Enhancing Distribution System Resilience by Unlocking Latent Flexibility in Large-Scale Small-Capacity Distributed Energy Resources

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***Abstract*—Escalating climate and cyber risks are triggering unforeseen outages and substantial economic losses across Distribution System (DS). This paper discusses a hierarchical DS restoration framework that fully utilizes the latent flexibility from large-scale small-capacity DERs to achieve a massive improvement in system resilience. To ensure the restoration model remains computationally manageable under extensive DER penetration, DERs are managed in clusters, and a Space Divisional Flexibility Modeling and Aggregation (SDFMA) method is proposed to accurately and efficiently formulate a low-dimensional equivalent flexibility model of each cluster. Case studies demonstrate the effectiveness and superiority of the proposed method.**

***Index Terms*--Distribution system, distributed energy resources, flexibility aggregation, resilience.**

I. Introduction

In recent years, the surge in natural disasters and human-induced attacks has imposed unprecedented challenges on power systems. The Distribution System (DS) is often the most susceptible segment due to its comparatively lower robustness and response capacity.

As the DS is fundamental to social and economic activities of modern society, many studies have been conducted to reduce power outage loss and accelerate system recovery rate under extreme events. Reference [1] considers the coupling effect between the DS and natural gas system to minimize the total load loss cost during system failures. Reference [2] treats mobile energy storage as an efficient backup resource for enhancing the resilience of DS. Reference [3] centers on hydrogen-facilitated resilience enhancement in DS, exploiting power–hydrogen coupling for post-disaster restoration. Reference [4] considers renewable distributed generations in the restoration of DS. Existing research primarily concentrates on leveraging large-capacity resources to enhance DS resilience. However, beyond these assets, DS hosts numerous small-capacity large-scale Distributed Energy Resources (DERs) whose potential remains insufficiently explored. Owing to their large scale, the aggregated flexibility of these small-capacity devices can materially reduce power outage loss and shorten restoration time.

However, tremendous amount of heterogeneous DERs can bring difficulties for system control and energy management due to the growing dimensionality of DS's restoration model. A promising solution is to cluster the DERs into a limited number of groups and formulate an equivalent model to represent the aggregated flexibility of each cluster. There have been a lot of studies on DERs' flexibility aggregation, which can be categorized as: 1) Operation Model Projection (OMP) method [5]; 2) Geometric Prototype Approximation (GPA) method [6]-[8]; 3) Geometric Adaptive Approximation (GAA) method [9]-[10]. The OMP method directly calculates the projection of the original operation model to obtain the aggregated flexibility model. As the operation model rises in complexity, the efficiency of the OMP method is limited. In order to improve modeling efficiency, GPA methods adopt geometric prototypes, such as inscribed hyperbox [6], ellipse [7] and zonotope [8], to model the flexibility of each DER in the cluster. The aggregated flexibility model is then obtained by calculating Minkowski Sum of the flexibility models. Formulating flexibility models as homothets of pre-defined prototypes may be of low accuracy. To improve modeling accuracy, GAA methods are proposed to represent DER's flexibility with shape-adaptive polytopes. Reference [9] presents DER's flexibility model with a set of hyperplanes. However, the disaggregation feasibility of the hyperplane-presented flexibility model can't be guaranteed. To guarantee disaggregation feasibility, [10] formulates ESS's flexibility model as a polytope with extreme vertices derived from the original operation region. Though the extreme-vertices-based flexibility aggregation method has been proven state-of-the-art in terms of efficiency, its accuracy still needs to be improved when applied to heterogeneous DERs other than Energy Storage Systems (ESSs).

To fill in the existing research gaps identified above, this paper discusses a hierarchical DS restoration framework that fully utilizes the latent flexibility in large-scale small-capacity

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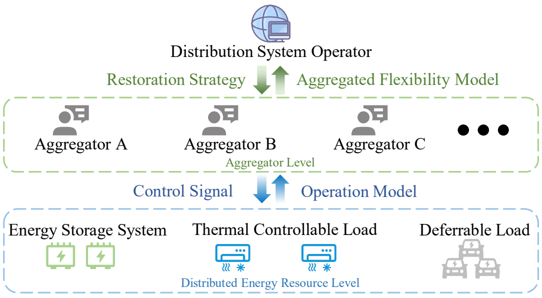

Figure 1. Schematic illustration of the hierarchical distribution system restoration framework.

DERs. In order to accurately and efficiently model and aggregate DERs' flexibility, a novel Space Divisional Flexibility Modeling and Aggregation (SDFMA) method is proposed to tackle the problems of state-of-the-art approaches. The proposed method formulates the aggregated flexibility model as a polytope with extreme vertices derived from the original operation region to guarantee disaggregation feasibility and modeling efficiency. As a result, the accuracy of the aggregated flexibility model typically increases with the number of extreme vertices. In order to improve modeling accuracy, a novel space divisional technique is proposed to increase the number of distinct extreme vertices.

The rest of the paper is organized as follows. The hierarchical DS restoration framework is introduced in Section II. The proposed SDFMA is presented in Section III. Then the proposed method is evaluated on a IEEE 37-bus test system and a real-world distribution system. Numerical results on solution optimality and computational efficiency are analyzed in Section IV. Finally, Section V concludes the paper.

II. Hierarchical Distribution System Restoration Framework

In this section, a hierarchical DS restoration framework is proposed to reduce power outage loss with the support of large-scale small-capacity DERs as shown in Fig. 1. We define the hierarchical restoration framework to have three levels, namely, DER level, aggregator level, and DS operator level.

A. Aggregated Flexibility Model of Distributed Energy Resources

DS incorporates a variety of DERs, which can provide significant flexibility to support power balance and regulation services. Due to the large number of small-capacity DERs, the restoration model can become prohibitively complex. To address this, DERs are managed in clusters. Each cluster's flexibility is represented by a reduced-order equivalent model as in (1)-(3). The operation model of each DER is formulated in (1). Equation (2) demonstrates the flexibility aggregation process. The aggregated flexibility model is shown in (3). The formulation of the aggregated flexibility model is discussed in detail in Section III.

$$\boldsymbol{p}_k = \Delta\boldsymbol{p}_k + \sum_{r=1}^{N_R}\sum_{j=1}^{T}\lambda_j^r \boldsymbol{v}_{k,j}^r \mid \sum_{r=1}^{N_R}\sum_{j=1}^{T}\lambda_j^r = 1 \\ ,\lambda_j^r \in \mathbb{R}^+, \forall k \in \mathcal{N}^{DER} \tag{1}$$

$$\boldsymbol{v}_j^{A,r} = \sum_{k\in\mathcal{N}^{DER}} \boldsymbol{v}_{k,j}^r,\ j=1,2,\dots,T;\ r=1,2,\dots,N_R \tag{2}$$

$$\boldsymbol{p}^A = \sum_{k\in\mathcal{N}^{DER}} \Delta\boldsymbol{p}_k + \sum_{r=1}^{N_R}\sum_{j=1}^{T}\lambda_j^r \boldsymbol{v}_j^{A,r} \mid \sum_{r=1}^{N_R}\sum_{j=1}^{T}\lambda_j^r = 1,\ \lambda_j^r \in \mathbb{R}^+ \tag{3}$$

where $\boldsymbol{p}_k$ / $\boldsymbol{p}^A$ indicate power consumption of each DER / DER cluster; $\Delta\boldsymbol{p}_k$ is a constant auxiliary vector; $\boldsymbol{v}_{k,j}^r$, $\boldsymbol{v}_j^{A,r}$ represent constant auxiliary vertices; N_R is the number of power consumption subspaces; T is the number of time slices; $\mathcal{N}^{DER}$ denotes the set of DERs.

The aggregated flexibility model of each cluster is formulated with a constant auxiliary vector and a positive combination of constant auxiliary vertices, whose complexity is unrelated to the number of DERs.

B. Restoration Model of Distribution System

The DS restoration model is holistically formulated considering aggregated flexibility of DERs and repair crew dispatching based on the intuition of minimum electricity shedding during power outage as in (4). By leveraging the aggregated flexibility of large-scale small-capacity DERs, DS can access additional local flexibility to offer power support during extreme events. As a result, service recovery is sped up, unserved energy is reduced, and outage losses are ultimately mitigated.

$$\min \sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{N}} P_{i,t}^S \Delta t \tag{4}$$

s.t. a) DERs' aggregated flexibility constraints (3);
b) distribution network constraints (5)-(15);
c) repair crew dispatching constraints.

where $P_{i,t}^S$ represents the load shedding at node i at time t.

1) Operation Model of Distribution System

The operation constraint of DS is described with the linearized DistFlow model as in (5)-(15).

$$\sum_{j:(i,j)\in\mathcal{L}} P_{ij,t} = P_{i,t}^G - (P_{i,t}^L + P_{i,t}^A - P_{i,t}^S),\quad \forall i\in\mathcal{N}, \forall t\in\mathcal{T} \tag{5}$$

$$\sum_{j:(i,j)\in\mathcal{L}} Q_{ij,t} = Q_{i,t}^G - (Q_{i,t}^L - Q_{i,t}^S),\quad \forall i\in\mathcal{N}, \forall t\in\mathcal{T} \tag{6}$$

$$-(1-\alpha_{ij,t})M \le U_{i,t} - U_{j,t} - (r_{ij}P_{ij,t} + x_{ij}Q_{ij,t}) \\ \le (1-\alpha_{ij,t})M,\quad \forall i\in\mathcal{N}, \forall t\in\mathcal{T} \tag{7}$$

$$0 \le P_{ij,t} \le \alpha_{ij,t} S_{ij}^{\max},\quad \forall (i,j)\in\mathcal{L}, \forall t\in\mathcal{T} \tag{8}$$

$$0 \le Q_{ij,t} \le \alpha_{ij,t} S_{ij}^{\max},\quad \forall (i,j)\in\mathcal{L}, \forall t\in\mathcal{T} \tag{9}$$

$$0 \le P_{i,t}^S \le P_{i,t}^L,\quad \forall i\in\mathcal{N}, \forall t\in\mathcal{T} \tag{10}$$

$$0 \le Q_{i,t}^S \le Q_{i,t}^L,\quad \forall i\in\mathcal{N}, \forall t\in\mathcal{T} \tag{11}$$

$$U_i^{\min} \le U_{i,t} \le U_i^{\max},\quad \forall i\in\mathcal{N}, \forall t\in\mathcal{T} \tag{12}$$

$$P_{i,t}^{G,\min} \le P_{i,t}^G \le P_{i,t}^{G,\max},\quad \forall i\in\mathcal{S}, \forall t\in\mathcal{T} \tag{13}$$

$$Q_{i,t}^{G,\min} \le Q_{i,t}^G \le Q_{i,t}^{G,\max},\quad \forall i\in\mathcal{S}, \forall t\in\mathcal{T} \tag{14}$$

$$P_{i,t}^G = Q_{i,t}^G = 0,\quad \forall i\in\mathcal{N}\setminus\mathcal{S}, \forall t\in\mathcal{T} \tag{15}$$

where $P_{ij,t}$ and $Q_{ij,t}$ are the active and reactive power flow of line (i,j) at time t; $P_{i,t}^G$ and $Q_{i,t}^G$ indicate the active and

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reactive output of power source at node i at time t; $P_{i,t}^{L}$ and $Q_{i,t}^{L}$ represent the active and reactive power consumption of un-controllable load at node i at time t; $P_{i,t}^{A}$ is the active power consumption of DERs at node i at time t; $P_{i,t}^{S}$ and $Q_{i,t}^{S}$ represent the active and reactive load shedding at node i at time t; $\alpha_{ij,t}$ is a binary variable indicating the on-off status of line (i,j) at time t; $U_{i,t}$ represent the voltage of node i at time t; r_{ij} and x_{ij} are the resistance and reactance of line (i,j); M is a large enough positive number; $S_{ij}^{\max}$ indicate the maximum capacity of line (i,j); $U_{i}^{\max}$ and $U_{i}^{\min}$ are the maximum and minimum voltage limit of node i; $P_{i,t}^{G,\max}$ / $P_{i,t}^{G,\min}$ and $Q_{i,t}^{G,\max}$ / $Q_{i,t}^{G,\min}$ are the maximum/ minimum active and reactive power output of power source at node i; $\mathcal{N}, \mathcal{L}, \mathcal{S}$ denote the set of nodes, lines, and power sources.

2) Dispatching Model of Repair Crew

Repair crews are rapidly dispatched to the faulted components for fault clearing. The repair crew dispatch model in [11] is adopted to develop a reasonable and efficient crew dispatching strategy for DS restoration.

C. Disaggregation Model of Aggregator

In compliance with DS operational requirements, the aggregators disaggregate the restoration strategy into implementable DER control signals by allocating the aggregate power profile among their affiliated DERs. This paper adopts the disaggregation method proposed in [10] to formulate an operation strategy for each DER as shown in (16)-(17). Equation (16) is the optimal power consumption of the aggregator from the DS restoration strategy. The power consumption of each DER is then formulated in (17).

$$\boldsymbol{p}^{real,A} = \sum_{k\in\mathcal{N}^{DER}} \Delta\boldsymbol{p}_k + \sum_{r=1}^{N_R}\sum_{j=1}^{T} \alpha_j^r \boldsymbol{v}_j^{A,r} \tag{16}$$

$$\boldsymbol{p}_k^{real} = \Delta\boldsymbol{p}_k + \sum_{r=1}^{N_R}\sum_{j=1}^{T} \alpha_j^r \boldsymbol{v}_{k,j}^r, \forall k\in\mathcal{N}^{DER} \tag{17}$$

where $\boldsymbol{p}_k^{real}$ / $\boldsymbol{p}^{real,A}$ are the optimal power consumption of each DER / DER cluster; α_j^r is a constant coefficient corresponding to the optimal power consumption.

III. Space Divisional Flexibility Modeling and Aggregation

In this section, a SDFMA method is proposed to model the aggregated flexibility of large-scale small-capacity DERs.

A. Universal Operation Model of Distributed Energy Resources

For simplicity, a universal model is formulated to describe the operation constraints of DERs in matrix form as shown in (18). The universal operation model is made up of power boundaries and cumulative energy boundaries. The power boundaries indicate DERs' temporal decoupling constraints. The temporal coupling constraints are described with the cumulative energy boundaries.

$$\Omega_k^O = \left\{ \boldsymbol{p}_k \in \mathbb{R}^T \left| \begin{array}{c} \boldsymbol{p}_k^{\min} \le \boldsymbol{p} \le \boldsymbol{p}_k^{\max} \\ \mathbf{A}_k \boldsymbol{p}_k \le \boldsymbol{b}_k \end{array} \right. \right\}, \forall k\in\mathcal{N}^{DER} \tag{18}$$

where Ω_k^O and $\boldsymbol{p}_k$ denote the operation region and power consumption vector of the k th DER; $\boldsymbol{p}_k^{\max}$ / $\boldsymbol{p}_k^{\min}$ are the upper / lower limit of the power boundaries; $\mathbf{A}_k$ and $\boldsymbol{b}_k$ indicate the parameters of the cumulative energy boundaries.

B. Flexibility Modeling and Aggregation

In this section, a SDFMA method is proposed to model and aggregate large-scale small-capacity DERs' flexibility based on the universal operation model. Firstly, DER's power consumption space is divided into multiple independent subspaces to improve modeling accuracy. Then, the flexibility model of each DER is described as a convex polytope with extreme vertices derived from the original operation region in each subspace. Lastly, the aggregate flexibility model is obtained by adding up extreme vertices along the same direction in each subspace.

1) Space Divisional Technique

DER's power consumption space is divided into 2^T independent subspaces with base vectors as in (19)-(22). The positive and negative coordinate axis vectors are adopted as base vectors to formulate the subspace generation matrix $\mathbf{G}$ as in (19) and (20). Each power consumption subspace is formulated by T base vectors extracted from $\mathbf{G}$ as in (21) and (22).

$$\left[\boldsymbol{g}_1, \boldsymbol{g}_2, \ldots, \boldsymbol{g}_T\right] = \mathbf{I}^{T\times T}, \quad \left[\boldsymbol{g}_{T+1}, \boldsymbol{g}_{T+2}, \ldots, \boldsymbol{g}_{2T}\right] = -\mathbf{I}^{T\times T} \tag{19}$$

$$\mathbf{G} = \left[\boldsymbol{g}_1, \boldsymbol{g}_2, \ldots, \boldsymbol{g}_T, \boldsymbol{g}_{T+1}, \boldsymbol{g}_{T+2}, \ldots, \boldsymbol{g}_{2T}\right] \in \mathbb{R}^{T\times 2T} \tag{20}$$

$$\left[\boldsymbol{g}_1^r, \boldsymbol{g}_2^r, \ldots, \boldsymbol{g}_T^r\right] = \mathbf{G}_{[:,\mathcal{J}]}, \quad \mathcal{J} \subseteq \{1,2,\ldots,2T\}, |\mathcal{J}| = T \tag{21}$$

$$\Omega_r^{\text{sub}} = \left\{ \boldsymbol{p}\in\mathbb{R}^T : \boldsymbol{p} = \sum_{j=1}^{T} \lambda_j \boldsymbol{g}_j^r \mid \lambda_j \in \mathbb{R}^+ \right\}, r = 1,2,\cdots,2^T \tag{22}$$

where $\mathbf{G}$ is the subspace generation matrix; $\boldsymbol{g}_i^r$ is the i th base vector of the r th subspace; Ω_r^{sub} denotes the r th power consumption subspace.

2) Space Divisional Flexibility Modeling of Individual DER

The flexibility model of each DER is formulated as a convex hull with extreme vertices derived from the operation region Ω_k^O as in (23)-(26). Equation (23) constructs an auxiliary operation region based on the original operation region to make the operation region evenly distributed among the subspaces to improve modeling accuracy and avoid infeasibility of the extreme vertices optimization problems as in (24). An auxiliary flexibility model is formulated with the obtained extreme vertices as shown in (25). Equation (26) obtains the flexibility model of the k th DER.

$$\tilde{\Omega}_k^O = \left\{ \tilde{\boldsymbol{p}}_k \in \mathbb{R}^T : \tilde{\boldsymbol{p}}_k = \boldsymbol{p}_k - \frac{\boldsymbol{p}_k^{\min} + \boldsymbol{p}_k^{\max}}{2} \mid \boldsymbol{p}_k \in \Omega_k^O \right\} \tag{23}$$

$$\boldsymbol{v}_{k,j}^r = \arg\left(\max_{\boldsymbol{v}_k^r \in \tilde{\Omega}_k^O \cap \Omega_r^{\text{sub}}} \left\langle \boldsymbol{g}_j^r + \varepsilon \sum_{i=1, i\ne j}^{T} \boldsymbol{g}_i^r, \boldsymbol{v}_k^r \right\rangle \right), \forall \boldsymbol{g}_j^r \in \Omega_r^{\text{sub}} \tag{24}$$

$$\tilde{\Omega}_k^F = \left\{ \tilde{\boldsymbol{p}}_k \in \mathbb{R}^T : \tilde{\boldsymbol{p}}_k = \sum_{r=1}^{N_R}\sum_{j=1}^{T} \lambda_j^r \boldsymbol{v}_{k,j}^r \mid \sum_{r=1}^{N_R}\sum_{j=1}^{T} \lambda_j^r = 1, \lambda_j^r \in \mathbb{R}^+ \right\} \tag{25}$$

$$\Omega_k^F = \left\{ \boldsymbol{p}_k \in \mathbb{R}^T : \boldsymbol{p}_k = \tilde{\boldsymbol{p}}_k + \frac{\boldsymbol{p}_k^{\min} + \boldsymbol{p}_k^{\max}}{2} \mid \tilde{\boldsymbol{p}}_k \in \tilde{\Omega}_k^F \right\} \tag{26}$$

where $\tilde{\Omega}_k^O$ and $\tilde{\boldsymbol{p}}_k$ denote the auxiliary operation region and auxiliary power consumption vector of the k th DER; $\boldsymbol{v}_{k,j}^r$ represents the extreme vertex of the k th DER corresponding to the jth base vector in the rth subspace; $\tilde{\Omega}_k^F$ and Ω_k^F are the auxiliary flexibility region and flexibility region of the kth DER.

3) Space Divisional Flexibility Aggregation

The aggregated flexibility model of multiple DERs is calculated with the auxiliary flexibility model as in (27)-(29). The extreme vertices of the auxiliary aggregated flexibility region are calculated by directly adding up vertices of individual DER as in (27). Equation (28) constructs the auxiliary aggregated flexibility model with the obtained extreme vertices. The aggregate flexibility model is formulated as in (29).

$$\boldsymbol{v}_j^{A,r} = \sum_{k\in\mathcal{N}^{DER}} \boldsymbol{v}_{k,j}^r,\ j=1,2,\ldots,T;\ r=1,2,\ldots,N_R \tag{27}$$

$$\tilde{\Omega}^A = \left\{ \tilde{\boldsymbol{p}}^A \in \mathbb{R}^T : \tilde{\boldsymbol{p}}^A = \sum_{r=1}^{N_R}\sum_{j=1}^{T} \lambda_j^r \boldsymbol{v}_j^{A,r} \mid \sum_{r=1}^{N_R}\sum_{j=1}^{T} \lambda_j^r = 1,\ \lambda_j^r \in \mathbb{R}^+ \right\} \tag{28}$$

$$\Omega^A = \left\{ \boldsymbol{p}^A \in \mathbb{R}^T : \boldsymbol{p}^A = \tilde{\boldsymbol{p}}^A + \sum_{k\in\mathcal{N}^{DER}} \frac{\boldsymbol{p}_k^{\min} + \boldsymbol{p}_k^{\max}}{2} \mid \tilde{\boldsymbol{p}}^A \in \tilde{\Omega}^A \right\} \tag{29}$$

where $\boldsymbol{v}_j^{A,r}$ represents the extreme vertex of the aggregated DERs corresponding to the jth base vector in the rth subspace; $\tilde{\Omega}^A$ and $\tilde{\boldsymbol{p}}^A$ denote the auxiliary flexibility region and auxiliary power consumption vector of the aggregated DERs; Ω^A and $\boldsymbol{p}^A$ denote the flexibility region and power consumption vector of the aggregated DERs.

As shown in (25) and (28), the accuracy of the flexibility model is positively correlated to the number of distinct extreme vertices. The calculation process of the extreme vertices in (24) indicates that the vertices with the largest projection onto the direction $\boldsymbol{g}_j^r + \varepsilon\sum_{i=1,i\neq j}^{T} \boldsymbol{g}_i^r$ are selected as extreme vertices, which are then used to construct the flexibility model. It occurs when the same vertex is selected for multiple directions because it has the largest projection in these directions, resulting in fewer distinct extreme vertices. By decomposing the power consumption space into independent subspaces and computing the extreme vertices in each subspace separately, the total number of extreme vertices can be significantly increased, thereby improving the accuracy of the flexibility model.

IV. Case Study

Three case studies are conducted to demonstrate the effectiveness and necessity of the proposed method. All DERs' parameters are randomly generated based on the probability distribution in [12]. It is assumed all faults can be cleared within 12 hours, and T is set to be 12 with one hour interval. All computational tasks are conducted via Python with the package *gurobi.py* and executed on a 64-bit PC with an *Intel(R) Core (TM) i7-10700F* CPU and 32 GB RAM.

A. Flexibility Aggregation Simulation

To demonstrate the superiority of the proposed SDFMA method over existing flexibility aggregation methods in terms of computational efficiency and modeling accuracy, this paper conducts flexibility modeling and aggregation simulations and compares SDFMA with three state-of-the-art methods, namely, OMP [5], GPA [8], extreme-vertices-based flexibility aggregation [10].

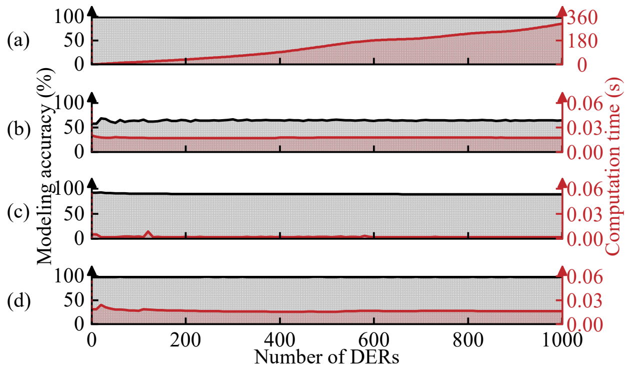

Figure 2. Flexibility aggregation simulation results. (a) Operation model projection [5]. (b) Geometric prototype approximation [8]. (c) Extreme-vertices-based flexibility aggregation [10]. (d) The proposed method.

The efficiency of the flexibility aggregated methods is measured by the computation time, while the modeling accuracy is measured by the Relative Volume (RV) between the flexibility region and the original operation region. The detailed RV calculation process is given in [9]. Simulation results are shown in Fig. 2. The OMP consistently achieves a modeling accuracy exceeding 99%. However, due to the increasing dimension of the operation model, its computation time grows rapidly with DER scale, limiting its scalability. By formulating each DER's flexibility model in parallel and aggregating them via the minkowski sum, the remaining methods can keep the computation time within one second regardless of the number of DERs. Accuracy of the GPA is just about 65%, because it assumes that each DER's flexibility region can be represented as a homothet of the given prototype, an assumption that does not hold in practice. By constructing DERs' flexibility models as shape-adaptive polytopes, the extreme-vertices-based flexibility aggregation method increases the modeling accuracy to around 90%. However, its accuracy improvement is ultimately limited by the number of unique extreme vertices retained in the aggregation. With the proposed SDFMA method, the modeling accuracy reaches approximately 99% while keeping the computation time within one second, indicating its effectiveness and superiority among state-of-the-art flexibility aggregation methods.

B. 37-Bus Test System Restoration Simulation

The modified IEEE 37-bus test system is illustrated in Fig. 3. Three DER Clusters (DCs) are respectively attached to node 6, 27, and 31, as shown in TABLE I. Following an extreme event, four transmission lines—F1, F2, F3, and F4—are assumed to be out of service, with repair time of 50, 65, 35, and 20 minutes, respectively. One repair crew is assigned for the restoration of the system. In order to illustrate the impact of DCs' flexibility, two cases are designed as follows:

- Case I: The flexibility of DCs is not considered in the restoration strategy.
- Case II: The aggregated flexibility model of DCs is incorporated into the restoration strategy.

Simulation results are shown in TABLE II. It can be seen that the flexibility of DCs greatly affects the repair crew sequence and power outage loss. In case II, F3 is repaired with priority because DC3 can serve as an emergency energy supply to local loads. F4 is repaired last, since the loads at nodes

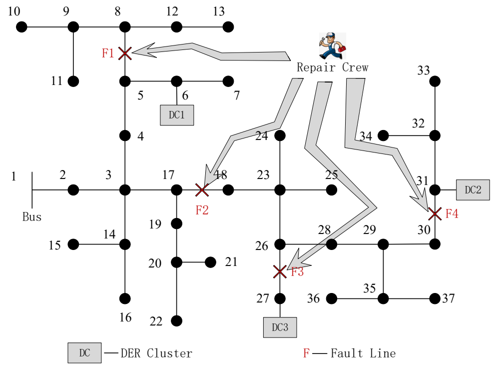

Figure 3. Structure of the 37-bus test system.

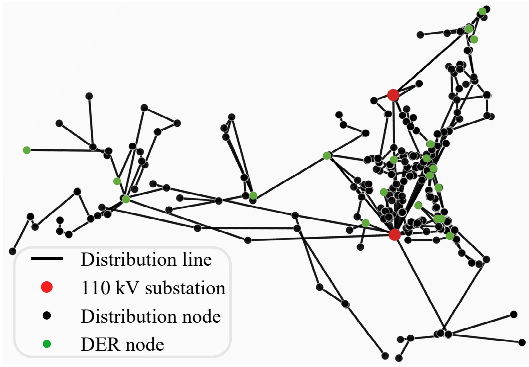

Figure 4. Visualization of the real-world distribution system.

TABLE I THE NUMBER OF DER IN EACH CLUSTER

	DC1	DC2	DC3
Number of DERs	50	500	5000

TABLE II 37-BUS SYSTEM RESTORATION SIMULATION RESULTS

	Case I	Case II
Repair Crew Sequence	F2 → F1 → F4 → F3	F3 → F1 → F2 → F4
Energy not supplied (MWh)	7.77	3.87

31–34 can be temporarily supported by DC2. As a result, the proposed hierarchical DS restoration framework reduces the total power outage loss by approximately 50% by leveraging the aggregated flexibility of large-scale, small-capacity DERs.

C. Real-World Distribution System Restoration Simulation

A real-world distribution system in Northeast China is used for simulation studies. The system consists of two 110kV substations and 400 distribution nodes, with a reference voltage level of 10kV. The studied distribution system is shown in Fig. 4. To investigate the contribution of the large-scale small-capacity DERs in improving distribution system resilience, 21 load groups with flexibility potential (e.g., residential communities, commercial buildings, and EV charging stations) are selected as DCs, which are located at the green-highlighted nodes.

Ten extreme event outage scenarios that occurred between January 1, 2020, and January 1, 2026, are used for restoration simulations. In practical restoration, the flexibility of the DCs was not fully exploited. To demonstrate the advantage of leveraging flexibility of the large-scale small-capacity DERs, restoration simulations incorporating DER clusters are conducted under the previous outage scenarios. Simulation results are shown in TABLE III. Compared with the practical restoration performance observed in the past extreme events, the proposed hierarchical DS restoration framework reduces the maximum load shedding by about 30% and the average energy not supplied by about 26%.

V. CONCLUSION

In this paper, the flexibility of large-scale small-capacity DERs is fully exploited to reduce power outage loss and enhance the resilience of the distribution system. To tackle the problem of model complexity, a SDFMA is proposed to effectively formulate an equivalent flexibility model of the DER

TABLE III REAL-WORLD SYSTEM RESTORATION SIMULATION RESULTS

	Case I	Case II
Maximum load shedding (MW)	16.12	11.22
Average energy not supplied (MWh)	95.23	70.65

cluster. In future work, the large-scale small-capacity DERs will be considered in the planning of the distribution system.

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