Distributed Multi-Period OPFs for Energy Storage in Distribution Systems via Temporal Approximation

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***Abstract*—Energy storage can improve power distribution system performance and lower overall energy costs through temporal energy shifting. Optimizing storage dispatches in distribution systems often involves solving large-scale, unbalanced Multi-Period Optimal Power Flow (MPOPF) problems. Existing methods are often not compatible with parallel computing and struggle to achieve efficient convergence. This paper presents a novel temporally decomposed approach that approximates the value function through temporal aggregations, enabling parallelizable subproblem decomposition while ensuring boundary consensus via dual coordination. The algorithm supports both sequential forward-backward sweep and parallel execution. Validation on the IEEE test systems demonstrate 4x computational speedup while achieving near-optimal solutions across diverse energy price profiles, supporting the effectiveness of the proposed temporal separability framework for large-scale energy storage optimization problems.**

***Index Terms*—Distributed multi-period opfs, distributed optimization, energy storage, temporal decomposition, unbalanced distribution system**

I. Introduction

Energy storage plays a critical role in power systems by addressing surging electricity demand, providing resource adequacy, and providing flexibility and resilience across multiple timescales [1]–[3]. In distribution systems, storage can manage local demand, reduce dependence on upstream transmission, improve operational efficiency, alleviate voltage and thermal violations while enhancing overall system reliability [3]–[5]. The MPOPF is crucial in achieving this, as it integrates both current system conditions and future forecasts, enabling storage to flexibly adapt to varying operational contexts while maximizing system and economic benefits [6].

On the other hand, solving MPOPF requires tackling large-scale problems, constrained by scalability issues [7]. These challenges primarily stem from (i) the temporal constraints across multiple periods and (ii) the complexity inherent in modeling large-scale unbalanced distribution systems, making the optimization process both intricate and resource-intensive. Recent advancements in MPOPF algorithms address computational challenges through decomposition, approximation, and distributed optimization. Most existing MPOPF approaches have primarily focused on transmission systems or single-phase equivalent models, leaving realistic unbalanced three-phase systems largely unaddressed [6], [8], [9]. Extended Dynamic Programming (EDP) [7] uses Hamilton–Jacobi–Bellman recursion to decompose economic dispatch into node-based stages, ensuring linear complexity for radial, balanced systems. However, EDP struggles with unbalanced and meshed networks, and needs major adaptations for large-scale systems with diverse constraints. Differential Dynamic Programming in [9] investigates multi-period settings using Forward-Backward Sweeps (FBSs) for temporal consensus but is validated only on single-phase networks, limiting scalability. Stochastic methods and linear approximations [10] address MPOPF but rely on balanced networks and lack efficient temporal decomposition. In [11], the three-phase OPF problem is reformulated into a second-order cone program (SOCP) and solved using time-decoupled subproblems over a rolling window, enabling adaptability to uncertainties. However, its reliance on fixed state of charges (SOCs) limits flexibility, restricting non-linear program subproblems to reactive power and voltage constraints.

Distributed optimization addresses scalability in active device management for distribution systems [12], [13], while [14], [15] focus on improving MPOPF efficiency. The Auxiliary Problem Principle (APP) method coordinates subproblems via parallel temporal decomposition [14], while ADMM exchanges information across periods using "temporal neighbors" [15]. Despite improved computational speed, these methods face challenges, such as slow convergence and non-guaranteed optimality, needing excessive iterations. A geographically decomposed multi-period approach [16] solves the entire time horizon while enforcing boundary matching but lacks temporal decomposition for adaptable computation. In brief, existing methods face three major limitations: (i) poor scalability in unbalanced systems, (ii) limited efficient temporal decomposition, and (iii) suboptimal approximations. This paper addresses optimal energy storage dispatch in distribution systems by introducing Temporal Equivalent Network Approximation (T-ENApp), a novel temporally distributed algorithm for solving MPOPF problems in unbalanced networks. T-ENApp is inspired by the network equivalence method developed for spatial decomposition in ENApp (addresses network scalability issues for single time steps) [17], [18]; however, T-ENApp addresses temporal coupling and multi-period coordination by applying temporal aggregation using dual variables and an augmented Lagrangian framework to maintain tractability over long horizons. The approach leverages temporal decomposition

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and value function approximation to form equivalent subproblems with boundary coordination, enhancing computational efficiency while preserving feasibility and near-optimal storage dispatch.

II. Multi-Period OPF Formulation

Let the graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ represent the network, where $\mathcal{V}$ and $\mathcal{E}$ are the sets of nodes and edges. $\mathcal{T}$ is the set of all time steps. Let Φ_j represent the set of phases present at node $j \in \mathcal{V}$, allowing for unbalanced network representation where not all nodes have all three phases. Let us assume the set of nodes with a battery are denoted by $\mathcal{B} \subseteq \mathcal{V}$. In this section, we formulate the central multi-period OPF (COPF) problem using the network and battery models.

A. Network Model

The network is modeled in (1), as a multi-phase distribution system using the LinDistFlow equations [18]. For simplicity and improved readability, temporal notation t is omitted, as this applies to each time step. Power flow variables ($P_{ij}^{\phi\phi}, Q_{ij}^{\phi\phi}$), net injections ($p_j^{\phi}, q_j^{\phi}$), battery power ($P_{j,\text{batt}}^{\phi}$), branch impedances ($z_{ij}^{\phi\psi}$), and voltages ($v_j^{\phi}$) are defined on the network for all $i, j \in \mathcal{V}$, with $\phi, \psi \in \Phi_i$ denoting bus phases.

$$\sum_{i:i\to j} P_{ij}^{\phi\phi} - p_j^{\phi} + P_{j,\text{batt}}^{\phi} - \sum_{k:j\to k} P_{jk}^{\phi\phi} = 0 \tag{1a}$$

$$\sum_{i:i\to j} Q_{ij}^{\phi\phi} - q_j^{\phi} - \sum_{k:j\to k} Q_{jk}^{\phi\phi} = 0 \tag{1b}$$

$$p_j^{\phi} = p_j^{\phi l} - p_j^{\phi pv}, \quad q_j^{\phi} = q_j^{\phi l} - q_j^{\phi pv} \tag{1c}$$

$$v_j^{\phi} = v_i^{\phi} - \sum_{\psi\in\Phi_j} 2\mathbb{R}\Big[(P_{ij}^{\phi\psi} + jQ_{ij}^{\phi\psi})(z_{ij}^{\phi\psi})^*\Big] \tag{1d}$$

B. Battery Storage Model

Equation (2) models battery storage for distribution systems. $P_{j,\text{dis}}^{\phi}$ and $P_{j,\text{ch}}^{\phi}$ are the discharging and charging power, $\overline{P_{j,\text{batt}}}$ denotes the maximum charging or discharging power rate, η_{ch} and η_{dis} are the battery charging and discharging efficiencies, $\text{SOC}_{j,t}$ represents the state-of-charge (MWh), and Δt is the time interval. The initial state-of-charge $\text{SOC}_{j,0}$ is specified as a given parameter.

$$P_{j,\text{batt}}^{\phi} - P_{j,\text{dis}}^{\phi} + P_{j,\text{ch}}^{\phi} = 0 \tag{2a}$$

$$\sum_{\phi\in\Phi_j}\left(\frac{P_{j,\text{dis}}^{\phi}}{\eta_{\text{dis}}} - \eta_{\text{ch}} P_{j,\text{ch}}^{\phi}\right) + \frac{\text{SOC}_{j,t} - \text{SOC}_{j,t-1}}{\Delta t} = 0 \tag{2b}$$

$$0 \leq P_{j,\text{dis}}^{\phi}, P_{j,\text{ch}}^{\phi} \leq \overline{P_{j,\text{batt}}} \tag{2c}$$

$$\text{SOC}_{j,\min} \leq \text{SOC}_{j,t} \leq \text{SOC}_{j,\max} \tag{2d}$$

C. Centralized Multi-Period OPF (COPF)

The COPF minimizes total energy cost while satisfying power flow equations (1), battery constraints (2), and voltage limits $\forall t \in \mathcal{T}$. Let $\mathbf{x_t}$ denote the vector of state and control variables (power flows, battery dispatch, voltages), and let C_t denote the energy price ($\$/MWh$) at time t. The COPF problem is formulated as (3). Let Φ_{sub} denote the set of phases

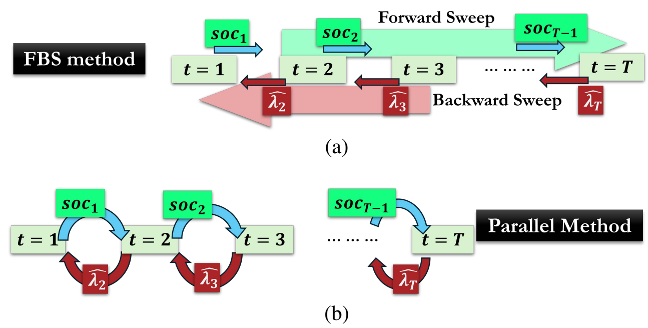

Fig. 1. Algorithmic view of T-ENApp variable exchange: SOCs and λ.

present at the substation, and $P_t^{\phi,\text{sub}}$ represent the power drawn from the substation on phase ϕ at time t.

$$\min \sum_{t\in\mathcal{T}} f_t(\mathbf{x_t}) = \sum_{t\in\mathcal{T}}\Big(C_t \cdot \sum_{\phi\in\Phi_{\text{sub}}} P_t^{\phi,\,\text{sub}}\Big) \tag{3a}$$

$$\text{s.t.}\quad \text{Equations (1) \& (2)}, \quad \forall t \in \mathcal{T} \tag{3b}$$

$$\underline{v} \leq v_j^{\phi} \leq \overline{v}, \quad \forall j \in \mathcal{V}, \forall \phi \in \Phi_j, \forall t \in \mathcal{T} \tag{3c}$$

III. Temporal Distributed OPF: T-ENApp

This section introduces T-ENApp, develops temporal decomposition and value-function approximation to define boundary variables for coordinating decomposed MPOPF subproblems.

A. Dual Variable Characterization

This section establishes the theoretical foundation for computing dual variables $\boldsymbol{\mu} \in \mathbb{R}^3$ associated with battery operation constraints and derives their relationship to marginal charging costs for each battery $b \in \mathcal{B}$ at time step $t \in \mathcal{T}$ during any iteration n. The key idea is that the battery-related dual variables collectively represent the marginal value of stored energy at each time step. For simplicity and improved readability, we omit the explicit notation for the battery b, the n^{th} iteration and the t time step in the equations.

Assumption 1. *The battery charge/discharge efficiency η is very close to 1.*

$$\eta_{ch} = \eta_{dis} = \eta \approx 1 \tag{4}$$

Assumption 2. *The energy storage system maintains phase-balanced operation across all phases $\phi \in \Phi_j$ at node j:*

$$P_{batt} = P_{batt}^{\phi} \tag{5}$$

The battery power variable $P_{\text{batt}} \in \mathbb{R}$ is subject to the constraint system $\mathbf{g}(P_{\text{batt}}, \boldsymbol{\xi}) = \mathbf{0}$ where $\boldsymbol{\xi}$ represents the state vector, and $\Delta\text{SOC} := \text{SOC}_t - \text{SOC}_{t-1}$ represents the state-of-charge increment. Now, under Assumption 1, the energy balance constraint (2b) reduces to:

$$\sum_{\phi\in\Phi_j} P_{\text{batt}} + \frac{\Delta\text{SOC}}{\Delta t} = 0 \tag{6}$$

Let $\boldsymbol{\mu} = (\mu_{\text{pb}}, \mu_{\text{batt}}, \mu_{\text{soc}})^{\top} \in \mathbb{R}^3$ be the dual variable vector associated with constraints (1a), (2a), and (2b) for each batteries. The first-order optimality conditions (KKT

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Algorithm 1 Forward-Backward Sweep Method

Initialize: Set $\hat{\lambda}^0_{b,t} = 0 \ \forall b \in \mathcal{B}, \forall t \in \mathcal{T}$, iteration $n = 0$
while not converged **do**
Forward Sweep:
  for $t = 1$ **to** T **do**
    Using eqn. (10) compute $\hat{\lambda}_{b,t+1}, \forall b \in \mathcal{B}$
    Solve temporal subproblem with $\hat{\lambda}_{b,t+1}$
    Update battery decisions and state-of-charge
  end for
Backward Sweep:
  for $t = T$ **downto** 1 **do**
    Extract dual variables from solved subproblems
    Update marginal costs $\hat{\lambda}_{b,t+1}, \forall b \in \mathcal{B}$
  end for
  Stop iteration if converged, else $n \leftarrow n + 1$
end while

conditions) yield (7) at any optimal solution P^*_{batt}, where $\mathcal{L}$ is the Lagrangian and $\mathbf{1}_3 = (1, 1, 1)^\top$.

$$\nabla_{P_{\text{batt}}} \mathcal{L}(P^*_{\text{batt}}, \boldsymbol{\mu}^*) = \left.\frac{\partial F}{\partial P_{\text{batt}}}\right|_{P^*_{\text{batt}}} + \boldsymbol{\mu}^\top \mathbf{1}_3 = 0 \tag{7}$$

Definition 1 (Marginal Energy Value)**.** The marginal charging cost $\lambda \in \mathbb{R}$ for $b \in \mathcal{B}$ are defined as the negative gradient of the objective function with respect to battery power.

$$\lambda := -\left.\frac{\partial F}{\partial P_{\text{batt}}}\right|_{P^*_{\text{batt}}} = \boldsymbol{\mu}^\top \mathbf{1}_3 = \mu_{\text{pb}} + \mu_{\text{batt}} + \mu_{\text{soc}} \tag{8}$$

The sign convention $P_{\text{batt}} > 0$ denotes discharge, which implies that λ quantifies the shadow price of energy storage, representing the marginal opportunity cost per unit of stored energy for the respective battery. Using temporal-iterative notation, let $\lambda^n_{t,\ b} \in \mathbb{R}$ denote the marginal energy value of battery b at time t during iteration n.

B. Subproblem Formulation for Temporal Decomposition

We use temporal decomposition to decouple the MPOPF problem while preserving intertemporal dependencies through value function approximation. The presence of a fixed point for this method ensures the iteration converges. For $t \in \mathcal{T}$, iteration $n+1$ and battery $b \in \mathcal{B}$, the temporal subproblem is formulated as (9). Let us define Ω_t as the set of local constraints for time step t, represented by (3b), and (3c); $\hat{\lambda}^n_{t+1,\ b} \in \mathbb{R}$ is the approximated future marginal cost for battery $b \in \mathcal{B}$. The second term in (9a) represents the value function approximation that aggregates the influence of future decisions into marginal price signals, enabling tractable coordination across time without solving the full horizon jointly.

$$\min_{\mathbf{x}_t \in \Omega_t} F^{n+1}_t(\mathbf{x}_t) := f_t(\mathbf{x}_t) - \sum_b \hat{\lambda}^n_{t+1,\ b} \cdot \frac{\text{SOC}_{t,b}}{\Delta t} \tag{9a}$$

$$\text{s.t.} \quad \text{SOC}^{\ init,\ n}_{\ t,\ b} = \text{SOC}^{\ n}_{\ t-1,\ b} \tag{9b}$$

The future marginal cost is approximated using the empirical average of subsequent period marginal costs:

$$\hat{\lambda}^n_{t+1} := \mathbb{E}^n[\lambda_s \mid s > t] = \frac{1}{|S_{t+1}|} \sum_{s \in S_{t+1}} \lambda^n_s \tag{10}$$

Algorithm 2 Parallel Temporal Decomposition Method

Initialize: Set $\hat{\lambda}^0_{b,t} = 0 \ \forall b \in \mathcal{B}, \forall t \in \mathcal{T}$, iteration $n = 0$
while not converged **do**
  Compute $\hat{\lambda}_{b,t+1}$ simultaneously
  for $t \in \{1, 2, \ldots, T\}$ in parallel **do**
    Solve temporal subproblem with $\hat{\lambda}_{b,t}$
    Extract battery decisions and marginal cost $\lambda_{b,t}$
  end for
  Collect results from all parallel processes
  Stop iteration if converged, else $n \leftarrow n + 1$
end while

where $S_{t+1} := \{t+1, t+2, \ldots, T\}$ and $|S_{t+1}| = T - t$.

Remark 1. To improve stability and convergence, marginal cost updates through fixed-point iteration method are regularized via weighted average, where approximated future marginal cost $\hat{\lambda}^n_{b,t+1}$ combines current and previous iterations with damping factor $\alpha \in (0, 1)$.

$$\hat{\lambda}^n_{b,t+1} := \alpha \cdot \hat{\lambda}^n_{b,t+1} + (1 - \alpha) \cdot \hat{\lambda}^{n-1}_{b,t+1} \tag{11}$$

C. FBS and Parallel Execution

This section introduces two approaches for solving the temporally decomposed optimization problem: FBS and parallel execution. The FBS alternates sequential forward and backward passes, updating subproblem solutions and marginal costs until convergence (see Alg. 1, Fig. 1a). In contrast, the parallel method solves all subproblems simultaneously across time steps, updating battery decisions and marginal costs globally (see Alg. 2, Fig. 1b). While both address multi-period optimization, FBS relies on sequential updates, whereas the parallel method improves efficiency via parallelism.

IV. Numerical Simulations

The proposed T-ENApp algorithm is validated on a modified, unbalanced IEEE 33-bus and IEEE 123-bus distribution system. The IEEE 33-bus system uses one 4.5 MWh BESS and an 8MW Distributed Generator (DG); the IEEE 123-bus scalability case includes four 1.675 MWh BESS units; the battery system is characterized by charging and discharging efficiencies of $\eta_{\text{ch}} = \eta_{\text{dis}} = 0.93$ and all rated at 2 MW. Inherent efficiency losses preclude simultaneous charging and discharging, maintaining convexity without binary variables. Simulations used a 24-h horizon ($\Delta t = 1$h) with $\alpha = 0.5$ (in equation (11)); stable convergence was observed for $\alpha \in [0.25, , 0.75]$. All tests used the Clarabel solver on an Intel i7-12800H (40GB RAM) with 24 parallel workers. Three test cases are analyzed to validate the algorithmic performance under varying energy price profiles:

- **Case 1**: IEEE 33-Bus system, Single price peak
- **Case 2**: IEEE 33-Bus system, Different initial SOC, Single price peak
- **Case 3**: IEEE 33-Bus system, Multi price peaks, high price volatility
- **Case 4**: IEEE 123-Bus system, 4 BESS, Single price peak

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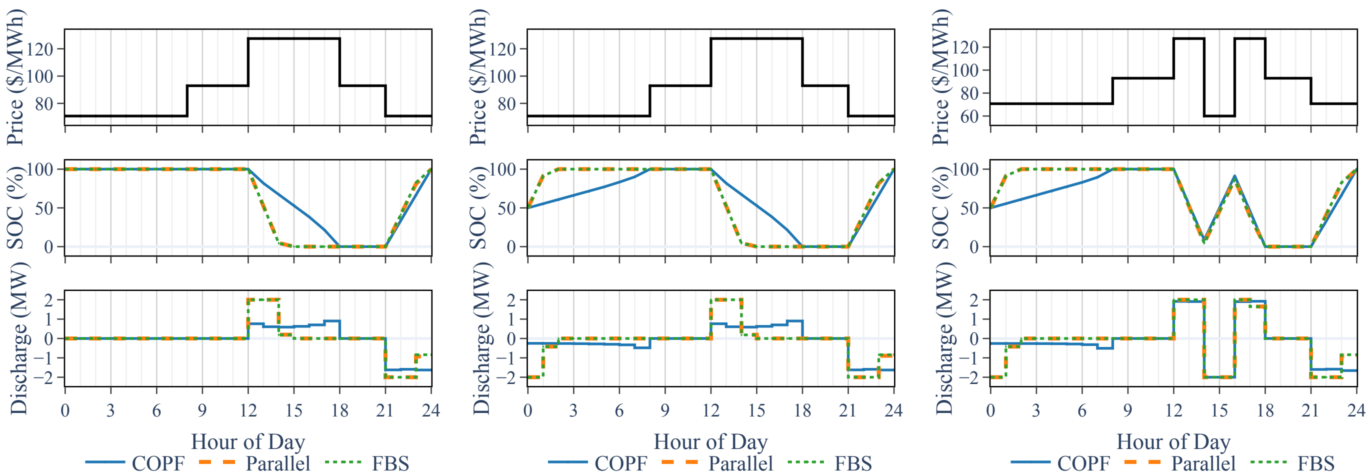
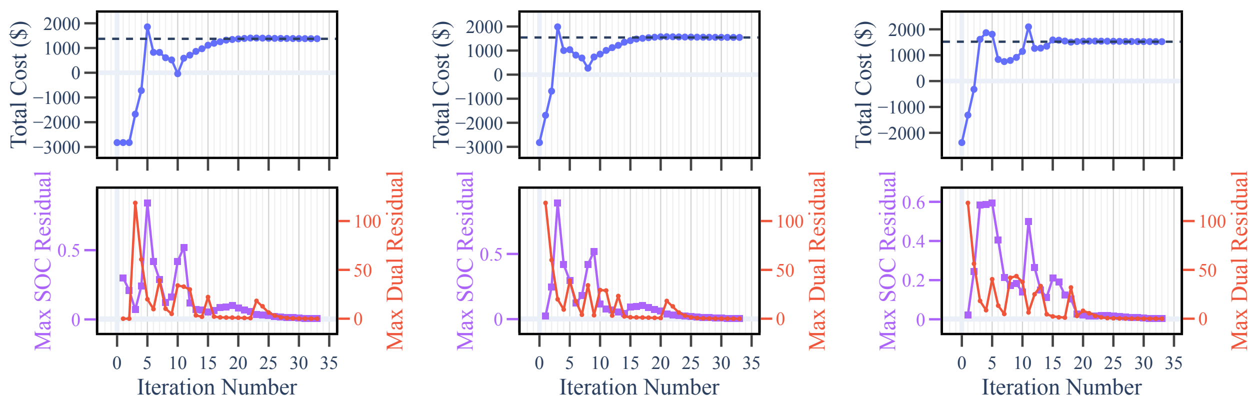


(a) Case 1 (b) Case 2 (c) Case 3

Fig. 2. SOC (%) and power injection w.r.t. time for both FBS and parallel execution of T-ENApp algorithm under different test case scenarios.



(a) Case 1: Parallel Execution (b) Case 2: Parallel Execution (c) Case 3: Parallel Execution

Fig. 3. Convergence properties of Parallel execution for T-ENApp algorithm

A. Optimal SOC and Power Dispatches

The computational results of the proposed T-ENApp algorithm are presented here for three distinct test cases. The performance comparison evaluates the FBS and parallel execution methods against the baseline (COPF). Fig. 2 illustrates battery SOC and power injection over time for Case1-3. Both methods demonstrate effective energy arbitrage, discharging during peak price periods (hours 12–17 in cases 1 and 2) and later fully recharging for the next day. In case 3, with high price volatility, the algorithms adapt with multiple charge-discharge cycles, reflecting responsiveness to price transitions while adhering to constraints. The close alignment of FBS and parallel trajectories highlights their theoretical equivalence, with a maximum deviation of less than 0.2%, validating convergence to near identical solutions.

B. Validation of T-ENApp

Table I presents a comprehensive evaluation comparing the FBS and parallel implementations of the T-ENApp algorithm against the COPF baseline across Case 1-3, highlighting solution quality, computational efficiency, and convergence characteristics. Both methods exhibit exceptional solution accuracy, with minimal deviation from the COPF benchmark. The parallel implementation shows strong alignment of the objective values with the COPF baseline, such as Case 1 achieves $1376.6 , representing a 0.31% gap relative to the $1372.3 COPF benchmark; In contrast, the FBS method achieves exact matching with the COPF solution across first 3 cases, producing identical objective values. We also have simulated Case 4 as a scalability test, where gap is 2.56% (gap reflects different final SOC approximations). These validate the

TABLE I
T-ENAPP ALGORITHM PERFORMANCE

Case	Execution	Time (s)	Cost ($)	COPF ($)	Gap	Speedup
Case 1	FBS	5.27	1372.3	1372.3	0.00%	–
	Parallel	1.24	1376.6		0.31%	4.24x
Case 2	FBS	5.12	1543.5	1543.5	0.00%	–
	Parallel	1.15	1547.3		0.25%	4.45x
Case 3	FBS	5.37	1520.8	1520.8	0.00%	–
	Parallel	1.13	1523.5		0.17%	4.74x
Case 4	Parallel	1.24	5,314.51	5,454.32	2.56%	4.52x

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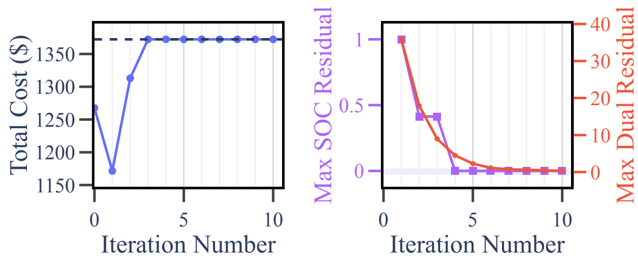

Fig. 4. Convergence properties of T-ENApp FBS execution: Case 1

effectiveness of the temporal value-function approximation and confirm that the proposed decomposition achieves near-optimal economic performance. While the FBS implementation yields COPF-equivalent solutions, its sequential nature limits computational efficiency and scalability. In contrast, the parallel implementation exploits simultaneous subproblem execution to achieve up to a $4\times$ speedup. This gain is enabled by the temporal decomposition framework, which distributes computational effort across processors and accelerates the resolution of complex price dynamics.

C. Convergence of T-ENApp

The criterion for convergence is defined as a change of less than 0.1% in consecutive iterations for both SOC and dual residuals (approximated future marginal costs $\hat{\lambda}_{t+1}$). The FBS method achieves convergence in 5 forward sweeps, while the parallel method requires 33 iterations (Fig. 3). FBS exhibits monotonic convergence due to its sequential structure, ensuring that each temporal subproblem is solved with complete information from earlier periods, as shown in Fig. 4. Similar behavior is observed across all cases but is omitted for brevity. On the other hand, the parallel method demonstrates more complex convergence dynamics arising from simultaneous subproblem solutions. Temporal coupling is maintained through approximated future value functions, introducing a fixed-point iteration process. Despite these complexities, the damping mechanism ensures stable convergence in all test cases. The parallel execution balances computational efficiency and solution accuracy, confirming its viability for large-scale multi-period OPF problems. Please note, for same test cases, ADMM-based consensus method [18] failed to converge within 300 iterations, with persistent oscillations.

V. Conclusion

Optimizing storage dispatches in distribution systems often involves solving large-scale, unbalanced MPOPFs. This paper develops T-ENApp, a novel temporally decomposed, distributed optimization framework that addresses temporal scalability challenges through value function approximation. The method leverages temporal separability to build an equivalent temporal network, enabling both parallel implementation and the FBS approach. Unlike ADMM-based methods, T-ENApp coordinates temporal coupling via network-equivalent boundary states informed by future marginal costs and storage dynamics. Computational tests on the IEEE 33-bus test system show that the parallel execution achieves approximately $4\times$ speedup over sequential FBS method while maintaining near optimal solution across diverse energy price profiles. Test on the IEEE 123-bus test system confirms that network scalability has negligible impact on solution time. The algorithm's ability to decompose complex temporal dependencies into parallelizable subproblems while preserving near-optimal economic performance makes it particularly valuable for applications where fast response of the energy storage are required. Future work will aim at extending the methodology for scalability to larger power systems with longer time horizons and include a mathematical analysis of convergence properties.

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