

Embedding Data-Driven Dynamic Efficiency Model into Day-ahead Bidding Strategy of Vanadium Redox Flow Batteries

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Abstract—Vanadium redox flow batteries (VRFBs) have attracted considerable attention due to their unique advantages. However, the efficiency of VRFBs varies dynamically with operating conditions and is influenced by complex mechanisms. Existing bidding strategies of VRFB participation in electricity markets fail to effectively account for dynamic efficiency effects, resulting in poor economic performance. To address this issue, this paper proposes a neural network-based instantaneous power loss model trained on experimental data, and develops an iterative decoupled optimization algorithm to incorporate this model into bidding strategies. First, based on operational data collected from practical VRFB systems under multiple power levels, a neural network is trained to jointly model internal losses and parasitic losses. Subsequently, an iterative optimization algorithm is designed to embed this model into the market bidding process. Case studies based on the PJM market demonstrate that the proposed bidding strategy can achieve higher economic returns.

Keywords—Vanadium redox flow battery, dynamic efficiency, machine learning, neural network, optimization

I. INTRODUCTION

Battery energy storage systems (BESS) are essential for ensuring the safe and efficient operation of modern power systems[1]-[2]. Among various battery storage technologies, the vanadium redox flow battery (VRFB) has attracted increasing attention due to its independently configurable power and energy capacities, long service life, and high safety[3]-[4]. However, compared with lithium-ion batteries, VRFBs exhibit significantly lower energy conversion efficiency—while lithium-ion batteries typically achieve efficiencies of 90%–98%, the efficiency of VRFBs generally ranges from 70% to 85%—and entail higher investment costs[5]. These challenges of high cost and low efficiency have created substantial economic barriers, severely limiting the large-scale commercialization of VRFBs. Against this background, expanding revenue streams through participation in diversified electricity market transactions has become an inevitable approach to improving VRFB economic viability and promoting industrial deployment. Consequently, accurately characterizing VRFB operational characteristics and determining optimal bidding strategies have become key issues of concern for VRFB investors and operators[6].

Various modeling methods have been proposed in existing studies to characterize the power loss of VRFBs. The traditional approach estimates efficiency as a constant value, defined as the ratio of the total discharged energy to the total charged energy over a complete charge–discharge cycle[7]. However, this method overlooks the strong nonlinearity of VRFB power losses, which are affected by the coupled effects

of electrochemical reactions, fluid dynamics, and thermal processes. In addition, the losses during charging and discharging are inherently asymmetric, and these discrepancies are masked when a constant efficiency is assumed[8]. The second approach, the semi-empirical method, models VRFB efficiency as a function fitted from experimental data. Reference [9] calculates the U–I–SOC curves by coupling hydrodynamic and electrochemical models, and derives the instantaneous efficiency for microgrid operation. In [10], a mathematical model that captures the nonlinear characteristics of VRFBs is developed and incorporated into a market optimization framework. Reference [11] develops an electrical circuit model of the VRFB and estimates the model parameters based on field experimental data. These semi-empirical methods replace complex mechanistic equations with low-dimensional, data-fitted functions that preserve key nonlinear relationships with variables such as SOC, power, and flow rate, as well as the asymmetry between charge and discharge processes. They offer low computational cost but limited accuracy, and cannot precisely capture the energy consumption of critical auxiliary subsystems such as cooling systems, electrolyte pumps, and power conversion systems (PCS). For grid-scale VRFBs participating in electricity markets, these parasitic losses are substantial and cannot be neglected. The third approach employs machine learning methods.

In [12], machine learning algorithms were applied to predict VRFB power losses, achieving prediction accuracies of around 98%. In [13], total loss models were constructed using artificial neural networks trained on measured data for system-level simulation and evaluation, and numerical computation was combined with equivalent circuit fitting to rapidly calculate instantaneous efficiency for microgrid energy management. However, these studies did not focus on modeling power losses in the context of electricity market bidding, and therefore cannot be directly applied to practical market-oriented optimization of trading strategies.

At present, studies on bidding strategies for VRFBs participating in electricity markets remain limited. In [14], a dynamic programming approach was employed to account for dynamic efficiency when VRFBs participated in the PJM energy and frequency regulation markets. In [15], VRFB dynamic efficiency and capacity degradation models were integrated into an optimization framework for bidding strategy design. However, both studies relied on semi-empirical efficiency models with limited accuracy. Overall, existing research on VRFB bidding strategies either neglects dynamic power losses or adopts overly simplified representations, making it difficult to accurately quantify

variations in dynamic efficiency. In this work, based on experimental data collected from an operating VRFB system, an instantaneous power loss model tailored for market bidding is developed. Furthermore, an iterative optimization algorithm is proposed, embedding the trained neural network – based power loss model into the bidding strategy optimization framework.

The main contributions of this paper are as follows:

- (1) Real-world operational data from a practical VRFB system under various conditions are collected and analyzed. Internal and parasitic losses are jointly treated, providing high-fidelity data support for instantaneous power loss modeling and addressing the limitations of previous studies that relied primarily on simulations or neglected parasitic losses.
- (2) A data-driven power loss model suitable for VRFB participation in the energy–frequency regulation markets is proposed, offering more accurate power loss predictions for bidding strategy optimization.
- (3) An iterative optimization algorithm is designed, in which the trained neural network–based power loss model is embedded into the bidding strategy solution process, achieving a unified framework that enables both the computability of complex nonlinear efficiency characteristics and the maximization of economic returns.

II. FRAMEWORK

The overall framework of the proposed strategy is illustrated in Fig.1. First, experimental data of the VRFB are collected and processed to quantify internal losses and parasitic losses, forming a unified dataset of total power losses. Based on this dataset, a data-driven instantaneous power loss model is developed using a neural network to characterize the dynamic variation of efficiency. Subsequently, an iterative decoupled heuristic algorithm is designed to embed this model into the bidding optimization framework. By repeatedly updating efficiencies, operating trajectories, and bidding constraints, the framework enables the dynamic efficiency characteristics of the VRFB to be fully reflected in the day-ahead bidding strategy, thereby enhancing the economic performance of the VRFB.

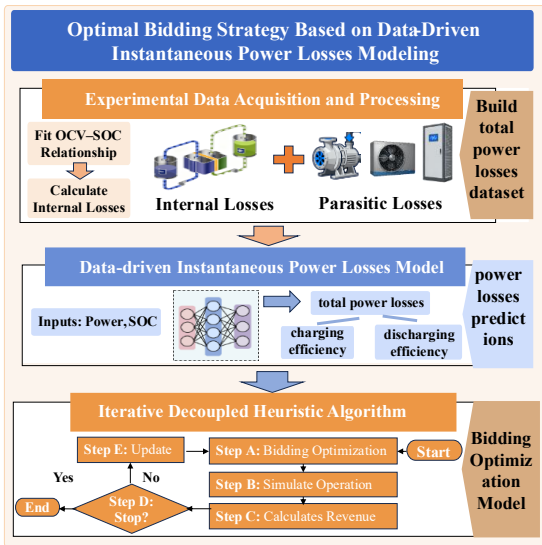


Figure 1 Framework of the proposed strategy

III. DATA-DRIVEN INSTANTANEOUS POWER LOSS MODELING

A. Battery cycle testing data

The data-driven power loss model developed in this paper is based on experimental data obtained from a real 168 kW / 504 kWh vanadium redox flow battery system, as shown in Fig.1. The cycling tests were conducted under power levels of 21 kW, 42 kW, 84 kW, and 126 kW. During the tests, key operational parameters—including voltage, current, temperature, active power setpoint, pump power, and electrolyte flow rate—were recorded with a temporal resolution of 5 seconds. First, the relationship between OCV, SOC, and temperature was derived through experimental fitting. The SOC at the beginning of each charge and discharge cycle was determined using the measured OCV and temperature during rest periods, and the SOC evolution during operation was subsequently calculated via the ampere-hour integration method. The internal battery losses were estimated from the current, terminal voltage, and OCV according to [18], while parasitic losses such as pump power were directly measured by sensors in real time. When participating in electricity market transactions, the economic performance of the system should be evaluated based on its *total* power losses. Therefore, in this work, the energy consumption of all auxiliary subsystems was aggregated with the internal battery losses to construct a unified total power loss metric, which serves as the basis for subsequent modeling and optimization analyses.



Figure 2 Real vanadium redox flow battery system.

B. VRFB instantaneous power losses modeling

Before introducing the instantaneous power loss model, the definition of instantaneous efficiency for the VRFB is first presented, as shown in (1) and (2). Due to the inherent asymmetry between the charging and discharging processes, the charging efficiency and discharging efficiency are defined separately.

$$\eta_t^{cha} = \frac{|P_t - L(P_t, SOC_t)|}{|P_t|} \quad (1)$$

$$\eta_t^{dis} = \frac{|P_t|}{L(P_t, SOC_t) + |P_t|} \quad (2)$$

As mentioned earlier, after processing the dataset, a data-driven power loss model for vanadium redox flow batteries applicable to market bidding was developed. Since the decision variable in day-ahead bidding is power rather than detailed electrochemical variables, the current and voltage of the VRFB cannot be precisely predicted. Therefore, macroscopic features such as power, SOC, electrolyte flow rate, and temperature are used as inputs, while the power loss at each time interval serves as the output. The preprocessed dataset is divided into training, validation, and testing subsets

in proportions of 70%, 15%, and 15%, respectively, to ensure the generalization capability of the model. A multilayer perceptron (MLP) is employed to approximate the nonlinear mapping relationship between the input features and the total power loss. The MLP is a feedforward fully connected neural network consisting of an input layer (6 neurons), two hidden layers (with 64 and 32 neurons, respectively), and an output layer that outputs the instantaneous power loss. Each hidden layer employs a ReLU activation function to enhance nonlinear feature learning. The network parameters are trained using the Adam optimization algorithm with a learning rate of 0.001, and the loss function is defined as the mean squared error (MSE) between the predicted and measured values. The trained MLP model is evaluated using standard metrics such as the coefficient of determination (R^2), root mean square error (RMSE), and mean absolute percentage error (MAPE). The results show that the model achieves an R^2 of 0.984 and a MAPE below 3%, indicating excellent prediction accuracy.

IV. BIDDING STRATEGY AND SOLUTION ALGORITHM

After obtaining the neural network model of the instantaneous power loss of the VRFB, the next key challenge is how to embed this nonlinear data-driven model into the market bidding optimization framework. However, incorporating a neural network directly into an optimization model poses significant challenges. The network structure is highly nonlinear and nonconvex, making it difficult to obtain analytical gradients or convex representations. To address these challenges, we propose a heuristic algorithm that decouples the neural network prediction from the optimization process. The algorithm enables effective coupling between the two modules through iterative information exchange: the neural network provides accurate instantaneous power loss predictions, while the optimization module dynamically adjusts the bidding strategy based on the predicted results. This decoupled framework significantly enhances computational efficiency while maintaining high prediction accuracy, allowing the neural network-embedded optimization problem to achieve fast and stable convergence.

A. Market settings

Without loss of generality, we refer to the PJM energy and frequency regulation market framework and introduces a settlement mechanism based on regulation performance. This mechanism introduces the dynamic regulation signal RegD, characterized by higher mileage, as a supplement to the traditional RegA signal. The compensation for regulation resources consists of two components—capacity payment and performance payment—whose detailed calculation rules are described later in this paper. Given that the capacity of the VRFB is relatively small compared to the overall market and its influence is negligible, it is reasonable to assume that the VRFB acts as a price taker. To account for the uncertainty of day-ahead electricity prices, a set of representative scenarios is constructed to perform stochastic optimization. As a price taker, the VRFB must optimally determine its bidding strategy in both markets based on price forecasts, with the objective of maximizing the expected total profit while satisfying all operational constraints. The VRFB is required to formulate its bidding strategy for the following day before the closure of the day-ahead market.

B. Mathematical Model

The optimization objective of the VRFB bidding model is to maximize the total revenue from participation in the

energy-regulation markets, as shown in (3). Given the long cycle life of VRFBs, degradation costs are not considered. The set of decision variables is given in (4) and includes the energy-market charging and discharging power $P_t^{\text{cha,da}}$ and $P_t^{\text{dis,da}}$ for each time interval i of the operating day, as well as the regulation capacity bid P_t^{reg} .

$$\max_X \sum_s \rho_s \left[\sum_t F_{t,s}^{\text{da}} + F_{t,s}^{\text{reg}} \right] \quad (3)$$

$$X = \{P_t^{\text{cha,da}}, P_t^{\text{dis,da}}, P_t^{\text{reg}}\} \quad (4)$$

Among them, $F_{t,s}^{\text{da}}$ and $F_{t,s}^{\text{reg}}$ represent the energy market revenue and the regulation market revenue in period i under scenario s , respectively. The detailed calculation formulas for $F_{t,s}^{\text{da}}$ and $F_{t,s}^{\text{reg}}$ are given in (5)–(10). The regulation revenue consists of two components: the capacity revenue $F_{t,s}^{\text{cap}}$ and the mileage revenue $F_{t,s}^{\text{mile}}$, both of which are influenced by the regulation performance score $K_{t,s}$. The term $\Delta R_{t,s}$ represents the mileage value of the regulation signal, and I denotes the number of regulation signal points within a time period. Since the PJM RegD signal has a temporal resolution of 2 seconds, I is set to 1800. The variable V represents the average absolute value of the regulation signal within the period.

$$F_{t,s}^{\text{da}} = \lambda_{t,s}^{\text{da}} (P_t^{\text{dis,da}} - P_t^{\text{cha,da}}) \Delta t \quad (5)$$

$$F_{t,s}^{\text{reg}} = (F_{t,s}^{\text{cap}} + F_{t,s}^{\text{mile}}) - \lambda_{\text{mis}} \frac{1}{I} \sum_{i=1}^I |P_{t,s,i} - P_t^{\text{reg}} R_{t,s,i}| \quad (6)$$

$$F_{t,s}^{\text{cap}} = \lambda_{t,s}^{\text{cap}} P_t^{\text{reg}} K_{t,s} \Delta t \quad (7)$$

$$F_{t,s}^{\text{mile}} = \lambda_{t,s}^{\text{mile}} \Delta R_{t,s} P_t^{\text{reg}} K_{t,s} \Delta t \quad (8)$$

$$\Delta R_{t,s} = \sum_{i=1}^{I-1} |R_{t,s,i+1} - R_{t,s,i}| \quad (9)$$

$$K_{t,s} = 1 - \frac{1}{I} \sum_{i=1}^I \left| \frac{P_{t,s,i}^{\text{reg}} - P_t^{\text{reg}} R_{t,s,i}}{V} \right| \quad (10)$$

The constraint conditions are given in (11)–(22).

$$0 \leq P_t^{\text{cha,da}} \leq \mu_t^{\text{da,cha}} P_{\text{max}} \quad (11)$$

$$0 \leq P_t^{\text{dis,da}} \leq \mu_t^{\text{da,dis}} P_{\text{max}} \quad (12)$$

$$\mu_t^{\text{da,cha}} + \mu_t^{\text{da,dis}} \leq 1 \quad (13)$$

$$P_t^{\text{reg}} \leq P_{\text{max}} \quad (14)$$

$$P_{t,s,i} = P_t^{\text{dis,da}} - P_t^{\text{cha,da}} + P_{t,s,i}^{\text{reg}} \quad (15)$$

$$|P_{t,s,i}| \leq P_{\text{max}} \quad (16)$$

$$\text{SOC}_{t,s} = \frac{E_{t+1,s} + E_{t,s}}{2E_{\text{rated}}} \quad (17)$$

$$E_{t+1,s} = E_{t,s} + \left[\eta_{cha,t} (P_t^{cha,da} + P_{t,s}^{reg,cha}) - \frac{P_t^{dis,da} + P_{t,s}^{reg,dis}}{\eta_{dis,t}} \right] \Delta t \quad (18)$$

$$0 \leq E_{t,s} \leq E_{rated} \quad (19)$$

$$SOC_{min} \leq SOC_{t,s} \leq SOC_{max} \quad (20)$$

$$E_{T,s} = E_0 \quad (21)$$

$$K_{t,s} \geq \rho_0 \quad (22)$$

Equations (11)–(13) represent the power bidding constraints of the energy market. The binary decision variables $\mu_t^{da,cha}$ and $\mu_t^{da,dis}$ denote the charging and discharging states at time t , respectively. Equations (15)–(16) describe the actual power constraints of the VRFB. Equations (17)–(21) define the SOC constraints, where $\eta_{cha,t}$ and $\eta_{dis,t}$ are obtained from the neural network outputs. To ensure controllability, the SOC of the VRFB is required to return to the same level at the beginning and end of each operating day. Equation (22) specifies the frequency regulation performance constraint. For each time period in every scenario, the VRFB's regulation performance must remain above a predefined minimum threshold.

C. Iterative Decoupled Heuristic

The previously proposed VRFB bidding model is difficult to solve directly due to the highly nonlinear and nonconvex nature of the embedded dynamic instantaneous efficiency neural network. To address this challenge, a heuristic algorithm is proposed to decouple the neural network from the bidding optimization problem, enabling stepwise iterative solution of the model. The overall procedure consists of the following five steps:

Step A: In the k -th iteration, fix the efficiencies $\eta_{cha,t}$ and $\eta_{dis,t}$, solve the bidding optimization model, and apply the “bidding constraint” generated in Step E. No such constraint is imposed in the initial iteration ($k = 0$). Step B: After obtaining the bidding strategy from the k -th iteration, compute the corresponding VRFB operational profile. Step C: Input the bidding strategy, SOC, and other relevant features into the efficiency neural network model to obtain the actual instantaneous efficiencies $\eta_{cha,t}$ and $\eta_{dis,t}$. Recalculate the total revenue based on these updated efficiencies, which serves as the true revenue of the updated iteration. Step D: If the improvement in true revenue over the previous iteration is below a specified threshold, or if a continuous decline is observed, terminate the iterative process and report the iteration with the best performance; otherwise, proceed to Step E. Step E: Update the bidding constraint for the next iteration as $(1 - \alpha)$ times the total bid quantity from the previous round, and return to Step A for the next iteration.

V. CASE STUDY

A 24-hour day-ahead simulation case is conducted to verify the effectiveness of the proposed strategy. The electricity price data for both the energy and frequency regulation markets are based on the real historical prices from the PJM markets in the United States during 2023 and 2024. The regulation signals are obtained from the publicly released

frequency regulation ancillary service operation data of the PJM market in 2024. In the simulation, the regulation market prices and mileage utilization rates are represented by the hourly average and cumulative values, respectively.

A. Analysis of the Proposed Bidding Strategy Results

This subsection presents the detailed experimental results of the proposed bidding strategy. The iterative revenue curve is shown in Fig.2. During the initial iterations, no efficiency constraints were imposed on the VRFB; as a result, the actual operation exhibited frequent SOC limit violations due to efficiency effects, leading to actual revenues significantly lower than the theoretical ones. In this stage, the actual revenue was only \$44.85. The revenue reached its maximum value of \$66.25 at the 12th iteration. As the operational range was further tightened in subsequent iterations, the revenue gradually decreased. Therefore, the 12th iteration is identified as the optimal round. Fig.3. illustrates the submitted power schedules in the energy market across different iterations. It can be observed that, overall, the VRFB discharges during high-price periods and charges during low-price periods. In the initial iteration ($k = 0^*$), since no efficiency constraints were imposed, the bidding strategy was the most aggressive. In reality, however, when efficiency effects are considered, arbitrage cannot be successfully achieved in certain time intervals. The bidding strategy obtained in the optimal iteration represents a more balanced approach, maintaining the VRFB operation mostly within 0.6–0.8 times of its rated power rather than operating continuously at full capacity. This behavior results from the fact that both charging and discharging efficiencies decline significantly when the output approaches the rated power limit. Fig.4. illustrates the submitted regulation capacities in different iterations. For the VRFB, the revenue obtained from the regulation market is higher than that from the energy market. Therefore, as the iterations proceed, the decline in the submitted regulation capacity is less pronounced than that in the energy market. The optimization model tends to first adjust the power outputs in the energy market to reduce power losses and achieve a more efficient operating state.

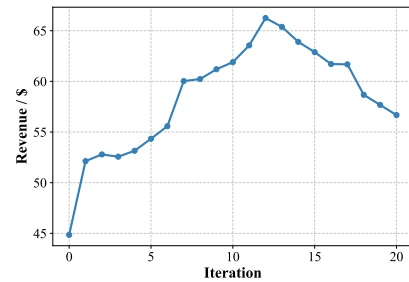


Figure 3 Iterative revenue curve.

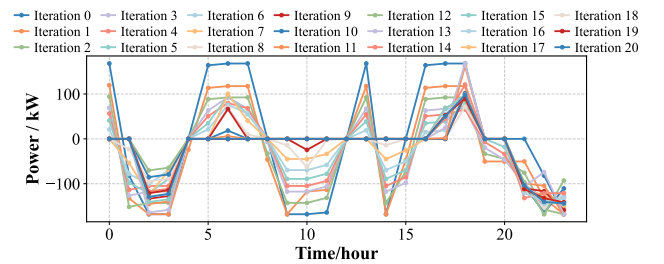


Figure 4 Power bids in the spot market across different iterations.

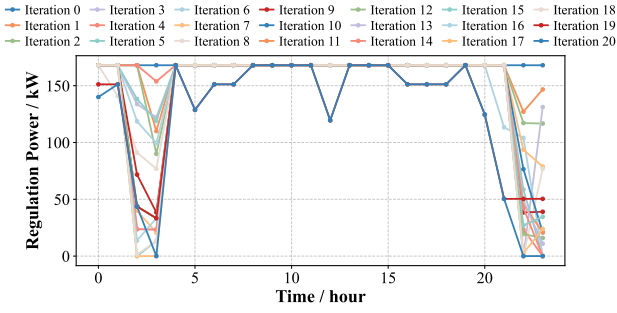


Figure 5 regulation capacity across different iterations.

B. Comparison with Conventional Efficiency Models

To evaluate the effectiveness of the proposed bidding strategy, a comparative analysis is conducted against bidding strategies based on the constant-efficiency model and the semi-empirical efficiency model.

Table I summarizes the actual revenues obtained under different models. Compared with the bidding strategy based on the constant-efficiency model, the semi-empirical model and the proposed model increased the total revenue by 20.2% and 37.39%, respectively. This demonstrates that the effects of power and SOC on efficiency have a significant influence on the economic performance of bidding strategies; therefore, accurately accounting for VRFB power losses during bidding is essential. Because the semi-empirical model exhibits relatively large errors—particularly at low power levels, where the modeling accuracy of power losses is poor—the resulting improvement in revenue is less significant than that achieved by the proposed model. In addition, since both the bidding strategy and efficiency influence the SOC trajectory of the VRFB, strategies based on constant- and semi-empirical efficiency models often experience SOC limit violations and fail to restore the SOC to its initial level by the end of the scheduling horizon. Consequently, these strategies require additional energy settlements in the real-time market at real-time prices, exposing the system to higher market volatility risks. This difference would become even more pronounced under greater fluctuations in real-time prices.

Table I Actual revenues under different models

Indicator	constant-efficiency model	semi-empirical efficiency model	proposed strategy
revenue	48.22	57.96	66.25
real-time market	-9.16	-3.25	-1.89

VI. CONCLUSIN

In this paper, a data-driven instantaneous power loss model and an iterative decoupled optimization algorithm based on neural networks are proposed to address the challenges of instantaneous efficiency modeling and optimization for VRFBs participating in electricity market bidding. By collecting experimental data from an actual VRFB system, a joint modeling framework for internal and parasitic losses is established. Incorporating instantaneous efficiency into the joint bidding of the energy and frequency regulation markets leads to significant changes in the optimal

bidding strategy of the VRFB. Simulation results based on real market data demonstrate that operating the VRFB under high-efficiency conditions yields greater economic returns. Compared with traditional constant-efficiency and semi-empirical models, the proposed strategy achieves more rational energy scheduling, higher profitability, and stronger operational controllability by explicitly considering dynamic efficiency effects.

REFERENCES

- [1] N. Padmanabhan, M. Ahmed, and K. Bhattacharya, "Battery energy storage systems in energy and reserve markets," *IEEE Trans. Power Syst.*, vol. 35, no. 1, pp. 215–226, Jan. 2020.
- [2] M. Kazemi and H. Zareipour, "Long-term scheduling of battery storage systems in energy and regulation markets considering battery's lifespan," *IEEE Trans. Smart Grid*, vol. 9, no. 6, pp. 6840–6849, Nov. 2018.
- [3] S. Wang, B. Xiong, C. Xie, and Z. Wei, "A prediction model of state of health for vanadium redox flow batteries based on variational mode decomposition and integrated neural network," *IEEE J. Emerg. Sel. Topics Ind. Electron.*, vol. 6, no. 4, pp. 1221–1230, Oct. 2025.
- [4] M. Jafari, A. Sakti, and A. Botterud, "Optimization of electrolyte rebalancing in vanadium redox flow batteries," *IEEE Trans. Energy Convers.*, vol. 37, no. 1, pp. 748–751, Mar. 2022.
- [5] B. Xiong et al., "Design of a two-stage control strategy of vanadium redox flow battery energy storage systems for grid application," *IEEE Trans. Sustain. Energy*, vol. 13, no. 4, pp. 2079–2091, Oct. 2022.
- [6] Y. Tohidi, M. Gibescu, and W. Kout, "Energy arbitrage of hydrogen-bromine flow battery between the day-ahead and imbalance electricity markets considering price uncertainty," in *Proc. IEEE Int. Energy Conf. (ENERGYCON)*, Limassol, Cyprus, 2018, pp. 1–6.
- [7] D. Li, N. Xiao, K. Zhan, Y. Jiang, J. Gao, and F. Yang, "Study on multi-timescale scheduling strategy based on large-capacity liquid flow battery for renewable energy consumption," in *Proc. IEEE 13th Data Driven Control and Learning Systems Conf. (DDCLS)*, Kaifeng, China, 2024, pp. 1859–1864.
- [8] M. Resch, J. Bühler, B. Schachler, et al., "Technical and economic comparison of grid supportive vanadium redox flow batteries for primary control reserve and community electricity storage in Germany," *Int. J. Energy Res.*, vol. 43, no. 1, pp. 337–357, 2019.
- [9] J. W. Ni, M. J. Li, and T. Ma, "The study of energy filtering management process for microgrid based on the dynamic response model of vanadium redox flow battery," *Appl. Energy*, vol. 336, Art. no. 120867, 2023.
- [10] T. A. Nguyen, D. A. Copp, R. H. Byrne, et al., "Market evaluation of energy storage systems incorporating technology-specific nonlinear models," *IEEE Trans. Power Syst.*, vol. 34, no. 5, pp. 3706–3715, 2019.
- [11] X. Qiu, T. A. Nguyen, J. D. Guggenberger, M. L. Crow, and A. C. Elmore, "A field validated model of a vanadium redox flow battery for microgrids," *IEEE Trans. Smart Grid*, vol. 5, no. 4, pp. 1592–1601, Jul. 2014.
- [12] R. Bryce, A. Latif, A. Nagarajan, et al., "Novel artificial neural network model for instantaneous power losses and operational efficiency mapping of MW-scale vanadium redox flow battery for improved technoeconomic analysis," *J. Energy Storage*, vol. 123, Art. no. 116635, 2025.
- [13] A. Verma, S. De, A. Gupta, et al., "Ensemble and deep learning based prediction of vanadium redox flow battery system power loss and a precision equivalent circuit model for parameter benchmarking," *J. Energy Storage*, vol. 123, Art. no. 116780, 2025.
- [14] T. A. Nguyen, R. H. Byrne, B. R. Chalamala, et al., "Maximizing the revenue of energy storage systems in market areas considering nonlinear storage efficiencies," in *Proc. Int. Symp. Power Electron., Electr. Drives, Autom. Motion (SPEEDAM)*, IEEE, 2018, pp. 55–62.
- [15] G. He, Q. Chen, C. Kang, et al., "Optimal operating strategy and revenue estimates for the arbitrage of a vanadium redox flow battery considering dynamic efficiencies and capacity loss," *IET Gener. Transm. Distrib.*, vol. 10, no. 5, pp. 1278–1285, 2016.