Differentially Private Distributed OPF via ADMM

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***Abstract*—While distributed optimization ensures optimal coordination across multiple operational domains in AC power networks, it also introduces critical privacy vulnerabilities and concerns through iterative information exchange. This paper presents a comprehensive framework for distributed Optimal Power Flow (OPF) that integrates Alternating Direction Method of Multipliers (ADMM) with formal Differential Privacy (DP) guarantees. Unlike previous approaches that offer theoretical privacy assurances, the proposed work delivers immediately implementable solutions with operational bounds. The proposed method partitions power networks into operational zones and injects calibrated Laplace noise, perturbing OPF sub-problem solutions at each iteration. Numeric results for the IEEE 14-bus and IEEE 118-bus systems demonstrate fundamental privacy-utility trade-offs, providing system operators with concrete optimistic and pessimistic operational bounds, thereby enabling robust privacy-aware decision-making.**

***Index Terms*—ADMM, Differential Privacy, Distributed Optimization, OPF, Privacy-Utility trade-off, Laplace Noise.**

I. Introduction

POWER networks have transformed due to integration of distributed energy resources, advanced metering infrastructure, and demand response programs [1], which have also led to privacy vulnerabilities. Conventional centralized Optimal Power Flow (OPF) requires full network data, exposing sensitive load information, costs, and other parameters. Distributed optimization techniques, notably Alternating Direction Method of Multipliers (ADMM), address this by decomposing OPF into subproblems, allowing collaboration without direct data sharing [2], [3]. However, recent studies in [2], [4] reveal that adversaries can still infer private information from ADMM coordination signals, underscoring the need for robust privacy-preserving mechanisms in distributed OPF.

Differential Privacy (DP) has emerged as a rigorous framework to safeguard sensitive data in power network operation and optimization [2]. Privacy concerns in distribution-level markets are mitigated via decentralized algorithms for distribution locational marginal pricing in [3]. Several studies have advanced privacy-preserving distributed economic dispatch over time-varying directed networks [5] and proposed differentially private projected subgradient OPF methods [4]. Consensus-based methods with optimality guarantees further enhance privacy-preserving distributed optimization [6]. DP-enabled dynamic pricing models for demand response mitigate privacy risks in real-time pricing [7], while mechanisms for pricing private data consider its value in state estimation [8]. Differentially private OPFs incorporating affine noise-based optimization ensure feasibility via chance constraints [9], and aggregated DCOPF data releases are analyzed for privacy-topology interactions [10]. DP frameworks allow agents to tune privacy based on local sensitivities [11], complemented by bus-level privacy-preserving dispatch [12]. Extensions to microgrids leverage state decomposition and time-varying communication strategies [13], [14], while event-triggered DP mechanism further balances privacy protection [15].

Despite significant advancements, several critical gaps remain in the literature. First, while various approaches guarantee theoretical DP, most offer limited practical implementation guidance with concrete numeric bounds for system operators. Second, there is insufficient analysis of how privacy noise differentially affects system variables and how these impacts translate to OPF decisions. Third, previous works largely focus on either privacy mechanisms or optimization efficiency without providing integrated frameworks that deliver immediately implementable solutions. In this regard, this paper addresses these gaps through the following contributions:

1) ADMM based distributed OPF with integrated ϵ-DP guarantee is proposed, which provides formal privacy protection while maintaining operational feasibility.
2) Unlike previous approaches that offer theoretical distributions, this paper provides implementable ACOPF solutions with concrete optimistic and pessimistic bounds.
3) A new operational perspective comparing privacy and utility-centric approaches is given, providing practical parameters selection guidance.

Numeric results for IEEE 14-bus and IEEE 118-bus systems demonstrate the method's efficacy in balancing DP protection with operational efficiency. The rest of the paper is as follows. Section II discusses the centralized ACOPF formulation and related DP framework. Section III discusses the proposed method and associated implementation. Section IV analyses the obtained numeric results. Section V discusses the implementation guidelines, while Section VI concludes the paper.

II. Problem Formulation

Let the AC power network be an undirected graph $(\mathcal{N}, \mathcal{E})$, where $\mathcal{N} = \{1, \ldots, N\}$ is set of buses and $\mathcal{E} \subseteq \mathcal{N} \times \mathcal{N}$ is set of transmission lines. Let $\mathcal{G} \subseteq \mathcal{N}$ be the set of buses, where a generator is connected. The centralized ACOPF model is

$$\min \sum_{i \in \mathcal{G}} C_i(P_{g,i}) = a_i P_{g,i}^2 + b_i P_{g,i} + c_i \quad (1a)$$

$$\text{s.t. } P_{g,i} - P_i = P_{d,i}, \forall i \in \mathcal{N} \quad (1b)$$

$$Q_{g,i} - Q_i = Q_{d,i}, \forall i \in \mathcal{N} \quad (1c)$$

$$\theta_{ij}^{\min} \leq \theta_{ij} \leq \theta_{ij}^{\max}, \forall (i,j) \in \mathcal{E}; \theta_{ref} = 0 \quad (1d)$$

$$P_{g,i}^{\min} \leq P_{g,i} \leq P_{g,i}^{\max}, Q_{g,i}^{\min} \leq Q_{g,i} \leq Q_{g,i}^{\max}, \forall i \in \mathcal{G} \quad (1e)$$

$$V_i^{\min} \leq V_i \leq V_i^{\max}, \forall i \in \mathcal{N} \quad (1f)$$

$$P_{ij}^{\min} \leq P_{ij} \leq P_{ij}^{\max}, \forall (i,j) \in \mathcal{E} \quad (1g)$$

where a_i, b_i, and c_i in (1a) are the cost coefficients of real power generation at bus i. $P_{g,i}$ ($Q_{g,i}$) is the unknown real

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(reactive) power generation at bus i. $P_{d,i}$ ($Q_{d,i}$) is known real (reactive) power demand at bus i. P_i and Q_i are the real and reactive power injections at bus i in (1b) and (1c), respectively, defined as $P_i = \sum_{j\in\mathcal{N}} V_iV_j(G_{ij}\cos\theta_{ij} + B_{ij}\sin\theta_{ij})$ and $Q_i = \sum_{j\in\mathcal{N}} V_iV_j(G_{ij}\sin\theta_{ij} - B_{ij}\cos\theta_{ij})$, where θ_i (V_i) is bus voltage phase angle (magnitude) of bus i with respect to angle reference bus ($\theta_{ref} = 0$), as in (1d). Also, $\theta_{ij} = \theta_i - \theta_j, \forall(i,j) \in \mathcal{E}$ in (1d). G_{ij} (B_{ij}) is the real (imaginary) part of $(i,j)^{th}$ element of bus admittance matrix of given AC power network. (1d) - (1f) are constraints on respective variables with lower and upper bounds as $()^{\min}$ and $()^{\max}$. (1g) is the constraint on real power flow (P_{ij}) between bus i and bus j, where $P_{ij} = V_i^2 g_{ij} + V_iV_j(g_{ij}\cos\theta_{ij} + b_{ij}\sin\theta_{ij})$, and g_{ij} (b_{ij}) is the conductance (susceptance) of line between bus i and bus j with $P_{ij}^{\min}$ usually equal to $-P_{ij}^{\max}$ in (1g).

DP provides a rigorous mathematical framework for quantifying and controlling privacy loss in data analysis [16].

Definition II.1 (ϵ-DP)**.** *A randomized mechanism $\mathcal{M} : \mathcal{D} \to \mathcal{R}$ satisfies ϵ-DP if for any two adjacent datasets $D, D' \in \mathcal{D}$ differ in at most one element, and for any subset of outputs $S \subseteq \mathcal{R}$, $\mathbb{P}[\mathcal{M}(D) \in S] \le e^{\epsilon}\mathbb{P}[\mathcal{M}(D') \in S]$.*

In the context of centralized ACOPF in (1), adjacent datasets correspond to load profiles that differ in a single user's demand pattern. Privacy parameter ϵ controls the trade-off between privacy and utility [16]. Low ϵ provides strong privacy guarantees but may degrade ACOPF's solution quality.

Definition II.2 (Sensitivity)**.** *L_1-sensitivity of function $f : \mathcal{D} \to \mathbb{R}^d$ is defined as $\Delta f = \max_{D,D'} \|f(D) - f(D')\|_1$, where maximum is taken over all pairs of adjacent datasets.*

For the Laplace mechanism, noise ξ is calibrated according to sensitivity Δf and privacy budget b as $\xi \sim \text{Laplace}(0, b)$, where $b = \Delta f/\epsilon$, and ϵ is defined in Definition II.1.

Theorem II.1 (Laplace Mechanism)**.** *For any function $f : \mathcal{D} \to \mathbb{R}^d$, the mechanism $\mathcal{M}(D) = f(D) + \xi$ satisfies ϵ-DP, where $\xi \sim$ Laplace$(0, \Delta f/\epsilon)$.*

Proof. Please refer page 32 of [16] for associated proof. □

III. Proposed Methodology

DP is relevant in the context of distributed ACOPF, unlike in (1), wherein the proposed method is applicable. The proposed method integrates DP in distributed ACOPF through a systematic approach that maintains both privacy guarantees and operational feasibility. The integrated ADMM-DP framework operates through

1) *Distributed OPF*: AC power network is divided into K operational zones based on geographical boundaries or organizational structures. Each zone solves its local ACOPF via ADMM with limited information exchange.
2) *DP integration*: Calibrated Laplace noise is injected in consensus signals to provide formal ϵ-DP guarantees.
3) *Statistical bounds*: Monte Carlo sampling establishes confidence bounds on power network's variables.

To enable decentralized ACOPF computation and address privacy concerns, ADMM is used, which combines dual decomposition aspect with convergence properties of method of multipliers [17], [18]. The network is partitioned into K disjoint zones, with each zone $k \in \mathcal{K} = \{1,\dots,K\}$ containing a subset of buses $\mathcal{N}_k$ and transmission lines $\mathcal{E}_k \subseteq \mathcal{E}$ such that $\mathcal{N} = \bigcup_{k=1}^{K}\mathcal{N}_k$, $\mathcal{N}_{k_1} \cap \mathcal{N}_{k_2} = \emptyset, \forall k_1 \neq k_2$, and $\mathcal{E}_k$ also includes the boundary lines directly connecting zone k to other adjacent zones. The boundary buses of zone k along with buses of adjacent zones (to which zone k is directly connected) form the set $\mathcal{M}_k$. At these buses, local copies of voltage phase angles are maintained to enforce coupling or global consensus between zone k and adjacent zones as

$$\theta_{i,k} = \hat{\theta}_i, \forall i \in \mathcal{M}_k \tag{2}$$

where $\mathbf{z}_k = \{\hat{\theta}_i, \forall i \in \mathcal{M}_k\}$ is the set of global consensus variables for zone k. Consequently, the local ACOPF for zone k in view of (1) is defined as

$$\min_{\Phi_k,\zeta_k} F_k = \sum_{i\in\mathcal{G}_k} C_i(P_{g,i}) \text{ s.t. } (2), \{\Phi_k, \zeta_k\} \in \mathcal{F}_k \tag{3}$$

where $\Phi_k = \{P_{g,i}\forall i \in \mathcal{G}_k, Q_{g,i}\forall i \in \mathcal{G}_k, V_i\forall i \in \mathcal{N}_k, \theta_i\forall i \in (\mathcal{N}_k - \mathcal{M}_k)\}$ and $\zeta_k = \{\theta_i \forall i \in \mathcal{M}_k\}$. $\mathcal{F}_k$ in (3) is the feasible space of local ACOPF of zone k, which is $\mathcal{F}_k = \{(1\text{b}) - (1\text{g})\}$ with sets $\mathcal{N}$, $\mathcal{G}$, and $\mathcal{E}$ replaced by $\mathcal{N}_k$, $\mathcal{G}_k$, and $\mathcal{E}_k$, respectively. Let ϱ_k be defined as $\varrho_k = \{\Phi_k, \zeta_k\}\forall k \in \mathcal{K}$.

In view of (3), the overall distributed ACOPF can be stated as a global consensus optimization problem as

$$\min_{\varrho_k \forall k\in\mathcal{K}} \sum_{k=1}^{K} F_k \text{ s.t. } \zeta_k = \mathbf{z}_k, \varrho_k \in \mathcal{F}_k, \forall k \in \mathcal{K} \tag{4}$$

The associated augmented Lagrangian of (4) is

$$\mathcal{L}_\rho = \sum_{k=1}^{K} \left[F_k + \lambda_k^T(\zeta_k - \mathbf{z}_k) + 0.5\rho\|\zeta_k - \mathbf{z}_k\|_2^2\right] \tag{5}$$

where λ_k is vector of Lagrange multipliers as per (5) and $\rho > 0$ is ADMM penalty parameter. Each iteration (t) of ADMM to solve (5) involves the following steps [2] $\forall k \in \mathcal{K}$.

$$\zeta_k^{t+1} \leftarrow \arg\min_{\varrho_k\in\mathcal{F}_k} [F_k + \lambda_k^{tT}(\zeta_k - \mathbf{z}_k^t) + \frac{\rho}{2}\|\zeta_k - \mathbf{z}_k^t\|_2^2] \tag{6a}$$

$$\mathbf{z}_k^{t+1} \leftarrow \arg\min_{\mathbf{z}_k} \sum_{k=1}^{K} [\lambda_k^{tT}(\zeta_k^{t+1} - \mathbf{z}_k) + \frac{\rho}{2}\|\zeta_k^{t+1} - \mathbf{z}_k\|_2^2] \tag{6b}$$

$$\lambda_k^{t+1} \leftarrow \lambda_k^t + \rho(\zeta_k^{t+1} - \mathbf{z}_k^{t+1}), t = t+1 \tag{6c}$$

ADMM in (6) iterates till primal and dual residual ($\|\zeta_k^{t+1} - \mathbf{z}_k^{t+1}\|_2, \|\mathbf{z}_k^{t+1} - \mathbf{z}_k^t\|_2, \forall k$) are less than desired tolerance.

While ADMM ensures decentralized ACOPF solution, the iterative exchange of consensus variables $\mathbf{z}_k$ and dual variables λ_k between zones may leak sensitive operational data. To mitigate this, DP is integrated by perturbing $\mathbf{z}_k \forall k \in \mathcal{K}$ in each ADMM iteration (as per Theorem II.1) before sharing with adjacent zones as

$$\tilde{\mathbf{z}}_k^t = \mathbf{z}_k^t + \boldsymbol{\xi}_k^t, \quad \boldsymbol{\xi}_k^t \sim \text{Laplace}(0, \Delta f_k/\epsilon_k), \forall k \in \mathcal{K} \tag{7}$$

where Δf_k is sensitivity for $\mathbf{z}_k$ mapping function $f_k : \mathcal{D}_k \to \mathbf{z}_k$ with $\mathcal{D}_k = \{P_{d,i}, Q_{d,i}\}_{i\in\mathcal{N}_k}$ being the local dataset and ϵ_k being the privacy budget. Δf_k, as per Definition II.2, is

$$\Delta f_k = \max_{\mathcal{D}_k,\mathcal{D}'_k} \|\mathbf{z}_k(\mathcal{D}_k) - \mathbf{z}_k(\mathcal{D}'_k)\|_1, \forall k \in \mathcal{K} \tag{8}$$

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where the maximum is taken over all pairs of adjacent datasets $\mathcal{D}_k$ and $\mathcal{D}'_k$ that differ in at most one load element. Additionally, Δf_k can be bounded by power network characteristics as $\Delta f_k \le \kappa \max_{i\in\mathcal{N}_k} |\Delta P_{d,i}|$ [19], where κ depends on network topology and impedances, and $\max|\Delta P_{d,i}|$ is the maximum possible load variation at any bus in zone k. With the above proposed integration of Laplace noise, formal ϵ-DP guarantee is ensured as discussed next (as per Definition II.1).

The probabilistic nature of above proposed DP requires a statistical characterization of solution uncertainty to provide practical operational guidelines. To achieve this, Monte Carlo based statistical bound computation is proposed, which treats each execution of proposed DP-integrated ADMM based distributed ACOPF for a single noise realization (uniform for all real power load injections) as a single trial. In each trial s, a perturbed but feasible ACOPF solution of (4), i.e., $\tilde{\mathbf{x}}^{(s)} = \{\varrho_k \forall k \in \mathcal{K}\}$ with $\tilde{x}_m^{(s)}$ being the m^{th} element of $\tilde{\mathbf{x}}^{(s)}$, is obtained. The collection of $\tilde{\mathbf{x}}^{(s)}$ over $s = 1,\dots,N$ trials of Monte Carlo forms a dataset that captures the statistical distribution of all ACOPF variables under the influence of DP, as per (7). Using these N samples, optimistic and pessimistic bounds for each ACOPF variable are evaluated as

$$x_m^{op} = q_{0.975}(\{\tilde{x}_m^{(s)}\}_{s=1}^N), x_m^{pe} = q_{0.025}(\{\tilde{x}_m^{(s)}\}_{s=1}^N), \forall m \quad (9)$$

$$\mathrm{CI}_{1-\alpha}(x_m) = [q_{\alpha/2}(\{\tilde{x}_m^{(s)}\}), q_{1-\alpha/2}(\{\tilde{x}_m^{(s)}\})], \forall m \quad (10)$$

where q_p is the p-th quantile of sample distribution, and α is the significance level for confidence interval $\mathrm{CI}_{1-\alpha}$.

These bounds provide operational envelopes that enable robust privacy-aware decision-making, allowing system operators to plan for worst and best case scenarios while maintaining operational feasibility across all DP realizations. The proposed DP and ADMM based distributed OPF framework and associated statistical bound computation is given in Algorithm 1.

Algorithm 1: Proposed statistical bound computation

Input: AC power network parameters, $C_i \forall i \in \mathcal{G}$ in (3), privacy budget $\boldsymbol{\epsilon}$, ADMM parameters (penalty ρ, tolerance τ, maximum iteration $t_{\max}$), and Monte Carlo samples N
Output: Optimistic and pessimistic bounds ($\mathbf{x}^{op}$, $\mathbf{x}^{pe}$)
Statistical Analysis via Monte Carlo;
for *each sample* $s = 1$ *to* N **do**
  Initialization: Initialize $\varrho_k^0, \mathbf{z}_k^0, \lambda_k^0, \forall k \in \mathcal{K}$, $t \leftarrow 0$;
  while $\|\mathbf{z}^{t+1} - \mathbf{z}^t\|_2 > \tau$ ***and*** $t < t_{\max}$ **do**
    for *each zone* $k \in \mathcal{K}$ ***in parallel*** **do**
      Solve (6a) for ζ_k^{t+1} and Φ_k^{t+1};
      Compute Δf_k as per (8);
      Evaluate perturbed consensus $\tilde{\mathbf{z}}_k^t$ from (7);
      Update $\mathbf{z}_k^{t+1}$, λ_k^{t+1}, and t via (6b), (6c) using $\tilde{\mathbf{z}}_k^t$;
    end
  end
  Obtain $\tilde{\mathbf{x}}^{(s)}$ as per Section III;
end
return the computed $\mathbf{x}^{op}$ and $\mathbf{x}^{pe}$ as per (9)

IV. Numeric Results and Discussion

The proposed method is solved via *fmincon* solver for IEEE 14-bus and IEEE 118-bus in MATLAB 2024b on an Intel Core i7-10700 32 GB RAM processor. Partitions in IEEE 14-bus system are chosen as per [17], where bus 2 is chosen as angle reference bus as it is a non-boundary bus located in zone 1, which simplifies the reference angle coordination across zones [17]. Numeric results for IEEE 118-bus system are also obtained, where bus 69 is the angle reference bus. Two perturbation magnitudes are applied, $\epsilon = 0.50$ and $\epsilon = 0.05$, as illustrated in Fig. 1(a) and Fig. 1(b), respectively.

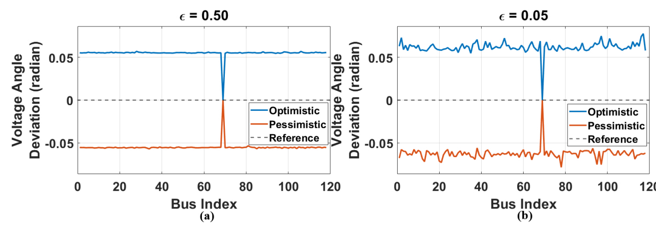

Fig. 1. Comparison of bus voltage phase angle deviation for IEEE 118-bus system

A uniform privacy budget of $\epsilon = 0.5$ is chosen for all real power loads in IEEE 14-bus system to demonstrate the system-wide impact of consistent privacy protection on bus voltage phase angles and associated magnitudes, as shown in Table I, where the relevant pessimistic values are the difference of respective optimistic values and associated variations. This analysis reveals significant but non-uniform phase angle variations across all buses, with bus 14 having the maximum variation of 0.1538 radians. The voltage magnitude at bus 4 has the maximum variation of 0.0142 in Table I. This non-uniformity stems from differences in network position and local sensitivity. Despite substantial angle perturbations, voltage magnitudes remain remarkably stable; a critical observation attributed to the selective DP integration strategy in Section III. Further, the results for different privacy levels at bus 14 (maximum voltage phase angle variation) and bus 4 (maximum voltage variation) are shown in Table II. The variations in bus voltage phase angles and magnitudes consistently decrease with increase in ϵ. This demonstrates the fundamental privacy-utility trade-off: low ϵ (more privacy) introduces larger deviations from non-private optimal solution. It can also be noted that with change in ϵ, the same respective buses tend to have the respective maximum voltage phase angle and magnitude variations, which is a critical observation demonstrating the robustness of the proposed method. An examination of Fig. 1 for IEEE 118-bus system provides additional insight of the impact of DP on solution accuracy. As expected, voltage phase angle for reference bus remains zero for all tested privacy budgets. For bus 80, which has relatively large phase angle variations in Fig. 1(b), the deviation from non-private reference is ≈ 0.12rad under the strict privacy setting of $\epsilon = 0.05$. With ϵ relaxed to 0.5, this deviation is significantly reduced and remains below 0.05rad. Similar is the case at other buses.

The active power variations reflect the fundamental privacy-optimality trade-off, where privacy protection introduces dispatch uncertainty. The bus voltage phase angle perturbations

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TABLE I
EFFECT OF PRIVACY BUDGET $\epsilon = 0.5$ ON BUS VOLTAGE PHASE ANGLES AND ASSOCIATED MAGNITUDES IN IEEE 14-BUS SYSTEM (REF. - REFERENCE: NON-PRIVATE ACOPF SOLUTION, OPT. - OPTIMISTIC, VAR. - VARIATION)

Bus	θ_i (radians)			V_i (pu)		
	Ref.	Opt.	Var.	Ref.	Opt.	Var.
1	-0.0167	0.0436	0.1384	1.0600	1.0654	0.0124
2	0.0000	0.0000	0.0000	1.0408	1.0463	0.0112
3	-0.1900	-0.1287	0.1262	1.0156	1.0211	0.0115
4	-0.1679	-0.1060	0.0963	1.0145	1.0208	0.0142
5	-0.1464	-0.0820	0.1399	1.0164	1.0209	0.0100
6	-0.2382	-0.1932	0.1107	1.0600	1.0640	0.0125
7	-0.2120	-0.1566	0.0990	1.0463	1.0514	0.0097
8	-0.1985	-0.1387	0.1196	1.0600	1.0672	0.0128
9	-0.2436	-0.1831	0.1170	1.0437	1.0495	0.0109
10	-0.2477	-0.1871	0.1173	1.0391	1.0449	0.0116
11	-0.2452	-0.1826	0.1113	1.0460	1.0510	0.0121
12	-0.2529	-0.1933	0.1166	1.0448	1.0517	0.0129
13	-0.2538	-0.1910	0.1333	1.0399	1.0455	0.0126
14	-0.2658	-0.1980	0.1538	1.0239	1.0281	0.0089

TABLE II
MAXIMUM VARIATIONS IN BUS VOLTAGE PHASE ANGLES (RADIANS) AND ASSOCIATED MAGNITUDES (PU) IN IEEE 14-BUS SYSTEM WITH RESPECT TO CHANGE IN ϵ

Bus	Privacy budget ϵ				
voltage	0.05	0.10	0.50	0.75	1.00
phase angle	0.8111	0.5195	0.1538	0.1170	0.1166
magnitude	0.1061	0.0675	0.0142	0.0129	0.0129

introduced for DP directly necessitate generation re-dispatch to maintain net power balance, even though all generator buses in IEEE 14-bus system are non-private. Table III shows the results of active and reactive power variations for different privacy levels in all real power loads, where it is evident that the non-zero generation variations (despite bus being non-private) are non-uniform. For active power dispatch (Table III),

TABLE III
VARIATIONS IN ACTIVE (P) AND REACTIVE (Q) POWERS FROM ADMM IN IEEE 14-BUS SYSTEM FOR DIFFERENT PRIVACY BUDGETS ϵ (REF. - REFERENCE, OPT. - OPTIMISTIC, VAR. - VARIATION)

Gen.	Active Power (P) & Reactive Power (Q)					
	$\epsilon = 0.05$ (pu)			$\epsilon = 0.50$ (pu)		
	Ref.	Opt.	Var.	Ref.	Opt.	Var.
P1	1.9433	1.9551	0.0297	1.9433	1.9572	0.0298
Q1	0.0000	0.0102	0.0202	0.0000	0.0166	0.0308
P2	0.3672	0.3804	0.0261	0.3672	0.3793	0.0272
Q2	0.2369	0.2478	0.0274	0.2369	0.2479	0.0261
P3	0.2874	0.3031	0.0357	0.2874	0.3021	0.0256
Q3	0.2413	0.2552	0.0275	0.2413	0.2536	0.0275
P4	0.0000	0.0119	0.0259	0.0000	0.0131	0.0286
Q4	0.1155	0.1283	0.0243	0.1155	0.1316	0.0274
P5	0.0849	0.0970	0.0258	0.0849	0.1036	0.0326
Q5	0.0827	0.0977	0.0291	0.0827	0.0955	0.0240

Generator 3 has the largest variation (0.0357 pu) at $\epsilon = 0.05$, while Generator 5 exhibits the highest variation (0.0326 pu) at $\epsilon = 0.50$. For reactive power dispatch (Table III), Generator 5 has the maximum variation (0.0291 pu) at $\epsilon = 0.05$, while Generator 1 shows the highest variation (0.0308 pu) at $\epsilon = 0.50$. Results reveal that generation variations are non-uniform, demonstrating strong dependence on network topology and generator location rather than privacy budget.

Overall, the results for IEEE 118-bus system are quantitatively consistent with those for IEEE 14-bus system, with average deviations for $\epsilon = 0.05$ for active and reactive power bus injections, and bus voltage magnitudes being 1.13%, 2.6%, and 2.34%, respectively, that reduce to 0.1867%, 1.22%, and 1.3%, respectively, at $\epsilon = 0.5$ in IEEE 118-bus system. The impact of DP on ADMM convergence is also evaluated for both IEEE 14-bus and IEEE 118-bus systems. Stricter privacy settings result in high convergence times, while increasing ϵ leads to faster convergence. Mean convergence times decrease from 0.3236s to 0.2951s (63.79s to 27.92s) for IEEE 14-bus (118-bus) system for $\epsilon = 0.05$ increase to $\epsilon = 0.5$, highlighting the inherent privacy-convergence trade-off. Privacy perturbations affect local solution accuracy, with network-wide impacts governed by power flow sensitivities and electrical coupling. Strongly connected buses exhibit larger deviations, while relaxing ϵ reduces both local and propagated effects, demonstrating predictable privacy-utility scaling.

The observed patterns highlight the operational uncertainty introduced by privacy constraints. **Optimistic** scenarios represent favorable noise realizations that allow more efficient operation, while **pessimistic** scenarios represent unfavorable conditions requiring costly generation re-dispatch. This underscores the importance of maintaining operational flexibility when implementing privacy-preserving mechanisms.

- *Privacy-utility trade-off*: Both voltage phase angle and magnitude variations decrease with increase in ϵ, confirming the fundamental trade-off between privacy strength and operational accuracy.
- *Topological dependence*: The impact of uniform DP is amplified in certain network locations due to structural sensitivity, demonstrating that network topology plays a crucial role in the privacy-utility trade-off.
- *Non-uniform variations*: Generator dispatch variations exhibit non-uniform patterns due to coupled effect of noise injection on absolute voltage phase angles and power flow dependence on relative angle differences.
- *Operational uncertainty*: DP introduces a range of possible operating conditions, necessitating generation flexibility to maintain system balance across all DP realizations.
- *Method's robustness*: The observed patterns and trade-offs are consistent with results on large networks.

V. ASSOCIATED IMPLEMENTATION GUIDELINES

Based on the comprehensive numerical analysis, we propose a few practical implementation guidelines for DP based distributed ACOPF. Table IV presents the fundamental trade-off between privacy protection and operational efficiency, providing a framework for selecting appropriate operational approaches based on specific application requirements.

The choice between these approaches should be guided by the operational context and privacy requirements. Based on the above numeric results, the following is recommended:

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TABLE IV
PRIVACY VS OPTIMALITY-CENTRIC OPERATIONAL PERSPECTIVES

Aspect	Privacy-centric approach	Optimality-centric approach
Objective	Maximizing privacy protection	Minimizing operational efficiency
Privacy budget	Low	High
Risk tolerance	High optimality loss acceptable	Low optimality loss required
Application context	Critical infrastructure, sensitive data	Economic dispatch, market operations
Confidence bounds	Wide operational ranges	Narrow operational ranges
Focus	Robust privacy guarantees	System efficiency

- *Privacy Budget Allocation*: Assign heterogeneous ϵ values based on bus sensitivity. Use $\epsilon \geq 10$ for generator buses and $\epsilon \leq 1.0$ for sensitive load buses, as demonstrated by voltage phase angle variations in Table I.
- *Operational Planning*: Use optimistic and pessimistic bounds (97.5% and 2.5% quantiles) for contingency planning and reserve allocation, leveraging the generation dispatch ranges observed in Table III.
- *System Monitoring*: Focus monitoring efforts on generation variables, which exhibit the largest variations under privacy constraints, as evidenced by the active and reactive power dispatch results.
- *Parameter Tuning*: Select ADMM penalty parameter ρ through systematic tuning to balance convergence speed and solution quality under privacy noise.
- *Adaptive Privacy*: Adjust ϵ values based on real-time system conditions and privacy requirements, using the demonstrated trade-offs between privacy protection and operational efficiency, as given in Table II.

These guidelines provide system operators a practical framework for implementing privacy-preserving OPF while maintaining grid reliability, using the optimistic-pessimistic bounds for robust decision-making across different scenarios.

VI. CONCLUSION

This paper has presented a comprehensive framework for privacy-preserving distributed OPF that integrates ADMM with DP. The numeric results for IEEE 14-bus and IEEE 118-bus systems demonstrate the practical efficacy of the proposed approach, showing that while privacy preservation introduces uncertainty in power dispatch decisions, voltage stability is maintained and operational feasibility is preserved. The optimistic and pessimistic bounds provide system operators with concrete, implementable ranges for operational planning under privacy constraints. The consistent patterns observed for both test systems validate the scalability and generalization of proposed method. The framework represents a significant advancement in privacy-aware power network operations, providing both theoretical rigor and practical implementation ability. By delivering concrete operational bounds rather than theoretical distributions, this work bridges the gap between academic research and practical system operation, enabling the secure and efficient coordination of modern power networks in the era of distributed energy resources and digital transformation. Future research includes extending the framework to stochastic optimization environments with renewable integration, developing adaptive privacy mechanisms that respond to real-time system conditions, and investigating the integration of the proposed approach with electricity market operations.

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979-8-3315-8138-1