A Novel Integrated Quantum Computing Approach for Unit Commitment

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***Abstract* – The transition toward a decarbonized power grid introduces increasing operational complexity driven by variable renewable, distributed energy resources, electric vehicles, and energy storage resources. These dynamics require rapid and efficient solutions to the Unit Commitment (UC) problem, which classical computing may struggle. Quantum Computing (QC) offers a promising alternative by exploiting qubits' exponential information capacity to accelerate large-scale optimization. This paper presents a novel integrated QC-based approach to solve UC through synergistic integration of Quantum Feasibility Labeling (QFL), Variational Quantum Search (VQS), and Quantum Global Minimum Finder (QGMF). It enhances feasible-state identification and global optimum discovery, addressing the limitations of existing algorithms such as the Quantum Approximate Optimization Algorithm. Two illustrative UC case studies are used to demonstrate the approach. Results indicate that the combined QFL-VQS-QGMF framework can identify the optimal UC solution accurately. This work highlights the potential of hybrid quantum–classical optimization as a foundational step toward scalable quantum solutions for future grid operations.**

***Index Terms* -- Mixed Integer Linear Programming, Quantum Computing, Unit Commitment**

I. Introduction

The power grid faces numerous challenges as it transitions toward decarbonization. Following Executive Order 14057, which targets a 50-52% reduction in greenhouse gas emissions by 2030, many U.S. states have adopted ambitious renewable goals. Integrating variable renewable energy like wind and solar introduces volatility, while FERC Order 2222 enables distributed resources to join wholesale markets, increasing coordination challenges [1]. Rising electric vehicle adoption adds further uncertainties on the demand side. Battery energy storage is becoming crucial for grid flexibility and congestion relief. As the dynamics on the power grid increase, the requirement for solving Unit Commitment (UC) becomes vital, a task classical computing struggles to achieve within reasonable time frames. Going forward, the future grid is anticipated to be more stochastic, requiring more measurement and data processing efforts for reliable grid maintenance.

Quantum Computing (QC) offers a promising pathway to address complex grid optimization challenges. By exploiting qubits' exponential information capacity, QC can accelerate problems like UC far beyond classical limits. Recent advances in both hardware (now reaching hundreds of qubits) and algorithms have driven rapid progress. Hybrid quantum-classical methods, such as Variational Quantum Algorithms (VQAs), have shown early success in optimization and machine learning tasks. Building on this progress, quantum optimization algorithms like the Quantum Approximate Optimization Algorithm (QAOA) and Grover's algorithm demonstrate the potential to solve complex problems faster than classical methods, though they remain constrained to specific formulations such as Quantum Unconstrained Binary Optimization (QUBO). However, current quantum systems remain limited in qubit count and circuit depth, restricting direct application to large-scale UC problems. As a result, researchers are exploring hybrid and decomposition-based frameworks to leverage quantum advantages within practical resource constraints while gradually advancing toward large-scale power system optimization.

In this paper, a novel integrated QC-based approach is developed to solve UC by synergistic integration of Quantum Feasibility Labeling (QFL), Variational Quantum Search (VQS), and Quantum Global Minimum Finder (QGMF). The QFL algorithm provides a systematic framework to classify candidate quantum states for UC solutions into feasible and infeasible ones, where the linear UC constraints and objective function are converted to logical ones and implemented via Quantum circuits. To address the 'Limited Theoretical Guarantee' challenge inherent in the QAOA, VQS leverages its proven capability to efficiently identify all feasible solutions. To further enhance the efficiency of optimal solution identification, QGMF expedites the discovery of the optimal solution. This innovative integration of QFL, VQS, and QGMF holds the potential to significantly advance the field of QC, addressing the limitations associated with QAOA.

To demonstrate the performance of the approach, two illustrative examples are presented. The first is a two-unit one-hour problem with unit generation capacity and system demand constraints, and the second is a one-unit two-hour example with unit generation capacity and start-up constraints. Results indicate that the combined QFL-VQS-QGMF framework can

This work is supported in part by the ISO-New England and by the National Science Foundation (NSF) under grant ECCS-2340095. Any opinions, findings, conclusions, or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the ISO-New England and NSF.

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979-8-3315-8138-1

identify the optimal UC solution accurately.

II. Literature Review

In this section, general Quantum Computing approaches are reviewed in Subsection A, and the Quantum Computing approaches for UC are reviewed in Subsection B.

A. Quantum Computing

In the past decade, the progress in quantum computer hardware development has been particularly impressive in recent years [2]. IBM has built a 1,121-qubit computer in December 2023 [2] and aims to reach 100k qubits by 2033. These quantum computers have demonstrated impressive performance on a variety of tasks [3, 4], and their capabilities are expected to continue to improve. Quantum algorithms, evolving alongside hardware, demonstrate notable success [5, 6]. It is expected that the development of quantum algorithms will continue to be an active area of research in the coming years [2]. VQAs [7] involve both classical and quantum computers, showing great promise in helping to realize systematic supremacy of QC since they typically require fewer qubits and lower circuit depth [7]. They have been successfully used in various areas such as optimization problems [8] and machine learning [9, 10], etc. Algorithms such as the QAOA and Grover's algorithms can handle only certain types of optimization problems. These algorithms require the optimization problem to be reformulated as a QUBO problem and can only solve for binary variables. Quantum Amplitude Amplification [11], a generalization of Grover's search algorithm, quadratically speeds up solutions to many important problems, such as NP-complete problems [12], etc. However, they face challenges with exponential circuit depth growth.

B. Quantum Computing for UC

With hybrid quantum and classical methods, large-scale UC problems are still difficult to solve. Most of the existing studies employ a combination of QC and decomposition and coordination approaches to solve UC [13-15], and a few on a combination of QC and heuristics. This review primarily focuses on the former. In [13], the key idea is to combine the Alternating Direction Method of Multipliers (ADMM) with QAOA to solve the UC problem. The UC problem is reformulated using the Augmented Lagrangian method, allowing the incorporation of quantum optimization techniques alongside classical solvers to compute optimal solutions. However, the scale of the UC problem solved is so small that it is not sufficient to validate the exponential speedup offered by the proposed methodology. In [14], a Quantum Surrogate Lagrangian Relaxation (QSLR) algorithm is developed by combining SLR and QAOA to address large-scale UC problems. The methodology involves developing a quantum formulation of the UC problem as QUBO using QSLR. In this QSLR framework, the unit-wise subproblems are solved sequentially. The results show that the QSLR method enables the use of QC techniques to solve large-scale UC problems. A novel quantum annealing techniques-based approach is explored by decomposing the UC problem into binary and linear programming subproblems using Bender's decomposition [15]. Bender's decomposition proves instrumental in addressing the challenge posed by a substantial number of constraints in the QUBO problem. The performance of the methodology is demonstrated through a small system.

III. Simple UC Formulation

In this section, the formulation of the deterministic hourly UC is summarized. The unit-level constraints are presented in Subsection A, system-level constraints in Subsection B, and the objective function in Subsection C. To apply the QC-based approach, continuous variables are discretized with binary variables as discussed in Subsection D.

A. Unit-level constraints

A power system with I units is considered, where i is the unit index. The UC is formulated over T periods with t as the time index. Here, two types of constraints are considered: generation capacity constraints (1) and start-up constraints (2-4):

$$P_i^{\min} x_{i,t} \le p_{i,t} \le P_i^{\max} x_{i,t}, i \in [1, I], t \in [1, T], \tag{1}$$

$$u_{i,t} \ge x_{i,t} - x_{i,t-1}, i \in [1, I], t \in [1, T], \tag{2}$$

$$u_{i,t} \le x_{i,t}, i \in [1, I], t \in [1, T], \tag{3}$$

$$x_{i,t-1} + u_{i,t} \le 1, i \in [1, I], t \in [1, T]. \tag{4}$$

In the above, the decision variables consist of binary commitment status $x_{i,t}$, binary start-up status $u_{i,t}$, and continuous generation level $p_{i,t}$. Unit initial conditions related to these decision variables are set as $x_{i,0}$. The minimum and maximum generation limits are P_i^{min} and P_i^{max}, respectively. To be accurate for the constraints, tightened start-up constraints (3) and (4) obtained in [16] are also considered in addition to the regular start-up constraint Eq. (2).

B. System-level constraints

For simplicity, only the system demand constraints are considered as below,

$$\sum_{i=1}^{I} p_{i,t} = L_t, t \in [1, T], \tag{5}$$

where L_t is the total demand at time t.

C. Objective function

As shown in (6), the objective is to minimize the total operating cost of all units, i.e.,

$$\sum_{i=1}^{I} \sum_{t=1}^{T} \left[C_{i,t}^{NL} x_{i,t} + C_{i,t}^{SU} u_{i,t} + C_{i,t}^{E} p_{i,t} \right]. \tag{6}$$

The costs include no-load cost $C_{i,t}^{NL}$, start-up cost $C_{i,t}^{SU}$, and generation cost $C_{i,t}^{E}$.

D. Discretized problem

To apply the QC-based approach, $p_{i,t}$ is discretized with M_i binary variables $p_{i,t,k}$,

$$p_{i,t} = \sum_{k=0}^{M_i - 1} w_k p_{i,t,k} \text{ where } M_i = \log_2 \left\lceil p_i^{\max} \right\rceil, w_k = 2^k D. \tag{7}$$

In the above, ω_k is the weight, and D is the scale or step size.

Then the UC problem can be converted to a binary linear programming problem as shown below,

$$\min \sum_{i=1}^{I} \sum_{t=1}^{T} \left[C_{i,t}^{NL} x_{i,t} + C_{i,t}^{SU} u_{i,t} + C_{i,t}^{E} \sum_{k=0}^{M_i - 1} w_k p_{i,t,k} \right] \tag{8}$$

$s.t.$

$$P_i^{\min} x_{i,t} \le \sum_{k=0}^{M_i - 1} w_k p_{i,t,k} \le P_i^{\max} x_{i,t}, i \in [1, I], t \in [1, T],$$

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$$u_{i,t} \ge x_{i,t} - x_{i,t-1}, u_{i,t} \le x_{i,t}, x_{i,t-1} + u_{i,t} \le 1, i \in [1, I], t \in [1, T],$$

$$\sum_{i=1}^{I} \sum_{k=0}^{M_i - 1} w_k p_{i,t,k} = L_t, t \in [1, T].$$

IV. QC-based Approach

In this section, a novel integrated QC-based approach is developed by synergistic integration of QGMF, QFL, and VQS as detailed below. The three components are introduced in the first three subsections, and the overall approach is summarized in the last subsection.

A. Quantum Global Minimum Finder (QGMF)

The QGMF [17] is a novel quantum algorithm designed to efficiently identify global minima of arbitrary functions, particularly those with multiple local optima. Many real-world optimization problems in engineering, finance, and artificial intelligence are characterized by highly non-convex landscapes, where classical optimization algorithms often become trapped in local minima. QGMF overcomes this limitation by combining a binary search mechanism with the VQS framework, thereby leveraging both logarithmic convergence and linear-depth quantum circuit efficiency.

QGMF integrates quantum binary search with a QFT-based adder and the oracle-free VQS to efficiently locate global minima of non-convex functions. By leveraging quantum parallelism for function evaluation, QGMF achieves logarithmic convergence and linear-depth circuit complexity O(n), providing a scalable foundation for noisy intermediate-scale quantum (NISQ) era optimization.

B. Quantum Feasibility Labeling (QFL)

The QFL algorithm provides a systematic framework to classify all candidate quantum states of a combinatorial optimization problem into feasible and infeasible solutions [18, 19]. Originally demonstrated for the vertex-coloring problem [20, 21], QFL transforms any NP-complete problem into the task of labeling an unstructured quantum database in which each possible solution is encoded by qubits and its feasibility is represented by a single label qubit. A label state $|1\rangle$ denotes a feasible solution, while $|0\rangle$ indicates an infeasible one.

Unlike Grover-based quantum search methods that depend on externally defined oracles to mark target states, QFL embeds the feasibility conditions directly within the circuit construction itself. The final global label qubit is obtained by cascading all local feasibility outputs through a sequence of quantum AND operations, yielding a single qubit that compactly represents the global feasibility of each state.

The computational resources required by QFL scale polynomially with problem size. For a k-colorable n-vertex g-edge vertex coloring instance, the circuit depth scales as O(k^2g) (with reset circuits) or O(kg) (without resets), and the total number of qubits is $mn + m + 2g$ (where $m = \lceil \log_2 k \rceil$). Thus, QFL provides a scalable method for labeling feasible solutions in polynomial depth and qubit count. When coupled with the VQS algorithm, which amplifies the probability of the $|1\rangle$ label states, this framework (QFL + VQS) can identify all feasible solutions efficiently without the need for an external oracle.

C. Variational Quantum Search

The VQS algorithm integrates the principles of VQAs with amplitude amplification to achieve efficient quantum search in the NISQ era [22, 23]. Unlike Grover's algorithm, whose circuit depth grows exponentially with the number of qubits, VQS employs a parameterized quantum circuit (Ansatz) optimized via classical feedback to maximize the probability of "good" solutions. Numerical and experimental validation up to 26 qubits demonstrates that a shallow depth-10 circuit can amplify the success probability from an initial uniform distribution to nearly unity, while the required circuit depth scales linearly with system size. This linear-depth scaling yields an exponential reduction in circuit complexity compared with Grover's approach (e.g., depth ≈ 56 vs. 270,989 for 26 qubits), making VQS a practical search framework for large-scale combinatorial optimization, such as UC, when coupled with QFL and the QGMF scheme.

D. Overall approach

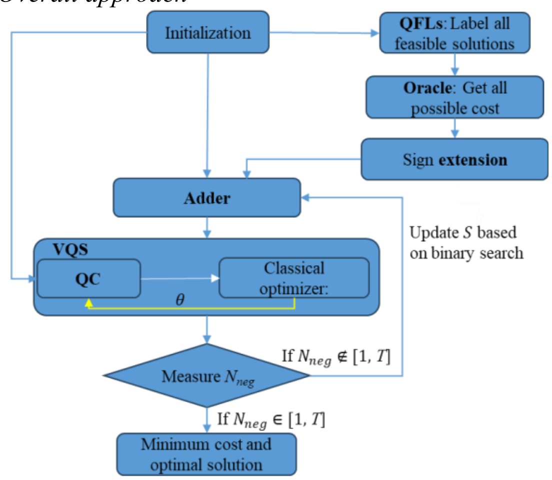

Figure 1: Flowchart of QGMF+QFL+VQS

The overall workflow of the proposed QGMF + QFL + VQS framework is illustrated in Figure 1. The process begins with QFL, where all candidate UC solutions (e.g., on/off, startup, and generation levels) are quantum-encoded and labeled as feasible that satisfy all constraints (e.g., capacity and startup constraints) or infeasible. The Oracle then evaluates the corresponding UC cost function, and a sign-extension step provides labels to feasible solutions with negative costs. The QGMF employs a QFT-based adder to iteratively shift the objective function along the y-axis. In each iteration, VQS, optimized by a classical feedback loop, measures the number of negative outputs (N_{neg}) and guides the binary-search-based update of the shift S_i. Once N_{neg} falls within [1, T], the global minimum ($G_m = M_r - S_i$) and the corresponding optimal UC solution are obtained. This hybrid framework combines polynomial-depth labeling (QFL), logarithmic-time convergence (binary search), and linear-depth quantum search (VQS), offering improved scalability and reduced noise sensitivity on NISQ-era hardware. The complexity of the quantum circuits depends on the number of qubits and circuit depth, with the former determined by the number of binary decision variables, unit parameters (e.g., cost, p^{min} and p^{max}), system-level data, feasibility checking, etc.

V. Numerical Testing

The above approach is implemented in Python 3.13, NumPy version 1.26.4, and PennyLane v0.40.0 on a laptop with Intel Core i7 with 32GB memory and Nvidia ADA 2000 GPU with 4GB memory. To demonstrate the performance of the approach, two illustrative examples are presented. The first is a two-unit one-hour problem with unit generation capacity and system demand constraints, and the second is a one-unit two-hour example with unit generation capacity and start-up constraints.

Example 1

In this example, consider a system with two generators and D = 3 for a period of one hour. Here, unit generation capacity and system demand constraints are considered, and start-up costs and constraints are not. The generator parameters are shown in Table 1 below.

Table I: Unit parameters

Generator	C_i^{NL}	C_i^E	P_i^{min}	P_i^{max}
1	2	4	1	3
2	4	2	1	3

For each unit, one qubit is needed for x, and two qubits for p. To realize the linear unit capacity constraints Eq. (1) through quantum circuits for QFL, they are converted to logical constraints as follows,

$$\text{if } x_i = 0, \text{then } p_{i0} = p_{i1} = 0; \tag{9}$$

$$\text{if } x_i = 1, \text{then } 1 \le p_{i0} + 2p_{i1} \le 3. \tag{10}$$

Eq. (9) is implemented by using *Controlled-Hadamard* gates, and Eq. (10) is implemented by using *Controlled Quantum integer comparators* with P_i^{min} and P_i^{max} as input parameters. The two sides of the capacity constraints for one unit are combined with a logic *AND*, implemented by using *CNOT* gates with an additional qubit, which is also controlled by x_i with a logic OR. The overall QFL circuit is shown in Figure 2 below (because the comparators can only handle "<", not "≤", P_i^{max} +1 is used instead of P_i^{max}).

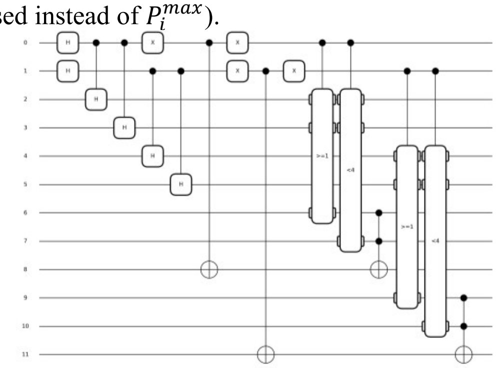

Figure 2: QFL for capacity constraints

The system demand constraint is rewritten as follows,

$$\left(p_{10} + 2p_{11}\right) + \left(p_{20} + 2p_{21}\right) - 3 = 0. \tag{11}$$

The addition of p related qubits is implemented by using QFT Addition with three additional qubits (based on the highest possible total generation without considering demand), taking ω_k as input parameters. The subtraction is implemented by using QFT Subtraction with L as input parameters. At the end, the overall feasibility of the demand constraints is checked through a logic *AND* implemented by using *CNOT* gates. The overall QFL circuit is shown in Figure 3 below.

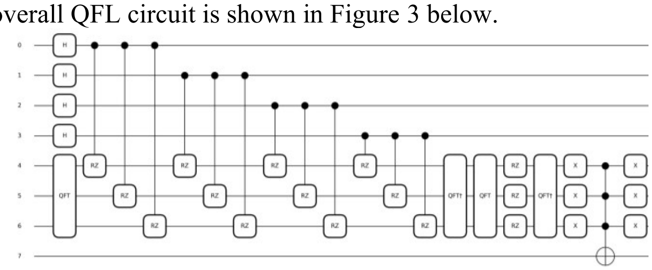

Figure 3: QFL for system demand constraints

At the end, the overall feasibility of the capacity constraints for the two units and the system demand constraints is combined with a logic *AND* implemented by using CNOT gates with an additional qubit.

The objective is given as,

$$F = 2x_1 + 4x_2 + 4\left(p_{10} + 2p_{11}\right) + 2\left(p_{20} + 2p_{21}\right). \tag{12}$$

This addition is also implemented by using QFT Addition with six additional qubits (based on the highest possible cost without considering demand), taking C_i^{NL}, C_i^E, and ω_k as input parameters. In addition, if the solution is not feasible, F = 0. The circuit is shown in Figure 4 below.

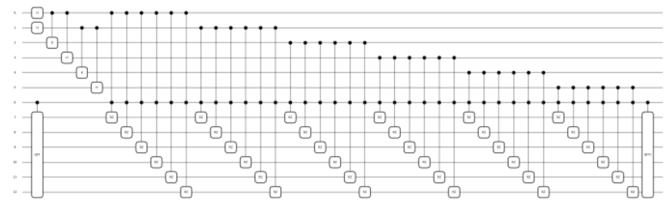

Figure 4: Oracle for cost function

As for the Adder in QGMF, the circuit is designed based on standard QFT Addition. In terms of VQS, the circuit is designed based on the standard QFT and RY gates following [22, 23]. For illustration purposes, a VQS circuit for 5-qubit searching is shown in Figure 5 below.

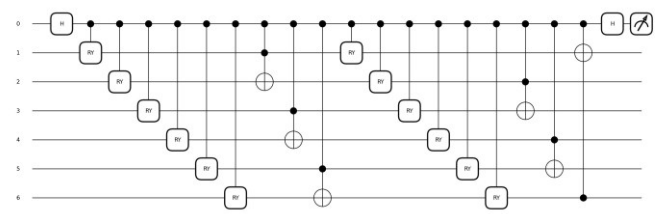

Figure 5: VQS for 5-qubit searching

For the overall circuit of QFL-VQS-QGMF, there are 26 qubits: 6 qubits for x and p; 5 qubits for F + 1 for the sign (positive or negative) of F; (1 qubit for P^{min} feasibility+ 1 qubit for P^{max} feasibility + 1 qubit for combined feasibility of capacity constraints)×2 units; 3 qubit for summation of p + 1 qubit for system demand feasibility; 1 qubit for overall feasibility; 1 qubit for the Adder; 1 qubit for combined feasibility and cost sign; and 1 qubit for H-test in VQS.

By using the QFL-VQS-QGMF, the measured bitstring with the highest probability of 97.25% is [010011 001010], where the first 6 qubits are for x and p, and the last 6 qubits are the corresponding cost F. Therefore, the optimal solution is x = [0 1] and p = [0 3], and the optimal cost is 10, which is the same as what obtained by using classical computing. The entire simulation time is 5.92 seconds via 5 steps of S updates.

Example 2

The second example is a one-unit two-hour problem with unit generation capacity and start-up constraints (generator 1 in Example 1 with C^E=2, C^{NL}=2, C^{SU}=3). The objective is to maximize the profit (i.e., payment - cost) as shown below,

$$\sum_{t=1}^{T}\left[\lambda_t\, p_t - \left(C_t^{NL} x_t + C_t^{SU} u_t + C_t^E p_t\right)\right]. \tag{11}$$

where λ is the payment, which is 3 in this example.

For each hour, one qubit is needed for x, one qubit for u, and two qubits for p. One more qubit is also needed for the initial condition x_0. To realize the linear start-up constraints Eq. (2-4) through quantum circuits for QFL, they are converted to logical constraints as follows,

$$\text{if } x_0 = 0, \text{then } u_1 = x_1; \text{if } x_0 = 1, \text{then } u_1 = 0; \quad (12)$$

$$\text{if } x_1 = 0, \text{then } u_2 = x_2; \text{if } x_1 = 1, \text{then } u_2 = 0. \quad (13)$$

For hour 1, Eq. (12) is implemented by using *CNOT* gates with an additional feasibility qubit, which is controlled by x_0, x_1, and u_1. For hour 2, Eq. (13) is implemented similarly with an additional feasibility qubit, which is controlled by x_1, x_2 and u_2. The overall QFL circuit is shown in Figure 6 below.

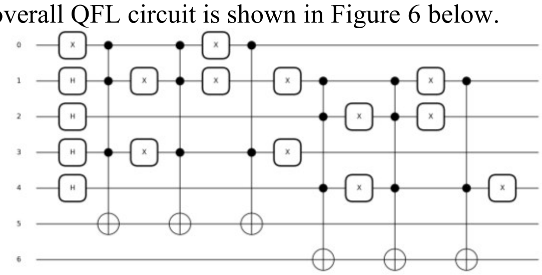

Figure 6: QFL for start-up constraints

For the overall circuit of QFL-VQS-QGMF, there are 27 qubits: 8 qubits for x, u and p; 1 qubits for x_0; 5 qubits for F + 1 for the sign (positive or negative) of F; (1 qubit for P^{min} feasibility+ 1 qubit for P^{max} feasibility + 1 qubit for combined feasibility of capacity constraints)×2 hours; (1 qubit for start-up feasibility)×2 hours; 1 qubit for overall feasibility; 1 qubit for the Adder; 1 qubit for combined feasibility and cost sign; and 1 qubit for H-test in VQS.

By using the QFL-VQS-QGMF, the measured bitstring with the highest probability is [00000000 000000] (first 8 for x, u, and p, and last 6 qubits for cost) when $x_0 = 0$ and [11001111 100010] when $x_0 = 1$. The probabilities are 73.23% and 73.17%, respectively. Therefore, for $x_0 = 0$, the optimal solution is $x = [0\ 0]$, $u = [0\ 0]$, and $p = [0\ 0]$, and the optimal cost is 0. For $x_0 = 1$, the optimal solution is $x = [1\ 1]$, $u = [0\ 0]$, and $p = [3\ 3]$, and the optimal cost is -2. The results are the same as those obtained by using classical computing, where the unit remains offline if initially off due to the start-up cost and remains online if initially online for profit. The entire simulation time is 42.50 and 32.45 seconds via 6 and 4 steps of S updates, respectively.

VI. Conclusion

This paper presents a novel integrated QC-based framework to solve UC through the synergistic integration of QFL, VQS, and QGMF. It enhances feasible-state classification by QFL, where the linear UC constraints and objective function are converted to logical ones and implemented via Quantum circuits. The limitations of existing algorithms, such as QAOA, are overcome by VQS, which leverages its proven capability to efficiently identify all feasible solutions. The global optimum discovery is guaranteed by QGMF. The two illustrative examples indicate that the combined QFL-VQS-QGMF framework can identify the optimal UC solution accurately. This work highlights the potential of hybrid quantum-classical optimization as a foundational step toward scalable quantum solutions for future grid operations. In future work, due to the qubit limit, a decomposition and coordination framework will be used to decompose the overall UC into unit subproblems, and the unit subproblem may need to be further decomposed into a few sub-subproblems of a few hours. This integrated QC-based approach will be used to solve the sub-subproblems.

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