A Practical Method for Constructing Dynamic Security Region Based on Maximum Lyapunov Exponent Criterion

Yuan Zeng
State Key Laboratory of Smart Power Distribution Equipment and System, Tianjin University
Tianjin, China
zengyuan@tju.edu.cn

Yuhui Ma, Junzhi Ren
State Key Laboratory of Smart Power Distribution Equipment and System, Tianjin University
Tianjin, China
mayuhui2024234059@tju.edu.cn

Pei Zhang, *Fellow, IEEE,* Xikai Liu
State Key Laboratory of Smart Power Distribution Equipment and System, Tianjin University
Tianjin, China
liuxikai@tju.edu.cn

***Abstract*—The maximum Lyapunov exponent (MLE) leverages power system response data to provide a model-free estimate of the transient stability margin. However, this approach is fundamentally constrained by its binary characteristic, offering a mere stability classification without quantifying the degree of stability. Therefore, this paper proposes a framework for MLE-based dynamic security region (DSR) construction. Firstly, the dynamic characteristics of the typical angular velocity deviation trajectories and MLE trajectories are analyzed to extract the MLE criterion and then a stability boundary function is constructed. Secondly, the linear mapping relationship between injection space parameters and criterion sensitivity is established for a single-machine system and then extended to multi-machine case. Furthermore, based on the sensitivity of the stability boundary function to variations in the injection space parameters, a DSR boundary characterization model of the dominant instable generators is proposed. Finally, the proposed method is validated on the IEEE39 bus system.**

Keywords—maximum Lyapunov exponent, transient stability, sensitivity, dynamic security region

I. INTRODUCTION

The secure and stable operation of power systems is a global issue that directly impacts economic development and social stability, which has long been a primary concern in the power industry [1]. Traditional transient stability analysis often employs the time-domain simulation method, which is widely used in power system planning, design, and operation due to its intuitive nature [2]. However, with the continuous low-carbon transformation of power systems and the increasing share of renewable energy, the development of power electronic-based systems has not only increased the complexity of stability analysis but also heavily relied on accurate model construction [3]. Due to discrepancies between simulation models and actual systems, the simulation results will inevitably exhibit certain deviations. Therefore, with the development of Phasor Measurement Units (PMUs) and Wide Area Measurement Systems (WAMS), real-time collection of generator response data has become feasible [4], and transient stability analysis based on response trajectories has gained increasing attention.

The Lyapunov Exponent (LE) characterizes the average exponential rate of convergence or divergence between neighboring trajectories in the system's operational space. It is a crucial tool for studying the stability and chaotic behavior of nonlinear dynamic systems [5]. Among them, the maximum Lyapunov exponent (MLE) can quantify the long-term dynamic behavior of a system after a disturbance [6], making it applicable for the stability analysis of power systems. Reference [6] calculates the MLE of the time-series power angle curves for each unit and observes the sign of the MLE of the relative power angle curves within a predefined time window to identify transient stability. Further, reference [7] estimates the MLE of the wide-area measured power angle trajectories using a recursive least squares method, proposing a data-driven transient stability assessment approach. Although substantial research has been achieved by using MLE for transient stability assessment in power systems, several challenges remain in its practical application:

- The MLE criterion can only passively reflect the instability state of a power system. Within the reliable stability assessment time window, transient instability may have already occurred between units, lacking sufficient predictive capability.
- Although the MLE criterion can accurately determine the system's dynamic behavior over a longer period after a disturbance, it lacks the ability to quantify stability margins and cannot provide reliable references for preventive or emergency control.

In recent years, the security region (SR) theory has been proven to be an effective tool for assessing the stability margin of power systems and predicting subsequent scheduling security [8]. The dynamic security region (DSR) theory is a theory to judge system stability based on the idea of the region. It transforms the dynamic model of a post-fault time-varying system into a pre-fault linear time-invariant model. By analyzing the distance between the actual operating point and the DSR boundary, the system's stability margin can be quantified, or the fastest adjustment direction for unstable operating points can be provided. Given the advantages of the security region theory in margin quantification, there is an urgent need to conduct research integrating MLE theory with DSR.

Based on the above analysis, this paper proposes a practical method for constructing DSR based on the MLE criterion. First, the MLE variation curves under different disturbances are analyzed to derive and summarize the stability criterion. Then, the mapping relationship between the DSR and the MLE criterion is established, enabling the rapid construction of the DSR boundary. The main contributions of this paper are summarized as follows:

*1)*By adjusting the MLE criterion that uses the rotor speed deviation at the first zero-crossing instant of the MLE trajectory instead of the MLE at infinity, the proposed criterion significantly improves the predictability of transient stability assessment.

XXX-X-XXXX-XXXX-X/XX/$XX.00 ©20XX IEEE
979-8-3315-8138-1

*2)*The method proposed in this paper establishes an effective mapping between the MLE criterion and the injection space parameters. Based on the strong linear effect of the MLE stability boundary function near critical points, a characterization model for the DSR boundary of the dominant unstable unit is developed. Compared with MLE-based stability assessment, the proposed model can capture the system's pre-fault instability state and enable a quantitative evaluation of the stability margin, which is of great significance for the real-time monitoring of secure and stable power grid operation.

II. Theoretical Foundation

A. Maximum Lyapunov exponent theory

Reference [6] summarizes the MLE-based stability criterion as follows: if the system's MLE is negative at the infinite moment after a fault, the system is dynamically stable, and vice versa. This paper employs the model-free method from Reference [7] to calculate the system's MLE. Starting from the fault-clearing moment t_0, the system's MLE is calculated by capturing the slope of the linear segment in Phase II:

$$\begin{aligned} MLE(k) &= \frac{1}{k\Delta t}\lg\left(\frac{LD(m(n),n,k)}{LD(m(n),n,0)}\right) \\ &= \frac{1}{k\Delta t}\lg\left(\frac{\left\|X_{m(n)+k}-X_{n+k}\right\|}{\left\|X_{m(n)}-X_{n}\right\|}\right) \end{aligned} \tag{1}$$

Where $MLE(k)$ denotes the MLE value calculated at period k, $m(n)$ and n denote the initial times of the neighboring and original trajectories, Δt denotes the sampling interval, $LD(m(n),n,0)$ and $LD(m(n),n,k)$ denote the distances between the two trajectories at the initial time and at period k, and ||A−B|| denotes the Euclidean distance between points A and B.

B. Dynamic security region theory

In a multi-machine system's injection power space, the DSR model can be defined as [9]:

$$\Omega_d = \left\{ \mathrm{P} \in R^{n-1} \;\middle|\; \begin{array}{c} \sum_{i=1}^{n-1} \alpha_i P_i < 1 \\ P_i^m < P_i < P_i^M, i = 1,2,\cdots,n-1 \end{array} \right\} \tag{2}$$

Where Ω_d represents the set of safe operating points in DSR, α_i represents the hyperplane coefficient of the i-th generator, P_i^m and P_i^M are the lower and upper limits of the active power output of the i-th unit, and R^{n-1} is an n-1 dimensional variable space, representing all generators except for the slack bus.

III. Effective Mapping Between the MLE Criterion and Dynamic Security Region

A. Maximum Lyapunov exponent stability criterion

For a single machine infinite bus (SMIB) system, neglecting the effects of the governor and exciter, a power imbalance occurs following a large disturbance, and the system motion equation can be expressed as:

$$\begin{cases} \dot{\delta} = \Delta\omega\omega_0 \\ \Delta\dot{\omega}_0 = \dfrac{1}{M}(P_{\mathrm{m}} \text{-} P_{e\max}\sin\delta) \end{cases} \tag{3}$$

Where M represents the inertia time constant. When applying the MLE to assess the transient stability of a power system, the selection of characteristic variables is critical. In this paper, the rotor angle of the generator is chosen as the observation variable, which leads to the following MLE calculation formula for the measured data:

$$MLE(k) = \frac{1}{k\Delta t}\lg\left(\left|\frac{\delta(m(n)+k)-\delta(n+k)}{\delta(m(n))-\delta(n)}\right|\right) \tag{4}$$

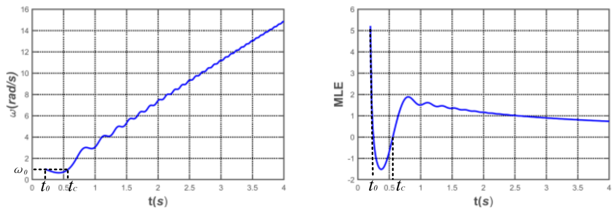
A separation degree (SD) of 1 between the original and neighboring rotor angle trajectories ensures good distinctness [7]. For the N-th measurement of the time-series response trajectory, the following equation holds:

$$\delta(N)-\delta(N-1) = \omega_0\Delta\omega(N-1)/f \tag{5}$$

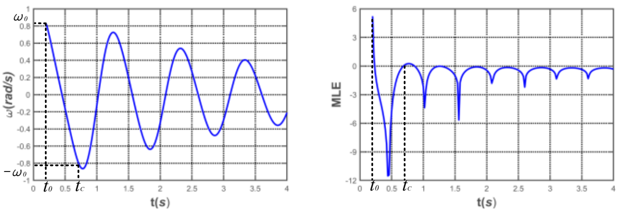
Where f represents the system frequency. Combining equations (3) and (4), the MLE stability criterion is transformed into a criterion based on the angular velocity deviation of the synchronous generator rotor.

$$MLE(k) = \frac{1}{k\Delta t}\lg\left(\left|\frac{\Delta\omega(m(n)-1+k)}{\Delta\omega(m(n)-1)}\right|\right) \tag{6}$$

The characteristic behaviors of synchronous generators, represented by angular velocity deviation trajectory and MLE trajectory following fault clearing, for both typically stable and unstable system scenarios, are illustrated in Figure.1. The angular velocity deviation at the fault-clearing moment t_0 is denoted as ω_0, and the time when the MLE first crosses the X-axis from the negative half-axis is denoted as t_c.



(a) ω and MLE curves of typical instability trajectory



(b) ω and MLE curves of typical stability trajectory

Figure.1. ω and MLE characteristic curves in typical scenarios

As shown in Figure.1.(a), under severe fault disturbances, the generator rotor experiences a brief deceleration phase, followed by acceleration, which leads to system instability. The MLE experiences a brief negative value, then remains positive after crossing the X-axis, with the rotor angular velocity deviation $\omega_{tc} = \omega_0 > 0$ at the moment when it crosses the X-axis. As shown in Figure.1.(b), after the fault occurs and is quickly cleared, the generator rotor immediately enters the deceleration phase, followed by damped oscillations that ultimately stabilize. The MLE first cross the X-axis with the rotor angular velocity deviation $\omega_{tc} = -\omega_0 < 0$, and then oscillates around the zero point until the system stabilizes.

Based on the above analysis, this paper proposes the following stability criterion: after fault clearing, the angular velocity eigenvalue at the moment when the MLE first crosses the X-axis from negative to positive can be used as a substitute for the MLE at infinity as the stability criterion. If $\Delta\omega(t_c) < 0$, the system is stable; otherwise, the system is unstable.

B. Sensitivity analysis of the MLE stability boundary Function

To facilitate the following description, the stability boundary function $f(t_c)$ is introduced, and the stability criterion can be expressed as:

$$\begin{cases} MLE(t_c) = \dfrac{1}{k\Delta t}\lg\left(\left|\dfrac{\Delta\omega(t_c)}{\Delta\omega(t_0)}\right|\right) = 0 \\ f(t_c) = \Delta\omega(t_c) < 0 \end{cases} \tag{7}$$

The sensitivity of the stability boundary function can be analyzed and calculated based on the angular velocity at the fault-clearing moment because of $|\Delta\omega(t_c)| = |\Delta\omega(t_0)|$.

Extensive experimental analysis indicates that when the fault-clearing time is less than the critical fault-clearing time, the angular velocity deviation is linearly related to t_0, and when $t_0 > t_c$ quickly transitions to another linear state, as the equation (7).

$$\Delta\omega(t_0) = \begin{cases} \Delta\omega(t_c) + \kappa_s(t_0 - t_c), & t \le t_c \\ \Delta\omega(t_c) + \kappa_u(t_0 - t_c), & t > t_c \end{cases} \tag{8}$$

Additionally, Reference [10] rigorously verified that, in a single-machine infinite bus system, the generator's active power output P_m is linearly related to the critical fault-clearing time t_c.

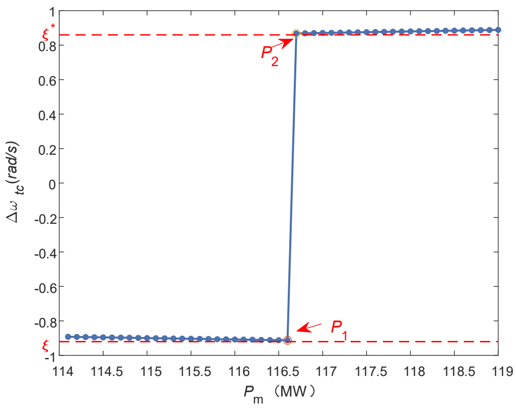


Fig.2. Schematic diagram of the relationship between $\Delta\omega(t_c)$ and P_m

Based on the above analysis, the stability boundary function based on the MLE criterion exhibits a piecewise linear relationship with the system's active power output near the critical stability point. As shown in Fig.2., as the generator's active power output increases, $\Delta\omega(t_c)$ linearity decreases, and when it reaches the minimum value ξ, the system is critically stable. Continuing to increase the generator's output, $\Delta\omega(t_c)$ shifts to another parameter ξ^*, and further increases with another slope. Therefore, the sensitivity of the transient stability boundary function based on the MLE trajectory can be defined by Equation (8):

$$\frac{df(t_c, P_m)}{dP_m} = \begin{cases} k & \xi \le \Delta\omega(t_c) < 0 \\ k^* & \Delta\omega(t_c) \ge \xi^* > 0 \end{cases} \tag{9}$$

C. Dynamic security region boundary

To maintain consistency of the criterion in multi-machine systems, based on the method proposed in Reference [11], the CCCOI-RM transformation technique is used to project the original system's n-dimensional time-series response curves onto n mutually independent planes, thus extending the MLE stability criterion to multi-machine systems.

$$\begin{cases} \Delta\hat{\omega}_i = \Delta\omega_i - \Delta\omega_{COI} \\ \Delta\omega_{COI} = \dfrac{1}{M_T}\sum_{i=1}^{n} M_i\Delta\omega_i \\ M_T = \sum_{i=1}^{n} M_i \end{cases} \tag{10}$$

For a multi-machine power system with n generators, the motion equations are typically expressed in the center of inertia (COI) coordinate system.

$$\begin{cases} \dfrac{d\hat{\delta}_i}{dt} = \Delta\hat{\omega}_i\omega_0 \\ \dfrac{d\Delta\hat{\omega}_i}{dt} = \dfrac{1}{M_i}(P_{mi} - P_{ei}) - \dfrac{\sum_{i=1}^{n}(P_{mi} - P_{ei})}{\sum_{i=1}^{n} M_i} \end{cases} \quad i = 1, 2\cdots, n \tag{11}$$

Where P_{mi} and P_{ei} represent the mechanical power and electromagnetic power of the i-th generator.

As shown in Equation (10), the rotor motion equations of a multi-generator system are similar to the rotor motion equations of a SMIB power system. Therefore, the MLE criterion sensitivity analysis is also applicable to multi-machine systems. Therefore, the stability criterion for the multi-machine system is:

$$f_i(t_c) = \Delta\hat{\omega}_i(t_c) < 0 \, \& \& \, \Delta\hat{\omega}_i(t_c) \ge \xi_i \quad i = 1, 2, \cdots, n \tag{12}$$

For a power system with n generators, the active power sensitivity matrix of the stability boundary function can be defined as:

$$F = \begin{bmatrix} \dfrac{\partial f_1}{\partial P_1} & \dfrac{\partial f_1}{\partial P_2} & \cdots & \dfrac{\partial f_1}{\partial P_n} \\ \dfrac{\partial f_2}{\partial P_1} & \dfrac{\partial f_2}{\partial P_2} & \cdots & \dfrac{\partial f_2}{\partial P_n} \\ \vdots & \vdots & \ddots & \vdots \\ \dfrac{\partial f_n}{\partial P_1} & \dfrac{\partial f_n}{\partial P_2} & \cdots & \dfrac{\partial f_n}{\partial P_n} \end{bmatrix} \tag{13}$$

In the critical point, when the generator output changes, the stability criterion is transformed into:

$$\begin{cases} \left[f_1, f_2, \cdots, f_n\right]^T = \left[f_1 - \xi_1, \cdots, f_n - \xi_n\right]^T \\ \left[f_1, f_2, \cdots, f_n\right]^T + F \bullet \left[\Delta P_1, \Delta P_2, \cdots, \Delta P_n\right]^T > 0 \end{cases} \tag{14}$$

Considering the active power balance constraint of the generators, equation (13) can be rewritten as:

$$\begin{cases} -\dfrac{1}{f_i}\sum_{j=1}^{n\text{-}1}\left(\dfrac{\partial \hat{f}_i}{\partial P_j}-\dfrac{\partial \hat{f}_i}{\partial P_s}\right)\Delta P_j<1 \\ f_i>0 \end{cases} \quad i=1,2,\cdots,n \qquad (15)$$

Where the subscript S denotes the balance machine. To further consider the upper and lower limits of generator active power outputs, the generator-dominated practical DSR boundary model can be expressed as:

$$\begin{cases} -\dfrac{1}{f_i}\sum_{j=1}^{n\text{-}1}\left(\dfrac{\partial \hat{f}_i}{\partial P_j}-\dfrac{\partial \hat{f}_i}{\partial P_s}\right)\Delta P_j<1 \\ f_i>0 \\ P_j^m-P_j\le \Delta P_j\le P_j^M-P_j \\ P_s^m-P_s\le -\sum_{j=1}^{n-1}\Delta P_j\le P_s^M-P_s \end{cases} \quad i=1,2,\cdots,n \qquad (16)$$

In practical power system operating scenarios, the number of generators is often quite large, making it difficult to construct DSR considering all generators based on the above formula. Given that the transient power angle instability in power systems manifests as the angular displacement between the key units dominating instability [12], when calculating the sensitivity matrix F, it is generally sufficient to consider only the critical unstable units. Therefore, the steps for obtaining the practical DSR boundary are as follows:

- Based on power system data, determine the fault line L and fault duration t , obtain real-time measurement data after the fault and use the MLE criterion to perform stability assessment.
- By increasing or decreasing the fault-clearing time, identify the leading unit, obtain the critical stable operating point, and identify the dominant unstable unit under the current fault. Perform CCCOI-RM transformation on the $\Delta\omega_i$ to obtain the transformed angular acceleration.
- In the initial fault scenario, gradually change the output of the dominant unstable units until the system becomes unstable, and calculate the stability reference parameters ξ or ξ^* of the MLE criterion for this fault.
- Perturb the generator output at the critical instability point, calculate the sensitivity matrix of the MLE stability boundary function F, and construct the DSR boundary according to Equation (16).

IV. CASE STUDY

To verify the effectiveness and applicability of the proposed method, tests are conducted on the New England 10-generator 39-bus system, with bus 31 set as the reference (slack) bus. A three-phase short-circuit fault is applied at the sending end of the transmission line between bus 17 and bus 18 at $t=0$s, and the fault-clearing time is set to 0.18 s.

A. Dynamic security region boundary construction

Perform time-domain simulation at the initial operating point to obtain measurement data, compute the MLE trajectory numerically, and use the MLE criterion to assess stability. The system is found to be in a stable operating state. When the fault-clearing time is increased to 0.22 s, the system becomes unstable. The power angle and MLE trajectories are shown in Fig.3., and the leading generator is identified as $\{G_{34}, G_{38}\}$.

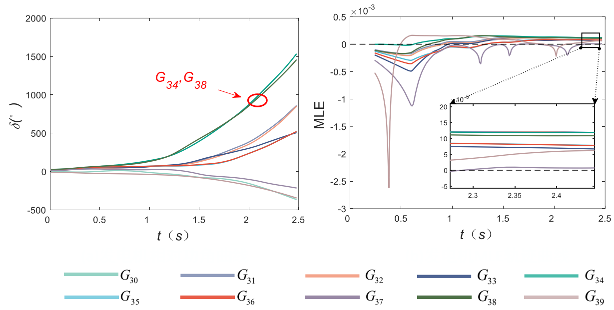


Fig.3. Unstable power angle curves and MLE trajectories.

TABLE I. SENSITIVITY OF GENERATORS

Generators	Stability boundary function sensitivity	
	$F_{G34}(\times10^{-3})$	$F_{G38}(\times10^{-3})$
G_{30}	-4.721	3.991
G_{32}	6.451	-8.876
G_{33}	-2.790	-2.565
G_{34}	-74.791	-9.668
G_{35}	-2.457	-5.465
G_{36}	-2.975	-6.192
G_{37}	6.505	-1.653
G_{38}	8.914	-23.262
G_{39}	8.611	6.557

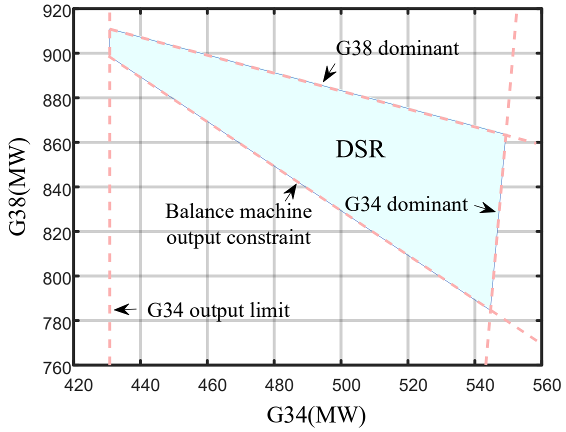


Fig.4. Practical dynamic security region dominated by G34 and G38

At the initial fault, the output of the leading generators is increased individually, and the system becomes unstable in each case, indicating that the fault is a multi-machine dominated instability. Keeping the outputs of the other generators unchanged, the outputs of generators G_{34} and G_{38} are increased until the system loses stability. The corresponding critical active powers are determined to be 545

MW and 910 MW, with the stability reference parameters $\xi_{G34} = -0.3876(rad/s)$, $\xi_{G38} = -0.4487(rad/s)$, respectively. Individually vary the active power output of each generator, calculate the sensitivity matrix of the dominant unstable units, and the results are shown in TABLE Ⅰ.

For intuitive visualization, the active power injections of generators G34 and G38, which are associated with the dominant instability mode, are selected as the two-dimensional parameter space of the PDSR. The resulting PDSR is shown in Fig.4. The entire PDSR construction process requires computational time $T = 1.51s$.

B. Model error comparison

In order to verify the accuracy of describing the security boundary, the results obtained by this method were compared with the traditional fitting method.

$$Err_j = \frac{LD_j}{\left\| \boldsymbol{p}_j \right\|^2} = \frac{\left| \sum_{i=1}^{n} \alpha_i p_{ji} - 1 \right|}{\sqrt{\sum_{i=1}^{n} \alpha_i^2} \sqrt{\sum_{i=1}^{n} p_{ji}^2}} \tag{17}$$

Where, LD_j represents the distance from the critical injection power to the security boundary, $\left\| p_j \right\|^2$ denotes the magnitude of the critical point, and α_i is the coefficient of the security region hyperplane.

In this case, 20 critical points are selected for verification. Using equation (17), the percentage error between each critical point and the security region hyperplane is calculated, and a comparison of the DSR boundary errors obtained by different methods is presented in Fig.5. The maximum errors of the dynamic security boundary obtained by the fitting method and the proposed method are 1.72% and 1.55%, both within the 5% tolerance range acceptable for engineering calculations. In terms of computational efficiency, the fitting method requires a large number of critical points to approximate the security region boundary, taking up to 10 minutes in this case. In contrast, the proposed method constructs the model using MLE trajectories and rotor speed deviation information on a second-level timescale, significantly reducing the time required for boundary characterization.

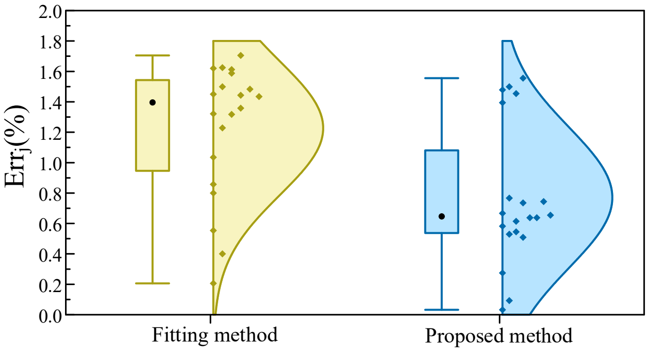


Fig.5. Comparison of DSR boundary errors

V. Conclusion

This paper extracts the MLE criterion and establishes a stability boundary function by analyzing the variations in generator rotor speed deviation and MLE trajectories during transient processes. An effective mapping between the MLE-based stability boundary function and the injection space parameters is constructed, and the DSR is developed at the critical stability point based on the sensitivity matrix of the MLE stability boundary function. Simulation results demonstrate that the proposed method achieves smaller errors and faster computation compared with the fitting method. The boundary derived from this model can effectively determine system instability, and the relative position between the operating point and the DSR boundary provides valuable insight for the early prediction of transient stability and the quantitative assessment of stability margins.

The following research will further analyze the applicability of the proposed DSR construction method in power systems with high renewable penetration, focusing on the impact of power electronic devices on the security boundaries.

Acknowledgment

The work was supported by National Natural Science Foundation of China (No. U23B200752).

References

[1] S. A. B. Mosavi, "Extracting Most Discriminative Features on Transient Multivariate Time Series by Bi-Mode Hybrid Feature Selection Scheme for Transient Stability Prediction," in IEEE Access, vol. 9, pp. 121087-121110, 2021.

[2] R. Ma, M. Zhan, K. Jiang, D. Liu, P. Hu and S. Cheng, "Transient Stability Assessment of Renewable Dominated Power Systems by the Method of Hyperplanes," in IEEE Transactions on Power Delivery, vol. 38, no. 5, pp. 3455-3468, Oct. 2023.

[3] Y. Zhao, H. Liao, Y. Zhao, and S. Pan, “Data-augmented trend-fluctuation representations by interpretable contrastive learning for wind power forecasting,” Applied Energy, vol. 380, p.125052, 2025.

[4] J. Yan, C. -C. Liu and U. Vaidya, "PMU-Based Monitoring of Rotor Angle Dynamics," in IEEE Transactions on Power Systems, vol. 26, no. 4, pp. 2125-2133, Nov. 2011.

[5] G. Jakimoski and K. P. Subbalakshmi, "Discrete Lyapunov Exponent and Differential Cryptanalysis," in IEEE Transactions on Circuits and Systems II: Express Briefs, vol. 54, no. 6, pp. 499-501, June 2007

[6] S. Dasgupta, M. Paramasivam, U. Vaidya and V. Ajjarapu, "PMU-Based Model-Free Approach for Real-Time Rotor Angle Monitoring," in IEEE Transactions on Power Systems, vol. 30, no. 5, pp. 2818-2819, Sept. 2015.

[7] S. Wei, M. Yang, J. Qi, J. Wang, S. Ma and X. Han, "Model-Free MLE Estimation for Online Rotor Angle Stability Assessment With PMU Data," in IEEE Transactions on Power Systems, vol. 33, no. 3, pp. 2463-2476, May 2018

[8] J. Ren, Y. Zeng and C. Qin, "Practical Dynamic Security Region Model: A Hybrid Physical Model-Driven and Data-Driven Approach," in IEEE Transactions on Power Systems, vol. 40, no. 1, pp. 728-739, Jan. 2025.

[9] Y. Zeng, H. Cao and Y. Hu, "A Practical Method for Building Dynamic Security Region Based on Extended Phase Trajectory," 2024 IEEE Power & Energy Society General Meeting (PESGM), Seattle, WA, USA, 2024, pp. 1-5.

[10] S. Sharma, S. Pushpak, V. Chinde and I. Dobson, "Sensitivity of Transient Stability Critical Clearing Time," in IEEE Transactions on Power Systems, vol. 33, no. 6, pp. 6476-6486, Nov. 2018.

[11] M. Jin, T. S. Sidhu and K. Sun, "A New System Splitting Scheme Based on the Unified Stability Control Framework," in IEEE Transactions on Power Systems, vol. 22, no. 1, pp. 433-441, Feb. 2007.

[12] Q. Zong, W. Yao, H. Zhou, H. Zhao, J. Wen and S. Cheng, "Transient Stability Assessment of Large-Scale Power System Using Predictive Maximal Lyapunov Exponent Approach," in IEEE Transactions on Power Systems, vol. 39, no. 3, pp. 5163-5176, May 2024.