A Physics-Informed Evaluation Method for Improving the Accuracy of Single-PMU Center of Inertia Frequency Estimation

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***Abstract*—The growing penetration of renewable energy sources reduces system inertia, making the accurate estimation of the Center of Inertia (CoI) frequency crucial for maintaining grid stability. While methods exist for estimating CoI frequency from a single Phasor Measurement Unit (PMU), they often yield multiple candidate curves due to parameter ambiguity, creating uncertainty about the true system state. This paper proposes a novel, physics-informed evaluation method to address this challenge. The method is founded on the fundamental principle that any single generator's frequency oscillates around the true CoI frequency. By quantifying this relationship through a mean bias metric, our framework systematically evaluates and selects the most accurate CoI curve from a set of candidates. Simulations conducted on the IEEE 39-bus system demonstrate that the proposed method successfully identifies the candidate curve that most closely matches the ground truth, thereby providing a vital validation layer and significantly improving the accuracy and trustworthiness of CoI estimations derived from single PMU measurements.**

***Index Terms*—Center of Inertia (CoI), estimation, single-PMU measurement, physics-informed methods.**

I. Introduction

The transition towards a sustainable energy landscape, marked by the large-scale integration of inverter-based resources (IBRs) like solar and wind power, presents profound challenges to power system stability [1]. The displacement of traditional synchronous generators by IBRs, which typically lack rotational inertia, leads to a decline in the overall system inertia. Consequently, the grid becomes more vulnerable to frequency excursions following disturbances, with higher Rates of Change of Frequency (RoCoF) [2]. In this new paradigm, the Center of Inertia (CoI) frequency, which represents the inertia-weighted average frequency of the entire system, has become the definitive indicator of system-wide frequency health. Accurate, real-time knowledge of the CoI frequency and its RoCoF is paramount for deploying advanced control actions, such as fast frequency response and emergency load shedding [3]. While Phasor Measurement Units (PMUs) provide the high-resolution data necessary for such monitoring, achieving full PMU observability across all generating units is often economically and practically infeasible. This has spurred research into estimating the CoI frequency from a limited number of PMUs, with single-PMU methods being particularly attractive due to their minimal data requirements.

Several techniques have been proposed to estimate the CoI frequency using local measurements [3]–[6]. A notable example is the method based on identifying the inflection points of the local frequency curve, followed by polynomial fitting to reconstruct a continuous CoI trajectory [5]. This method enables CoI frequency estimation using only data from a single machine and has been subsequently leveraged to enable the estimation of total system inertia [7]. While powerful, it faces a critical and often overlooked challenge: non-uniqueness. The choice of key parameters within the estimation algorithm—for instance, the order of the polynomial used for fitting—can lead to the generation of multiple, distinct, yet plausible CoI candidate curves from the exact same input data.

This ambiguity presents a significant barrier to practical application. Without a systematic process to validate these candidate curves, an operator or an automated control system is left to make an arbitrary selection, which could lead to a flawed assessment of the system's state and potentially incorrect control actions. This lack of a reliable validation step undermines the trustworthiness of the entire single-PMU estimation approach. To address this critical gap, this paper introduces a physics-informed evaluation method designed to systematically assess multiple CoI candidate curves and identify the one that most accurately reflects the true system dynamics. The physics-informed nature of our approach stems from its reliance on a fundamental law of power system electromechanical dynamics: an individual generator's frequency is physically constrained to oscillate around the system's CoI frequency.

The primary contributions of this work are twofold. First, a framework is proposed for validating and selecting the optimal CoI frequency curve from a set of candidates generated by single-PMU estimation techniques. Second, a physics-informed metric is introduced that employs the mean bias

This work was supported by JSPS KAKENHI Grant Number JP25K23458.

979-8-3315-8138-1

between the measured generator frequency and each candidate curve as a clear and physically meaningful indicator, directly quantifying how well a candidate curve serves as the "center of oscillation".

The proposed evaluation framework is subsequently used to validate the CoI estimation strategy presented in [5], demonstrating its ability to resolve the aforementioned ambiguity. The remainder of this paper details the theoretical foundation of our method, presents the evaluation framework, validates its effectiveness through a comprehensive case study, and concludes with a discussion of its implications.

II. Theoretical Background

A. Core Physical Principle

The Center of Inertia (CoI) frequency is a fundamental concept used to represent the average frequency of an entire power system or a coherent group of generators. It is formally defined as the inertia-weighted average of the rotor speeds of all synchronous machines in the system [8]. Its instantaneous value, $f_{\text{CoI}}(t)$, is given by:

$$f_{\text{CoI}} = \frac{\sum_{i=1}^{N_g} H_i S_i f_i(t)}{\sum_{i=1}^{N_g} H_i S_i} \tag{1}$$

where N_g is the total number of synchronous generators, S_i is the capacity of machine i, while $f_i(t)$ and H_i are the instantaneous frequency and inertia constant of generator i, respectively.

Physically, the CoI frequency represents the motion of the system's center of mass in terms of kinetic energy. Its rate of change (RoCoF) is directly proportional to the net power imbalance (ΔP_{sys}) across the entire system. For this reason, $f_{\text{CoI}}(t)$ serves as the most accurate and reliable indicator of the overall system frequency stability.

The dynamic relationship between the frequency of an individual synchronous generator and the system's CoI frequency is governed by the electromechanical swing dynamics. For any generator i, its instantaneous frequency, $f_{\text{gen},i}(t)$, can be precisely described as the superposition of the system-wide CoI frequency, $f_{\text{CoI}}(t)$, and a local oscillatory component, $\Delta f_{\text{osc},i}(t)$ [9]:

$$f_{\text{gen},i}(t) = f_{\text{CoI}}(t) + \Delta f_{\text{osc},i}(t) \tag{2}$$

where $\Delta f_{\text{osc},i}(t)$ represents the frequency deviation of generator i corresponding to its rotor angle oscillations with respect to the system's center of inertia.

A key consequence of this physical relationship is that the time-average of the oscillatory component, $\Delta f_{\text{osc},i}(t)$, approaches zero over the duration of an electromechanical transient, assuming the system remains stable. This implies that:

$$\frac{1}{T}\int_{t_0}^{t_0+T} \Delta f_{\text{osc},i}(t)\,dt \approx 0 \tag{3}$$

where the interval $[t_0, t_0+T]$ covers the transient frequency response period. Therefore, an accurate estimate of the CoI frequency, $\hat{f}_{\text{CoI}}(t)$, must serve as the true "center line" for the measured generator frequency.This fundamental principle forms the bedrock of our proposed evaluation method.

B. Review of the Inflection Point Polynomial Fitting Method

The estimation strategy proposed in [5], which we aim to validate, provides a means to estimate $f_{\text{CoI}}(t)$ using only a single local frequency measurement, $f_{\text{gen}}(t)$. The method is based on the inflection point principle, which posits that, under idealized conditions, the local frequency curve intersects the true CoI frequency curve at its inflection points (i.e., where $d^2 f_{\text{gen}}/dt^2 = 0$).

The estimation process consists of two main steps:

1) **Discrete Point Identification:** The algorithm numerically processes the measured $f_{\text{gen}}(t)$ signal to identify the timestamps t_k of its inflection points. These points, $(t_k, f_{\text{gen}}(t_k))$, are then treated as discrete samples of the true CoI frequency.
2) **Continuous Curve Reconstruction:** A continuous CoI frequency trajectory, $\hat{f}_{\text{CoI}}(t)$, is reconstructed by fitting a polynomial of a chosen order to the set of identified discrete points.

The critical challenge, and the source of ambiguity, lies in the second step. The selection of the polynomial order is a non-trivial decision that directly impacts the resulting curve. A low-order polynomial may oversmooth the trajectory and miss important dynamics, while a high-order polynomial risks overfitting the discrete points and introducing spurious oscillations. Consequently, different choices for the polynomial order will yield a set of distinct candidate curves, $\{\hat{f}_{\text{CoI},1}(t), \hat{f}_{\text{CoI},2}(t), \ldots, \hat{f}_{\text{CoI},N}(t)\}$. This inherent sensitivity to a key parameter creates the central problem addressed by this paper: given this set of candidates, a systematic method is required to determine which one is the most accurate representation of the true CoI frequency.

III. Proposed Method

To resolve the ambiguity inherent in single-PMU CoI estimation, this section introduces a systematic evaluation framework. The framework is built upon the core physical principle of oscillation and is fortified with a sensitivity analysis to ensure the robustness of its final selection.

A. Evaluation Criterion: Mean Bias

Building upon the physical principle established in Section II-A, we can formulate a direct and physically meaningful evaluation metric. For any given candidate CoI curve, $\hat{f}_{\text{CoI},k}(t)$, its accuracy can be assessed by examining the deviation signal, $\Delta f_k(t)$, relative to the measured generator frequency, $f_{\text{gen}}(t)$:

$$\Delta f_k(t) = f_{\text{gen}}(t) - \hat{f}_{\text{CoI},k}(t) \tag{4}$$

According to (3), if $\hat{f}_{\text{CoI},k}(t)$ were a perfect estimation of the true CoI frequency, the time-average of $\Delta f_k(t)$ over a transient period would be zero. Therefore, we define the Mean Bias, B_k, for each candidate curve as the primary evaluation criterion:

$$B_k = \frac{1}{T_w}\int_{t_{\text{start}}}^{t_{\text{end}}} \Delta f_k(t)\,dt \tag{5}$$

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where the analysis window $[t_{\text{start}}, t_{\text{end}}]$ spans the duration of the primary frequency response, and $T_w = t_{\text{end}} - t_{\text{start}}$. In discrete-time implementation with a sampling interval of Δt, this is calculated as:

$$B_k = \frac{1}{N} \sum_{n=1}^{N} \left(f_{\text{gen}}[n] - \hat{f}_{\text{CoI},k}[n] \right) \tag{6}$$

where N is the number of samples in the window.

B. Robustness Enhancement via Aggregated Bias Score

The selection of a single analysis window, particularly the end time t_{end}, can introduce arbitrariness. To create a more robust evaluation metric that is less sensitive to this choice, the performance of each candidate curve is aggregated across a range of plausible analysis windows.

The analysis is performed as follows:

1) **Defining the Evaluation Range:** An evaluation range of M different analysis windows, $\mathcal{W} = \{W_1, W_2, \ldots, W_M\}$, is created. This is achieved by varying the end time t_{end} over a predefined range from a minimum value ($t_{\text{end,min}}$) to a maximum value ($t_{\text{end,max}}$) with a set increment.
2) **Calculating Biases:** For each candidate curve $\hat{f}_{\text{CoI},k}$, its Mean Bias, $B_k(W_j)$, is calculated for every window $W_j \in \mathcal{W}$.
3) **Aggregating the Score:** The Aggregated Bias Score is defined as the mean of the absolute values of the biases calculated across all windows:

$$B_k^{\text{agg}} = \frac{1}{M} \sum_{j=1}^{M} |B_k(W_j)| \tag{7}$$

It rewards curves that maintain a consistently low bias across all evaluation windows and penalizes those that perform well in some windows but poorly in others. The use of the absolute value prevents positive and negative biases from canceling each other out, which would otherwise provide a misleadingly optimistic score. The final optimal curve is the one with the minimum Aggregated Bias Score.

C. Overall Evaluation Procedure

The complete, structured workflow of the proposed evaluation and selection framework, incorporating the aggregated bias metric, is formally presented as pseudocode in Algorithm 1.

IV. Case Study

A. Simulation Setup

The proposed framework is validated via dynamic simulations on the IEEE 10-machine, 39-bus New England test system as presented in Fig.1, modeled in MATLAB/Simulink. The system loads are represented using the composite ZIP model, comprising 50% constant impedance (Z), 30% constant current (I), and 20% constant power (P) components. Furthermore, the model incorporates a diverse mix of generator types: Generators G3, G6, and G9 are modeled as hydro turbine-governor systems [10]; Generators G7 and G8 are implemented as Virtual Synchronous Generators (VSGs) to represent modern inverter-based resources [11]; and the remaining units are conventional thermal generators with steam turbine-governor models [12]. To represent the decline in system inertia caused by the high penetration of renewable energy, the inertia constants of all synchronous generators were set to a low level. The inertia constants (H) for each of the ten synchronous generators are detailed in Table I.

Algorithm 1 Physics-Informed CoI Curve Selection

Require:
- $f_{\text{gen}}(t)$: Measured generator frequency data
- $\mathcal{F}_{\text{CoI}} = \{\hat{f}_{\text{CoI},1}(t), \ldots, \hat{f}_{\text{CoI},N_c}(t)\}$: Set of N_c candidate CoI curves
- $t_{\text{start}}, t_{\text{end,min}}, t_{\text{end,max}}, t_{\text{step}}$: Window parameters

Ensure:
- $\hat{f}^*_{\text{CoI}}(t)$: Optimal CoI curve

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Initialize 'agg_scores' array of size N_c to all zeros.
Define evaluation range T = {t_end,min, t_end,min + t_step, ..., t_end,max}.
Let M be the number of elements in T.
for k = 1 to N_c do                          ▷ Loop through each candidate curve
    total_abs_bias ← 0
    for each t_end in T do                   ▷ Loop through each window
        Define analysis window W = [t_start, t_end].
        Calculate Mean Bias B_k(W) for f̂_CoI,k over W.
        total_abs_bias ← total_abs_bias + |B_k(W)|
    end for
    agg_scores[k] ← total_abs_bias/M
end for
k* ← arg min_k (agg_scores)                  ▷ Find index of min score
return f̂_CoI,k*(t)
```

TABLE I
System Inertia Distribution

Gen No.	1	2	3	4	5	6	7	8	9	10	Sys
Base MVA	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	10000
H	4.93	2.12	2.51	2.00	1.82	2.44	1.85	1.70	2.42	2.94	2.472

A disturbance is initiated by applying a 600 MW step load increase at Bus 27 at $t = 0.0$ s(denoted as Case A). A set of candidate CoI curves is generated using the method from [5]. The minimum set of consecutive inflection points is selected from the frequency trajectory of the generator (starting from the disturbance) such that the timestamp of the final point exceeded the maximum evaluation end time of $t_{\text{end,max}}$ to serve as discrete estimated CoI samples. To simulate parameter uncertainty, polynomials of varying orders, ranging from 6th to 15th, are fitted to these points, yielding ten distinct candidate curves, denoted as $\{\hat{f}_{\text{CoI},k}(t) | k = 6, \ldots, 15\}$.

The evaluation framework is configured using the frequency of Generator G4 as the single measurement input, $f_{\text{gen}}(t)$. For final validation, the ground truth CoI frequency, $f_{\text{CoI,true}}$, is

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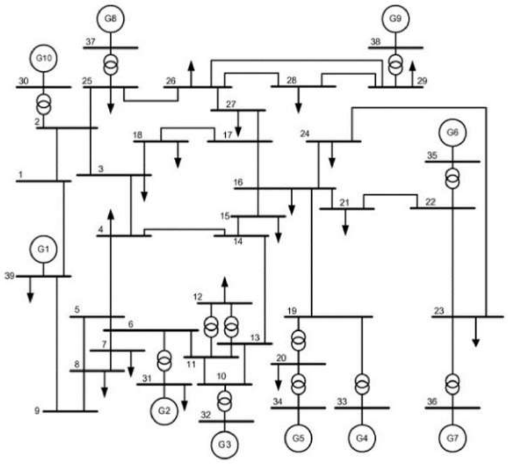

Fig. 1. Single-line diagram of the IEEE 39-bus New England test system.

calculated from all system generators but is withheld from the evaluation process itself. The analysis window is defined with a start time of $t_{\text{start}} = 0.0$ s. The analysis is then conducted over an end-time range of [4.0 s, 6.0 s] in 0.5 s increments. This configuration provides a rigorous test of the framework's ability to identify the optimal estimate from a set of ambiguous candidates.

B. Evaluation Results

The quantitative results of the proposed evaluation framework, presented in the bar chart in Fig. 3, provide a clear and decisive ranking of the candidate curves. The 9th-order polynomial fit achieved the lowest Aggregated Bias Score among all candidates and is marked with an asterisk for emphasis, identifying it as the optimal estimate. This score is remarkably close to the theoretical minimum represented by the 'Actual CoI' bar, which serves as a performance benchmark and confirms the high fidelity of the selected curve. This

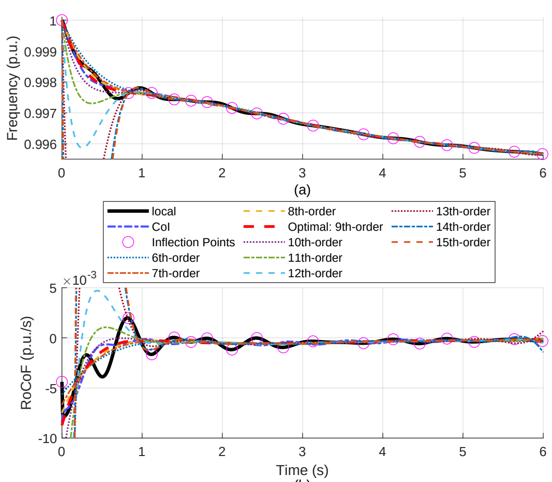

Fig. 2. (a) Frequency and (b) RoCoF trajectory comparison for each candidate curve and actual CoI for Case A

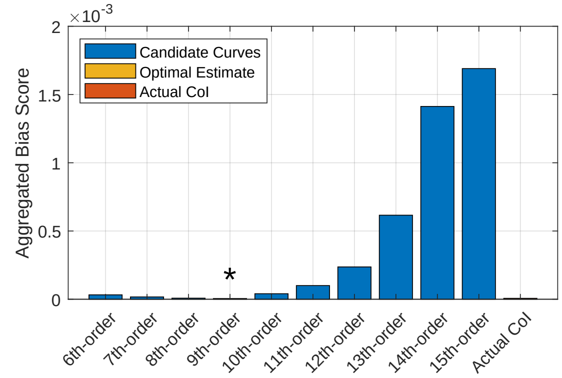

Fig. 3. Aggregated Bias Score for each candidate polynomial order and actual CoI for Case A

quantitative selection is strongly corroborated by the visual evidence in Fig. 2, where the 9th-order polynomial's frequency and RoCoF curves (Fig. 2(a) and Fig. 2(b), respectively) precisely track the acutal CoI. The fit accurately captures the initial frequency dynamics and the volatile RoCoF transient, demonstrating its superiority.

The framework's ability to penalize both under-fitting and over-fitting is also evident. Lower-order polynomials (e.g., 6th-8th order), while achieving low bias scores, exhibit slightly oversmoothed frequency and RoCoF profiles. Conversely, higher-order polynomials (12th-order and above) show clear signs of overfitting, reflected in their escalating bias scores in Fig. 3 and the introduction of spurious, high-frequency oscillations in their RoCoF curves, which do not represent the true system behavior.

It is noteworthy that the estimation utilized 18 inflection points, as shown by the magenta markers in Fig. 2. To ensure the polynomial fit covered the entire evaluation range up to the maximum end time of 6.0 s, it was necessary to include a final inflection point located just beyond this mark. Crucially, the selection of the 9th-order polynomial as the optimal choice aligns perfectly with the model selection criteria proposed in [5].

However, the conservative nature of the strategy in the original work can be a limitation under some challenging conditions. To demonstrate this and highlight the superior robustness of the proposed method, a second case was investigated(denoted as Case B). In this scenario, the system inertia was reduced by an additional 10%, and the disturbance was a 500 MW load increase at Bus 18. The frequency of Generator G7 was used as the single PMU input.

The results for Case B are shown in Fig. 4 and Fig. 5. The estimation process utilized 20 available inflection points. Our physics-informed framework unequivocally identifies the 11th-order polynomial as optimal, as confirmed by its minimal Aggregated Bias Score in Fig. 5. In contrast, the conservative selection strategy from [5] would have selected the 10th-order polynomial,as it suggests choosing a polynomial order of approximately half the number of identified inflection points (20 points in this case). While the 10th-order curve provides a reasonably accurate estimate, a closer inspection reveals

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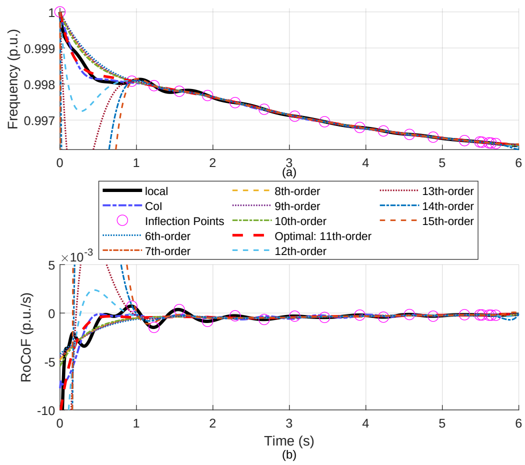

Fig. 4. (a) Frequency and (b) RoCoF trajectory comparison for each candidate curve and actual CoI for Case B

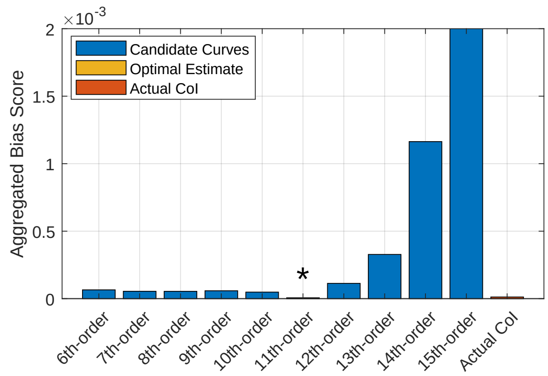

Fig. 5. Aggregated Bias Score for each candidate polynomial order and actual CoI for Case B

its deficiencies. In Fig. 4(b), the estimation of the 10th-order fit is visibly oversmoothed compared to the actual CoI RoCoF, failing to capture the true depth of the initial CoI frequency nadir and RoCoF. The 11th-order curve, selected by the proposed method, tracks both the frequency and RoCoF of the true CoI with significantly higher fidelity.

This comparison demonstrates a key advantage of our proposed method. While the original strategy can be acceptable in some cases, it is not guaranteed to be optimal and may introduce non-negligible errors.The proposed physics-informed method, however, provides a reliable means to identify the true best-fit curve, thereby enhancing the overall accuracy and trustworthiness of the single-PMU estimation technique.

V. Conclusions

This paper has addressed the parameter ambiguity inherent in single-PMU CoI frequency estimation by proposing a novel, physics-informed evaluation method. By proposing an Aggregated Bias Score, the method provides a quantitative and robust means to identify the most accurate candidate curve from a set of ambiguous estimates. Through case studies on a low-inertia test system, the method was shown to not only ratify the choice of existing method under standard conditions but, more importantly, to select a superior estimate in a challenging scenario where the original method proved suboptimal. The primary contribution is thus a systematic validation layer that significantly enhances the trustworthiness of single-PMU estimation techniques for critical applications like real-time inertia monitoring. Moreover, the method is general in nature and has the potential to be applied to other local measurement based CoI estimation methods that produce multiple candidate curves.

Future research will focus on proposing the validated CoI frequency estimates to perform quantitative system stability assessments based on local measurements.

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