and the results are presented in Table III.

TABLE III. CALCULATION RESULTS OF BTM' AND RTM' IN N−1 VERIFICATION OF SDN

Time (h)	BTM'(MVA)	RTM'(MVA)
9:00	1.6948	0.3846
10:00	1.9021	1.5693
11:00	2.0997	1.5249
12:00	2.6911	1.9326
13:00	3.9093	1.6889
14:00	2.7070	1.2051
15:00	2.9620	1.7568
16:00	3.3937	1.4850
17:00	2.4759	1.2459
18:00	7.4591	1.6559

However, since the traditional enumeration method requires repeated path searches and power-flow calculations for each fault scenario , leading to a computational complexity on the order of $O(n^{n/3+1})$. In contrast, the proposed method directly evaluates the transfer margins through matrix operations with a complexity approximately proportional to $O(n^{n*2\log(n)})$. A quantitative runtime comparison between the two methods is provided in Table IV.

TABLE IV. QUANTITATIVE RUNTIME COMPARISON

Method	Enumeration Method	Proposed Method
Runtime(s)	0.150	0.019

In addition, N−1 contingencies were validated using AC power flow, and the resulting transfer limits were compared with the analytical margins. The two methods show good agreement, with relative errors within 7%, confirming the physical validity of the proposed approach.

VI. CONCLUSION

This paper addresses the problem of evaluating the security level of SDNs under N−1 conditions and proposes an analytical expression method based on a topological correlation matrix. The main conclusions are as follows:

(1) A unified feeder correlation matrix model is developed to characterize intra- and inter-station connections within the SDN, providing a structural basis for describing load transfer in complex network topologies.

(2) A maximum transfer capability matrix is derived by considering the margins of main transformers and feeders, enabling fast computation of system-wide transfer capability without relying on fault/path enumeration.

(3) A structure-oriented security index system is established to quantitatively assess the security margin and load balance of SDNs under N−1 contingencies, using transfer margin, balance degree, and balance ratio to reflect system reliability and operational robustness.

Although the proposed analytical formulation is derived based on the snow-shaped distribution network, the method itself is not limited to this specific topology. By redefining the network coupling matrix, the proposed framework can also be extended to large ring or meshed distribution systems. In such cases, the ring structure can be decomposed into multiple equivalent coupling paths, and the same transfer margin indices can be evaluated without enumerating all N−1 contingencies.

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