From Field Measurements to online Digital Twin: Real-Time Distribution System State Estimation in Low-Voltage Grid

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***Abstract*—The ongoing integration of renewable energy sources, particularly at the low-voltage level, increases the loading of distribution grids and highlights the need for continuous system state monitoring to avoid critical operating conditions. This paper presents a low-voltage Distribution System State Estimation (DSSE) approach based on a weighted least squares (WLS) algorithm. Measurement data are acquired from a transformer station and approximately 38.5 % of the households within the study area. These data are recorded locally in the households, transmitted to a central database, and subsequently retrieved, preprocessed, and utilized to compute the overall grid state. Based on the estimated node voltages and phase angles, the individual household power injections are determined and integrated into a online Digital Twin, which is executed on a real-time simulation platform. The resulting Digital Twin state is validated against the measured data to ensure accuracy and reliability.**

***Index Terms*—Distribution System State Estimation, Low-Voltage Grids, Measurement Data, Information and Communication Technology, Digital Twin.**

I. Introduction

The increasing decentralization and volatility of modern power systems, driven by the large-scale integration of renewable energy sources, the electrification of heat and transport, and the active participation of consumers in electricity markets, fundamentally challenge the traditional operation of distribution grids. These developments lead to higher uncertainty in grid conditions and require advanced monitoring and control strategies to ensure stability, reliability, and efficiency of grid operation [1], [2]. A key method for obtaining the necessary real-time information is state estimation, a data processing algorithm that uses measurements and other available information to determine the systems' operating state. In transmission grids, such algorithms are well established as Transmission System State Estimation (TSSE). In contrast, Distribution System State Estimation (DSSE) is less common due to the limited availability of measurements in distribution grids [3]. Nevertheless, DSSE offers significant potential to support secure and efficient grid operation under the growing complexity of modern distribution systems. Typical state estimation algorithms include Weighted Least Squares (WLS), Kalman Filter, and Moving Horizon Estimation (MHE) [4], [5]. In [6], Distribution System State Estimation is analyzed with respect to different distribution grid topologies, such as radial, ring, and meshed networks [7], and the corresponding number of measurement nodes required within each grid section. The authors concluded that, for the grid topology applied in this study, meshed grid with one transformer station, at least 5 - 10 % of measurement nodes, as well as measurements at the transformer station, are required to achieve reliable state estimation performance.

II. Establishment of an online Digital Twin in the project Energetic Optimization of an Energy-Plus Settlement at System Level

The project Energetic Optimization of an Energy-Plus Settlement at the System Level (EOS) is carried out by the Institute of Electrical Energy Systems at the Friedrich-Alexander-Universität Erlangen-Nürnberg and local project partner such as the local distribution grid operator the "Erlanger Stadtwerke AG". The goal of the project is to optimize the settlement regarding electricity, heating and mobility. The study area consists of 70 grid-connected buildings and one transformer station. About 38.5 % of the houses are equipped with measurement technology. Buildings without measurement devices are assumed by pseudo-measurements. The measured data are sent by MQTT and uploaded to a database, from which they are retrieved by a Python script and processed by the state estimation algorithm. For this project, the Weighted Least Squares (WLS) method is applied. Once all states have been computed, the corresponding power flows are determined and transferred to the online Digital Twin, which is implemented on a real-time simulator at the laboratory of the Institute of Electrical Energy Systems [8] to analyze and optimize the grid. Fig. 1 shows the data flow from the field measurements to the Digital Twin and the corresponding services. The field measurement and the detailed grid area as well as the measurement setup is explained in more detail in [9]. A preprocessing routine to filter corrupted data and provide high-quality input for the Distribution System State Estimation investigated in this paper is presented in [10].

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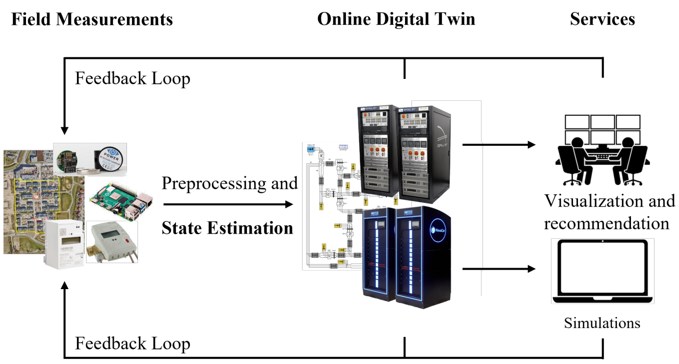

Fig. 1. Data Flow and Digital Twin Architecture

III. Weighted Least Squares Algorithm

The most commonly used algorithm for state estimation is the Weighted Least Squares (WLS) method [4], [11], in which the state vector x represent either bus voltages or branch currents, expressed in Polar or Cartesian coordinates [5]. The algorithm starts with (1).

$$\boldsymbol{r} = \boldsymbol{z} - \boldsymbol{h}(\boldsymbol{x}) \tag{1}$$

The measurement data is summarized in the measurement vector z. The corresponding vector of calculated measurements is $h(x)$, and r is the residual vector. In expanded form (1) can be written as (2).

$$\begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_m \end{bmatrix} = \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_m \end{bmatrix} - \begin{bmatrix} h_1(x_1, \ldots, x_n) \\ h_2(x_1, \ldots, x_n) \\ \vdots \\ h_m(x_1, \ldots, x_n) \end{bmatrix} \tag{2}$$

The Weighted-Least-Squares objective function $J(x)$ is defined in (3).

$$J(x) = r^T R^{-1} r \tag{3}$$

The weighting matrix R is determined by the measurement accuracy. It defines how trustworthy the measurement results are compared to the pseudo-measurements. Differentiating (3) with respect to x yields to (4).

$$\frac{\partial J(x)}{\partial x} = -2\, H^T(x)\, R^{-1}\, r \tag{4}$$

Where $H(x)$ represents the Jacobian matrix. At the optimal solution, the gradient vanishes, leading to (5).

$$H^T(x)\, R^{-1}\, r = 0 \tag{5}$$

Since $h(x)$ is generally nonlinear, it is linearized about an initial estimate x_0 using the first-order Taylor expansion (6).

$$h(x) \approx h(x_0) + H(x_0)\, \Delta x \tag{6}$$

Substituting (6) into (5) leads to (7).

$$G(x_0)^{-1} H^T(x_0)\, R^{-1}\, r = \Delta x \tag{7}$$

Where the gain matrix $G(x_0)$ can be expressed as (8).

$$G(x_0) = H^T(x_0)\, R^{-1}\, H(x_0) \tag{8}$$

Equation (7) forms the basis of the Distribution System State Estimation (DSSE) and is solved iteratively using the Newton–Raphson method until $\|\Delta x\|$ falls below a predefined tolerance ϵ. The measurement vector z and the calculated vector $h(x)$ must have the same structure.

The algorithm follows the steps, depicted in Fig. 2:

1) Initialization of the state vector x
2) Computation of the measurement function $h(x)$
3) Construction of the Jacobian $H(x)$
4) Computation of the gain matrix $G(x)$
5) Solution of (7) for Δx
6) Checking convergence
7) If not converged: $x = x + \Delta x$, repeat from Step 2

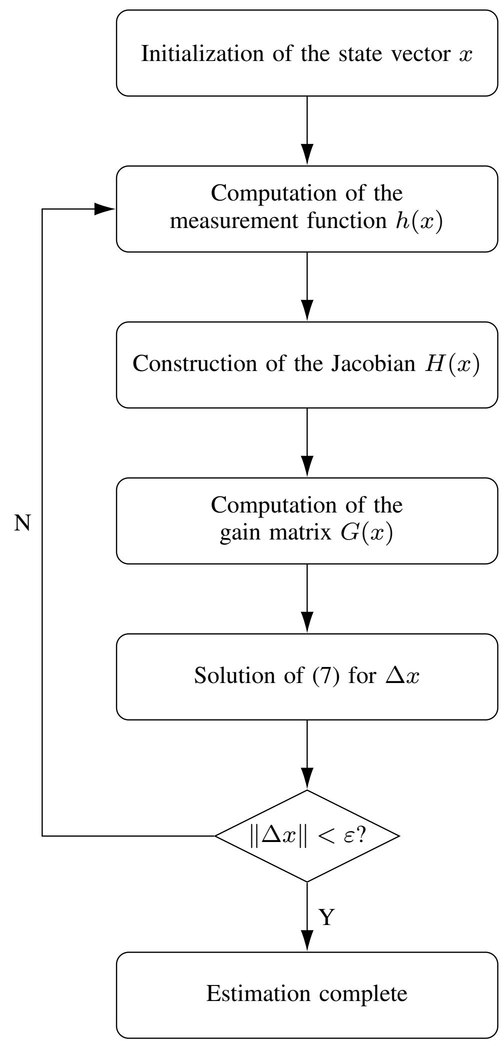

Fig. 2. Weighted Least Squares (WLS) state estimation process

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IV. State Estimation within the Residential Low-Voltage Grid in Project EOS

This chapter first highlights the distribution of measurement points and the locations of data acquisition units, and then explains the WLS-based DSSE algorithm in the context of the project area.

A. Measurement Infrastructure

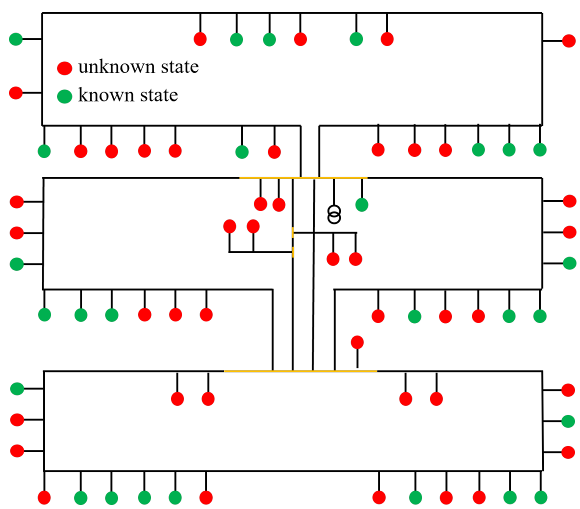

Fig. 3. Grid Topology within the study area

Out of the 70 houses in the grid, 27 are equipped with measurement devices, shown in Fig. 3. For each of the three phases (L1, L2, and L3), the quantities voltage V, current I, active power P, reactive power Q, and phase angle φ are recorded at ten-second intervals and stored in a database. The values, which are used for the state estimation are calculated to a moving average with a sample rate of five-minutes. Additional measurements from the transformer, including e.g. the single phases and total active and reactive power values, are also stored. The data are retrieved in Python and assembled into the measurement vector z.

For buildings without measurement devices, pseudo-measurements are generated in Python to initialize the algorithm. The voltage is assumed to be 224.25 V due to the fixed tap changer position of -2.5 % and the voltage angle is shifted by 120° between phases. The active and reactive power per phase are computed as the difference between the measured transformer power and the aggregated measured power of the monitored buildings. The resulting power is then allocated to the unmonitored buildings proportionally to their respective floor areas, assuming that larger floor areas correlate with higher electrical energy consumption or generation. Finally, the current of each building is obtained from the assigned active power and the assumed voltage.

The considered state variables are the voltage magnitudes V and voltage angles θ. Accordingly, the state vector contains both measured and pseudo-measured voltage values.

B. Data Preparation and Model Initialization for WLS Distribution System State Estimation

This section describes the project-specific data preparation process required for the WLS-based state estimation, as depicted in Fig. 4.

The data preparation starts with a database query in Python. Measurement data are extracted, checked for quality, and discarded if corrupted [10]. Valid data are subsequently averaged over the last five minutes. Based on the available measurements, unknown or not valid grid nodes are equipped with pseudo-measurements, and the measurement vector z and state vector x are constructed.

In parallel, topological data, particularly the cable parameters, are imported into Python, and the admittance matrix Y is computed. The resulting admittance matrix Y, the measurement vector z, and the state vector x are then used to evaluate the measurement function $h(x)$ and its Jacobian $H(x)$.

The calculated values of P, Q, and I for $h(x)$ can be derived from (9), (10), and (11)

$$P_i = \sum V_i V_j \left[G \cos(\Delta\theta_{ij}) + B \sin(\Delta\theta_{ij})\right] \tag{9}$$

$$Q_i = \sum V_i V_j \left[G \sin(\Delta\theta_{ij}) - B \cos(\Delta\theta_{ij})\right] \tag{10}$$

$$I_i = \sum Y_{ij} V_j \tag{11}$$

The partial derivatives of $h(x)$ lead to the Jacobian matrix $H(x)$. They are obtained as follows:

$$\frac{\partial P_i}{\partial \theta_i} = \sum V_i V_j \left[-G_{ij} \sin(\Delta\theta_{ij}) + B_{ij} \cos(\Delta\theta_{ij})\right] \tag{12}$$

$$\frac{\partial Q_i}{\partial \theta_i} = \sum V_i V_j \left[G_{ij} \cos(\Delta\theta_{ij}) + B_{ij} \sin(\Delta\theta_{ij})\right] \tag{13}$$

$$\frac{\partial P_i}{\partial V_i} = \sum V_j \left[G_{ij} \cos(\Delta\theta_{ij}) + B_{ij} \sin(\Delta\theta_{ij})\right] \tag{14}$$

$$\frac{\partial Q_i}{\partial V_i} = \sum V_j \left[G_{ij} \sin(\Delta\theta_{ij}) - B_{ij} \cos(\Delta\theta_{ij})\right] \tag{15}$$

$$\frac{\partial I}{\partial \theta} = \sum \Re\left(\frac{\overline{I_{\text{complex}}}}{|I|} Y_{ij}\, \mathrm{j}\, V_{c,j}\right) \tag{16}$$

$$\frac{\partial I}{\partial V} = \sum \Re\left(\frac{\overline{I_{\text{complex}}}}{|I|} Y_{ij} \frac{V_{c,j}}{V_j}\right) \tag{17}$$

Finally, the WLS algorithm for DSSE is executed, and the resulting state vector is obtained. Based on the estimated voltages, the corresponding household power consumption or generation is derived and fed into the real-time Digital Twin of the residential area.

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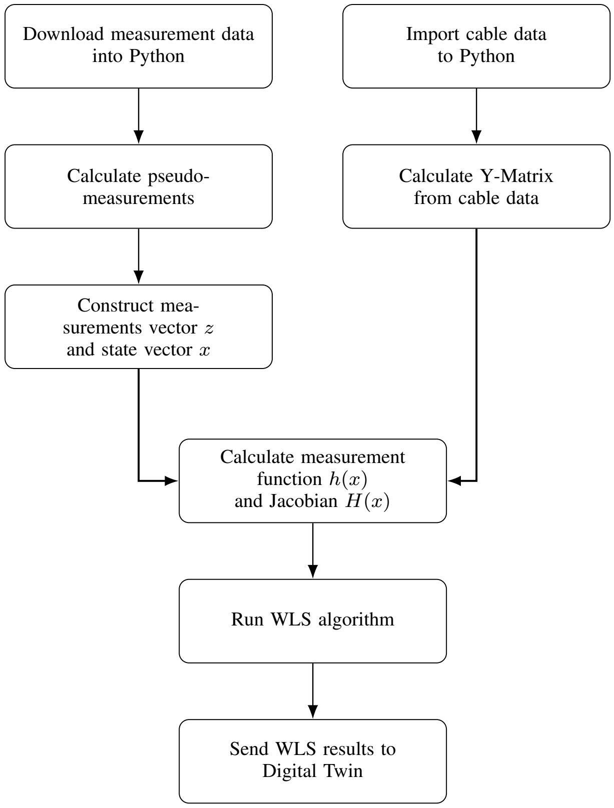

Fig. 4. State estimation in project EOS

V. State Estimation - Results and Validation

The following section presents the results of the Distribution System State Estimation, including a comparison between the measurements, the estimated states, and the simulation. The valid data are fed into the online Digital Twin with a cycle time of five minutes, resulting in 288 values per day for evaluating the deviations. The day shown in the plots is November 4th, 2025, which represents a sunny day. Fig. 5 illustrates the active and reactive power of each phase at the transformer station, comparing the field measurements (DB - Database) with the state estimation results (SE). A strong overall correlation is evident. The reactive power exhibits a mean daily deviation of 0.72 %. A slight deviation can be observed around noon in the active power values, which is associated with the high photovoltaic generation at that time. The maximum deviation amounts to 17.91 %, whereas the mean deviation over the entire day is approximately 3.07 %.

The plots in Fig. 6 compare the phase voltages and reveal a strong overall agreement among the three datasets: field measurements (DB), state estimation (SE), and simulation (sim). The corresponding deviations are summarized in Tab. I. The results show the mean daily deviation remains small, below 0.6 %. The maximum deviation at any time during the day is listed in Tab. II. These deviations stay below 2.4 %, with the discrepancies between the simulation and the measured database values being slightly larger than those between the state estimation and the database, which are below 2.1 %.

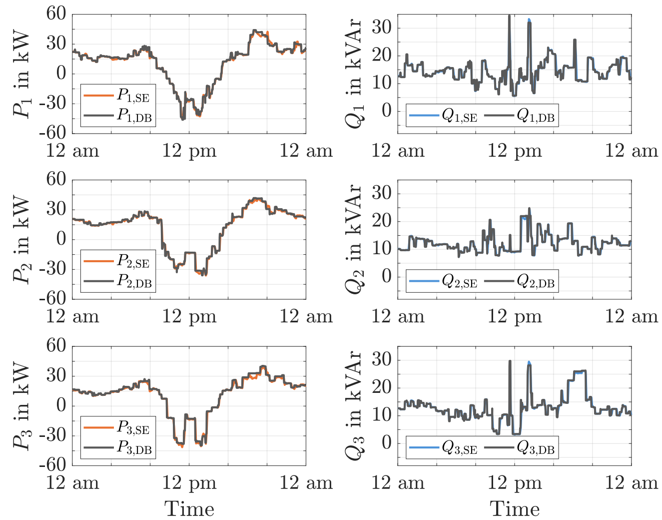

Fig. 5. Active and reactive power comparison between measurements and state estimation at the transformer station

TABLE I
Mean daily deviation of voltages of state-estimated and simulated values compared to field measurements (%)

	sim referred to DB			SE referred to DB		
	V1 [%]	V2 [%]	V3 [%]	V1 [%]	V2 [%]	V3 [%]
Street 1	0.29	0.33	0.35	0.37	0.30	0.47
Street 2	0.44	0.39	0.38	0.40	0.41	0.37
Street 3	0.37	0.37	0.49	0.33	0.31	0.53
Street 4	0.36	0.34	0.42	0.36	0.34	0.55
Street 5	0.35	0.32	0.35	0.46	0.40	0.40
Street 6	0.38	0.28	0.33	0.41	0.46	0.55

VI. Summary and Outlook

This paper presents the implementation of a Distribution System State Estimation (DSSE) based on the Weighted Least Squares (WLS) algorithm in the low-voltage grid. The input data consist of measurements from 27 out of 70 buildings (38.5 %) within an urban district grid, as well as measurements from the grid-feeding transformer station. Based on these measurements, pseudo-measurements are generated and assigned to the corresponding buildings. The resulting measurement vector serves as input for the DSSE. From the estimated state variables, voltage magnitude and phase angle, the corresponding active and reactive power values are derived and fed into the online Digital Twin. The update cycle time of the Digital Twin is five minutes, which is mainly limited by the database read performance and the associated IT infrastructure. The resulting states within the Digital Twin are exported and compared with the direct field measurements and the estimated values. The mean daily error for the simulated and analyzed day is below 0.6 %. Ongoing work focuses on replacing the WLS-based DSSE with more advanced approaches such as Kalman filtering. Subsequently, the Digital Twin will be

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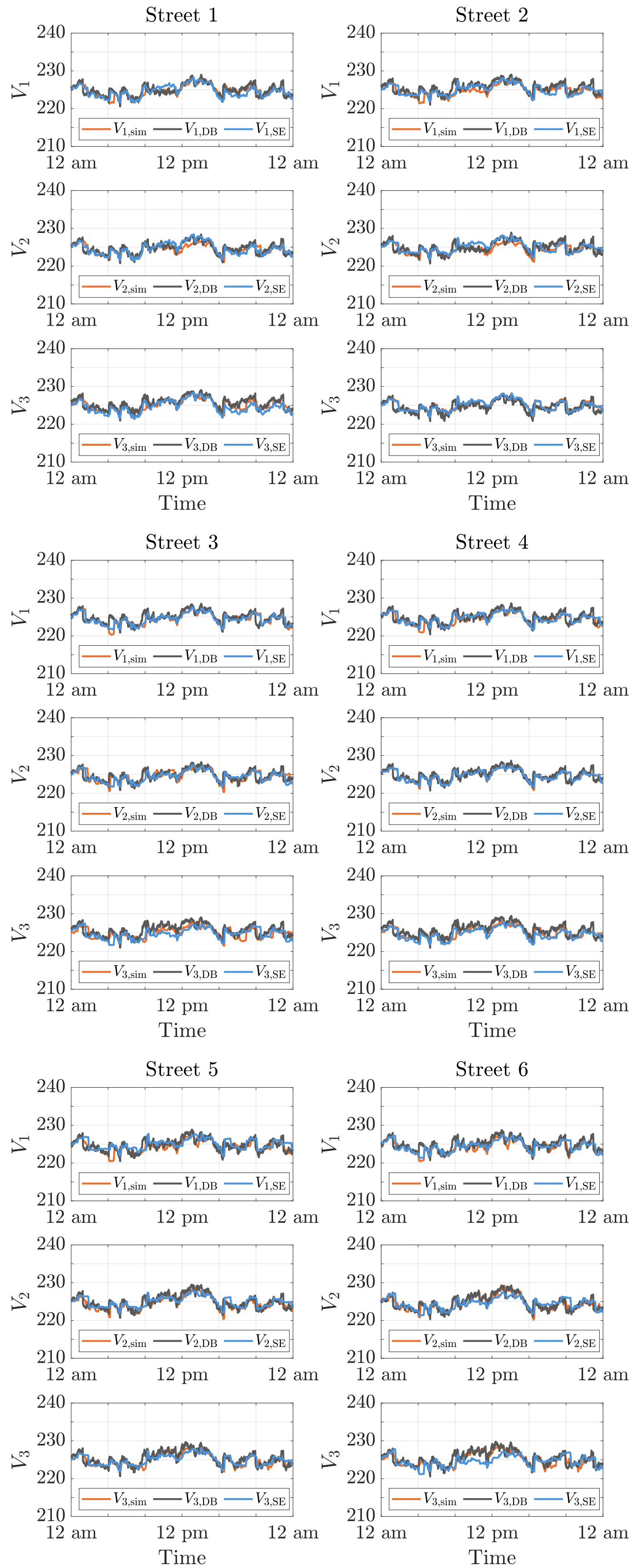

Fig. 6. Voltage comparison between Measurement, State Estimation, and Simulation

TABLE II
MAXIMUM DEVIATION OF VOLTAGES OF STATE-ESTIMATED AND SIMULATED VALUES COMPARED TO FIELD MEASUREMENTS (%)

	sim referred to DB			SE referred to DB		
	V1 [%]	V2 [%]	V3 [%]	V1 [%]	V2 [%]	V3 [%]
Street 1	1.80	1.91	2.00	1.69	1.14	1.70
Street 2	2.01	2.37	1.48	1.70	1.70	1.43
Street 3	2.26	2.37	2.39	1.63	1.56	1.96
Street 4	1.94	2.16	2.15	1.95	1.79	2.11
Street 5	2.20	2.38	2.30	1.77	1.58	1.82
Street 6	2.16	2.26	2.23	1.93	1.87	2.08

connected to a research control center for real-time monitoring and control purposes.

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