

Towards Efficient Probabilistic Simulations of Cascading Outages in Power Systems

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Abstract—Cascading outages that lead to blackouts have become a significant threat to the operation of a power system. Uncertain renewable generation and loads alter the original operational states of power systems. These uncertain inputs significantly affect the propagation of cascading outages, leading to uncertain sizes of cascading outage events. Therefore, simulating and estimating the probability of rare cascading outage events is crucial to assessing the operational safety of power systems. Cascading outages resulting from the numerous tripped lines have an extremely low probability of occurrence, but their impact is considerable. However, Monte-Carlo simulation (MCS) is computationally inefficient for estimating rare events. To address these issues, we propose a subset simulation method to effectively search for rare cascading outages. The subset simulation decomposes the rare cascading outage events into a series of intermediate events with larger probabilities, which are easier to estimate. In addition, we investigate the impact of renewable energy penetration levels on cascading outages and simulation performance. The simulation results reveal the excellent performance of the proposed method.

Index Terms—Cascading outages, probability estimation, rare events, subset simulation.

I. INTRODUCTION

CASCADING outages in power systems are a chain of events triggered by an initial failure that leads to multiple-stage outages that ultimately evolve into a widespread blackout [1]. Recent research has focused on models of the evolution of cascading outages. Park *et al.* [2] developed a novel cascading outages model considering the impact of transient dynamics of power systems on the cascading outages process. Li *et al.* [3] studied the topological properties of the power system during the propagation of cascading outages. Guo *et al.* [4] proposed the importance sampling method to efficiently estimate the probability of cascading high-impact outage events, resulting in high load shedding levels in power systems. However, the aforementioned works only consider the deterministic initial states of the power system. The increasing integration of highly uncertain renewable generation and loads results in uncertain operation states [5]. The evolution of cascading outages in the power system is heavily influenced by initial operating states. The same initial failure may lead to

different cascading paths under different load and generation conditions [6], [7].

Probabilistic power system failure analysis is widely advocated in the literature as a means to account for operational risk induced by uncertain inputs in a power system [8]. Xu *et al.* [9] proposed the multi-surrogate method for each $N-1$ event, while the remaining $N-k$ events are still assessed by the probabilistic power flow model. Ly *et al.* [10] proposed the quantile lite-polynomial chaos expansion to quantify the probability density functions of the operation states under $N-1-1$ outages. However, the aforementioned works model only multiple initial outages and do not account for the entire cascading outage propagation process after those initial outages. Henneaux *et al.* [11] proposed a probabilistic cascading outage model with MCS to simulate possible probabilistic evolutions. However, simulating rare cascading outage events with MCS requires many samples. For example, searching for rare cascading outage events with a probability of 10^{-3} requires approximately 10^5 MCS samples to achieve the desired accuracy. Therefore, a large number of scenarios should be considered in the cascading outage simulation, resulting in an extremely high computational burden. This highlights the growing need for an effective method for simulating rare cascading outages. The splitting method partitions the original simulation into a sequence of immediate events, defined by the number of tripped lines, thereby improving sampling efficiency [12], [13]. In this framework, the uncertainty lies in the stochastic evolution of the cascading process itself, such as random line outages. However, uncertainty in initial power system operating states is also a significant driving factor in the evolution of a cascading outage [14].

To address the aforementioned challenges, we propose a novel cascading-outage model that accounts for the uncertainty in power system operating states arising from stochastic generation and load variations. Specifically, we develop the subset simulation method to achieve high efficiency in capturing high-impact, low-frequency cascading outage events. The main contributions of this paper include:

- We formulate a novel probabilistic cascading outage model that starts with a specific initial failure. We simulate the uncertain number of line outages, accounting for

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the uncertainty in renewable generation and load.

- We develop the subset simulation method to alleviate the computational burden of simulating rare cascading outage events and estimate their probability. The subset simulation accelerates the process by decomposing rare cascading outage events into a series of intermediate events with higher probabilities, which are more tractable.

The remainder of the paper is organized as follows: Section II establishes the probabilistic cascading outage model with uncertain inputs. Section III proposes the subset simulation method for efficiently simulating rare cascading outage events. Section IV presents a case study on the IEEE 118-bus power system. Section V draws the conclusion.

II. POWER SYSTEMS CASCADING OUTAGE SIMULATION CONSIDERING UNCERTAIN INPUTS

In this section, we build the probabilistic model for loads and renewable generation. Then, we describe the uncertainties in the power system for its probabilistic analysis.

A. The Probabilistic Power Flow of Power Systems

1) *Uncertain Loads*: The uncertain loads are assumed to follow the Gaussian distribution [10]

$$p(\psi) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{(\psi - \mu)^2}{2\sigma^2}\right], \quad (1)$$

where p is the probabilistic density function; ψ is the load; μ and σ are the mean and standard deviation, respectively.

2) *Uncertain Wind Power Generation*: The wind speeds are assumed to follow the Weibull distribution [15]

$$p(v) = \frac{k}{\epsilon} \left(\frac{v}{\epsilon}\right)^{k-1} \exp[-(v/\epsilon)^k], \quad (2)$$

where v is the wind speed, ϵ and k are the scale and location parameters, respectively.

The relationship between the wind speed and active power of a wind farm is assumed to satisfy a piecewise function.

3) *Uncertain Photovoltaic (PV) Generation*: The solar radiations are assumed to follow the Beta distribution [16]

$$p(R) = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} \left(\frac{R}{R_{\max}}\right)^{\alpha-1} \left(1 - \frac{R}{R_{\max}}\right)^{\beta-1}, \quad (3)$$

where α and β are shape parameters of the Beta distribution, R and $R_{\max}$ are the actual and maximum solar radiations, respectively. The relationship between the solar radiation and the active power of a PV is assumed to satisfy a piecewise function.

Till now, we can analyze the probabilistic power flow by

$$\mathbf{g} = \mathbf{f}(x_1, \dots, x_n), \quad (4)$$

where $\mathbf{x} = \{x_1, \dots, x_n\}$ represents n -dimensional uncertain input variables, such as loads and renewable generations; $\mathbf{g}$ represents uncertain output variables of the system states, such as the power flow of branches.

B. Power Systems Cascading Outage Model with Uncertain Inputs

To quantify the uncertainty of cascading outages induced by uncertain initial states, stochastic analysis techniques are needed. The detailed procedure of the proposed probabilistic cascading outage model is as follows:

Step 1 Generate uncertain initial operating states: First, we sample the uncertain loads and renewables. Then we calculate the uncertain initial operating states using the probabilistic power flow shown in (4).

Step 2 Simulate cascading outages: The cascading outage simulation starts with an initial line outage. The bus admittance matrix is updated by removing the tripped branches. We calculate the power flow of the power system after the topology update. We then check whether all lines operate within safety limits. The line with the most serious power exceedance is tripped if branch overloads occur.

Step 3 Analyze statistical characteristics: We collect the number of tripped lines in each of the cascading outage scenarios. The probability density function of the uncertain cascading outage size is then obtained.

The flowchart for the probabilistic simulation of cascading outages is shown in Fig. 1, which outlines a double-loop process for uncertainty quantification and cascading outage simulation.

Remark 1. We consider the number of tripped lines in a cascading outage event to be the output of the power flow. Motivated by the idea of probabilistic power flow, the traditional deterministic cascading outage model is reformulated in a probabilistic manner.

III. THE PROPOSED METHOD FOR EFFICIENT RARE CASCADING OUTAGE EVENT SIMULATION

In this section, we propose the subset simulation to search for low-frequency cascading outage events at the tail of the probability density function of the uncertain cascading outage size.

A. Definition of Rare Cascading Outage Events

We quantify the size of cascading outages by the total number of fault branches involved, with those exceeding a certain threshold considered rare. The domain of rare cascading outages Ω is defined as

$$\Omega = \{\mathbf{x} \in \mathbf{\Omega} \mid h(\mathbf{x}) > \bar{\epsilon}\}, \quad (5)$$

where $h(\cdot)$ represents the total number of tripped lines in a cascading outage event; $\bar{\epsilon}$ represents the threshold for the target number of tripped lines; $\mathbf{x}$ represents uncertain variables of the power system; and $\mathbf{\Omega}$ represents the space for uncertain variables.

The probability of rare cascading outages is given by

$$P_{\Omega} = P(h(\mathbf{x}) > \bar{\epsilon}) = \int_{\Omega} \Lambda_{\Omega}(\mathbf{x})p(\mathbf{x}) d\mathbf{x}, \quad (6)$$

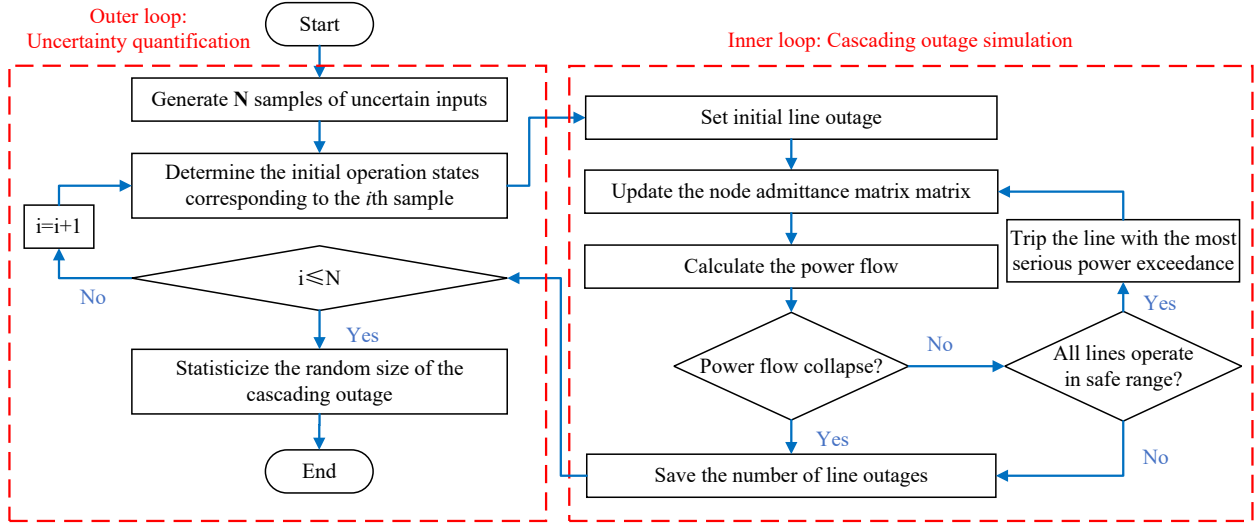


Fig. 1. Flowchart of probabilistic cascading outage simulation.

where $\Lambda_{\Omega}(\cdot)$ denotes an indicator function that defines rare cascading outages. When the number of fault lines exceeds the threshold $\bar{\varepsilon}$, $\Lambda_{\Omega}(\mathbf{x}) = 1$; $\Lambda_{\Omega}(\mathbf{x}) = 0$, otherwise.

Methods for simulating rare events are introduced in the following subsection.

B. Drawbacks of the Traditional MCS

Traditional MCS is a common benchmark for rare-event simulation. The probability of rare cascading outages can be estimated by traditional MCS as

$$\hat{P}_{\Omega}^{\text{MCS}} = \frac{1}{N_{\text{MCS}}} \sum_{i=1}^{N_{\text{MCS}}} \Lambda_{\Omega}(\mathbf{x}^{(i)}), \quad (7)$$

where $\mathbf{x}^{(i)}$, $i = 1, \dots, N_{\text{MCS}}$, represents the Monte-Carlo samples of $\mathbf{x}$ estimating rare events.

Despite its straightforward procedure, the error in probability estimation increases as the event becomes rarer. The coefficient of variation reflects the quality of the estimation, which is given by [17]

$$\delta^2 \left(\hat{P}_{\Omega}^{\text{MCS}} \right) = \frac{D \left[\hat{P}_{\Omega}^{\text{MCS}} \right]}{E \left[\hat{P}_{\Omega}^{\text{MCS}} \right]^2} = \frac{1 - P_{\Omega}}{N_{\text{MCS}} P_{\Omega}}. \quad (8)$$

The formula in (8) shows that MCS becomes computationally impractical when identifying rare events. For example, accurately capturing events with a probability of 10^{-3} typically requires around 10^5 samples to achieve $\delta \left(\hat{P}_{\Omega}^{\text{MCS}} \right) = 0.1$. For simulating cascading outage events with a lower probability, the number of required samples increases rapidly.

C. The Subset Simulation for Effectively Searching Rare Cascading Outage Paths

Compared with traditional MCS, subset simulation is an effective method for obtaining rare-event samples. [8], [18]. The detailed procedure for the subset simulation to search for rare cascading outage events is as follows:

1) *Define intermediate events*: First, we decompose the rare event into a series of nested intermediate events ending with the target rare event

$$\Omega = \cap_{i=1}^L \Omega_i, \quad \Omega_{i+1} \subseteq \Omega_i \quad \forall i = 1, \dots, L-1. \quad (9)$$

The i th intermediate event Ω_i is defined as

$$\Omega_i = \{h(\mathbf{x}) > \bar{\varepsilon}_i\}, \quad (10)$$

where the corresponding thresholds of the intermediate events satisfy $\varepsilon_1 < \varepsilon_2 < \dots < \varepsilon_L = \bar{\varepsilon}$.

With intermediate events, the probability of rare events can be expressed as a product of the conditional probability system as

$$\hat{P}_{\Omega}^{\text{SS}} = \prod_{i=1}^{L-1} \hat{P}(\Omega_{i+1} | \Omega_i), \quad (11)$$

2) *Generate samples of intermediate events with the Modified Metropolis algorithm*: To estimate $\hat{P}_{\Omega}^{\text{SS}}$, samples are drawn from the intermediate events. We adopt the modified Metropolis algorithm [18]. To generate samples in Ω_i , we draw samples with the conditional PDF $p(\mathbf{x} | \Omega_i)$. First, we transform the input variable x_i into a standard independent Gaussian variable ζ_i through the Gaussian copula as $\zeta_i = \Phi_i^{-1}(T_i(x_i))$; $i = 1, \dots, n$, where Φ_i^{-1} is the inverse cumulative distribution function of ζ_i , and T_i is the cumulative distribution function of x_i .

Then, we sample from the standard independent Gaussian space, conditional on the previous event F_{i-1} as [18]

$$\zeta^{(j)} = \rho \zeta^{(j)} + \sqrt{1 - \rho^2} \mathbf{Z}^{(j)}; \quad j = 1, \dots, N_{\text{SS}}, \quad (12)$$

where $\mathbf{Z}^{(j)}$ is the j th sample from the standard Gaussian distribution; $\rho \in (0, 1)$ is the correlation parameter; and $\zeta^{(j)}$ is the j th sample in the last intermediate event. N_{SS} is the number of samples for each intermediate event. We transform standard independent Gaussian variables ζ back into

the physical variables x through the Gaussian copula. To select samples x_d in $\mathcal{F}_i$, x' are accepted or rejected via

$$x_d^{(j)} = \begin{cases} x'^{(j)}, & \text{if } x'^{(j)} \in \mathcal{F}_i \\ x^{(j)}, & \text{if } x'^{(j)} \notin \mathcal{F}_i \end{cases} \quad j = 1, \dots, N_{SS}. \quad (13)$$

3) *Estimate the probability of rare cascading outage events*: The conditional probability between adjacent intermediate events are set as p_0 . ε_i is the p_0 quantile of the samples in Ω_i . Step 2 is repeated until $\varepsilon_i \geq \bar{\varepsilon}$. Then, the probability of rare cascading outage events is expressed as

$$\hat{P}_{\Omega}^{SS} = \frac{p_0^{M-1}}{N_{SS}} \sum_{j=1}^{N_{SS}} \Lambda_{\Omega}(x_{\Omega_{M-1}}^{(j)}), \quad (14)$$

where M is the number of the intermediate events; $x_{\Omega_{M-1}}$ stands for samples generated from Ω_{M-1} through Step 2; $\sum_{i=1}^{N_{SS}} \chi_f(x_{\Omega_{M-1}}^{(j)})/N_{SS}$ is the probability of the rare events sampled in the last intermediate event.

The procedure of the subset simulation is shown in Fig. 2.

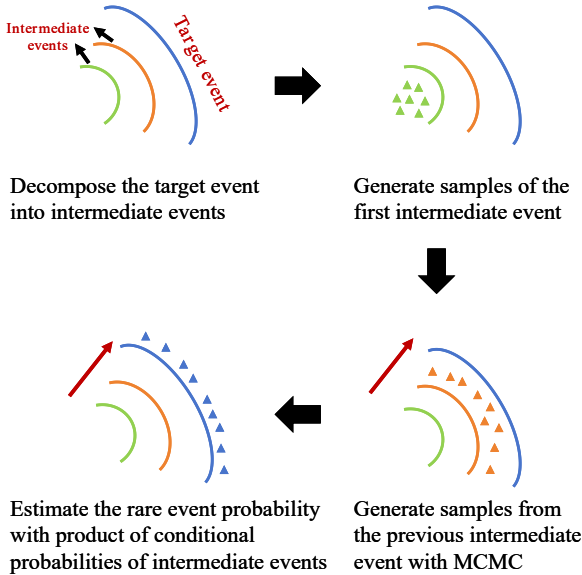


Fig. 2. The procedure of the subset simulation.

Remark 2. Estimating the probability of rare cascading failures is difficult. The subset simulation method initially generates cascading failures on a reduced scale, where probability assessments are more feasible. Subsequently, larger-scale cascading failures are simulated using the modified Metropolis algorithm, which is informed by smaller-scale simulations.

IV. CASE STUDIES

In this section, numerical experiments are conducted on the IEEE 118-bus system with two added wind farms and two added PVs. The capacities of wind farms and PVs are all 50 MW. The wind farm location parameters are 2 and 1.5, respectively. The scale parameters of wind farms are 7.6 and 8.5, respectively. The location parameters of PVs are set to

0.9 and 0.8, respectively. The scale parameters of PVs are 0.9 and 0.8, respectively. The mean value of the loads is equal to the base value, and the standard deviation is equal to 5% of the mean value [5]. The correlation coefficient for uncertain inputs is set to 0.2. Line 11-12 tripped as the initial outage.

A. Probability Distributions of the Size of Cascading Outages

To verify the efficiency and accuracy of the proposed subset simulation, we use the probability distributions of the size of cascading outages obtained by MCS with 10^5 samples as the baseline. The parameters of the subset simulation are as follows: N_{SS} is set to 2,000. The quantile probability p_0 is set to 0.2. The correlation parameter of the Modified Metropolis algorithm ρ is set to 0.8.

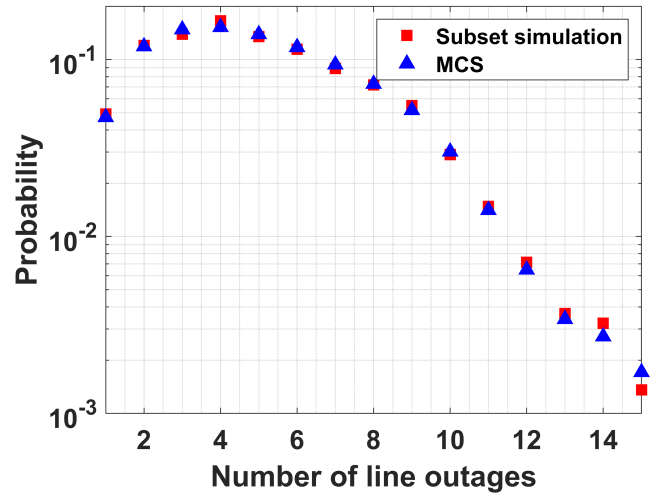


Fig. 3. Comparison of probability distributions of the number of line outages in cascading outage events estimated by MCS and by the subset simulation.

Here, we assume that the rare cascading outages have more than 14 line outages. The subset simulation method requires 4 intermediate events to estimate rare cascading outages. The computation times for the subset simulation and MCS are 103 and 956 seconds, respectively. The result indicates that the subset simulation improves the efficiency of searching for rare cascading outages by a factor greater than 9.

Figure 3 shows the probability distributions of the cascading outage sizes estimated by MCS and by subset simulation. The results show that the size of cascading outages simulated by the subset simulation has statistical features very similar to those simulated by MCS. The probability initially increases with the number of line outages from 1 to 6, suggesting a higher probability of experiencing minor cascading outages. However, as the number of outages continues to rise beyond this point, the probability declines, indicating a lower probability of cascading outages at larger scales. The results indicate that the proposed method can accurately and efficiently estimate the probability of rare cascading outages.

B. The Impacts of the Renewable Generation Capacities on the Cascading Outages

To comprehensively evaluate the impact of renewable generation capacity on the probability of rare cascading outages with more than 14 line outages, we consider 5 capacity levels: 50 MW, 80 MW, 100 MW, 150 MW, and 200 MW. Table I shows the probability of rare cascading outages with different capacity levels. The results indicate that the probability of rare cascading outages is related to renewable generation capacities. The reason is that different levels of renewable generation capacities will change statistical characteristics of the uncertain initial operating state, thereby further altering the probability distribution of sizes of cascading outage events.

TABLE I
THE PROBABILITY OF RARE CASCADING OUTAGES WITH DIFFERENT CAPACITY LEVELS

	50 MW	80 MW	100 MW	150 MW	200 MW
Probability	0.0058	0.045	0.053	0.032	0.014

C. Cascading Outages Simulation with Different Initial Failures Caused by Voltage Issues

Voltage issues cause component failures [19]. We simulate cascading outages triggered by initial tripping due to overvoltage or undervoltage conditions. The results of the probabilistic power flow indicate that the mean values of voltage magnitudes at Bus 76, Bus 53, and Bus 118 are below 0.95 p.u., while the mean values of voltage magnitudes at Bus 25 and Bus 66 are over 1.05 p.u. For each bus, one of the connected lines is selected to be tripped as the initial fault. Then, cascading outages simulations are conducted for each initial fault, namely, T1 to T5. Note that different initial faults correspond to different N-1 contingencies. Since this study focuses on the uncertainties in load and renewable energy, the probabilities are calculated separately for each contingency. Table II shows the probability of rare cascading outages with different initial failures. The results indicate that the probability of cascading outages is related to the locations of initial failures.

TABLE II
THE PROBABILITY OF RARE CASCADING OUTAGES WITH DIFFERENT INITIAL FAILURES CAUSED BY VOLTAGE ISSUES

	T1	T2	T3	T4	T5
Probability	0.0016	0.0011	0.0004	0.0477	0.0021

V. CONCLUSION

This study introduces a novel probabilistic model for cascading outages, aiming to quantify the probability of infrequent cascading outage incidents. To alleviate the computational burden, we develop the subset simulation technique, which decomposes the target events into a sequence of more tractable and smaller-scale cascading outages, making their

probabilities easier to compute. The simulation results confirm the accuracy and efficiency of the subset simulation for estimating the probabilities of rare cascading outages. The results highlight the low-probability but high-impact nature of large-scale cascading events. The result also indicates that the probability of cascading outages is related to the locations of initial failures and the level of renewable penetration.

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