Enhanced Demand-Side Flexibility Through Super-Twisting Sliding Mode Control of Linear Oscillatory Machine Compressors

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***Abstract*— The widespread electrification of loads and the integration of variable renewable resources necessitate unprecedented levels of demand-side flexibility. linear oscillatory machine (LOM) compressors, used in refrigeration and air conditioning, represent a massive, untapped reservoir of grid-responsive load due to their inherent thermal storage capacity. However, unlocking this potential requires precise, robust, and responsive piston stroke control to enable safe and reliable load modulation. This paper proposes a robust piston stroke control strategy for LOMs based on super-twisting sliding mode control (STSMC) to address this need. While proportional-integral (PI) control is commonly used for its simplicity, it suffers from slow dynamics and steady-state error, making it unsuitable for high-performance grid services. The proposed STSMC technique enables a 35% faster dynamic response, eliminates overshoot, and reduces steady-state error by 2%, thereby ensuring precise stroke regulation. Furthermore, STSMC operation results in a lower input current amplitude, enhancing system efficiency. The stability of the closed-loop system is formally proven using Lyapunov theory. By providing faster and more accurate control, the STSMC transforms the LOM compressor from a passive load into a highly responsive asset, facilitating its integration into demand response and frequency regulation programs for a more resilient and decarbonized power system.**

***Index Terms*— Linear oscillatory machines (LOM), proportional and integral control (PI), sliding mode control (SMC), Lyapunov stability function**.

I. Introduction

Traditional compressors are frequently motorized by rotary motors, which change rotary motion into linear motion through the crankshaft mechanism. These compressors have the drawbacks of low efficiency, high mechanical loss, and an un adjustable piston stroke because of their limited mechanical construction. In the past several years, a lot of researchers have focused their attention on linear compressors as a way to increase the efficiency of traditional compressors [1], [2], and [3]. The linear oscillatory machine (LOM) directly converts electrical energy into linear mechanical motion, eliminating the need for a crankshaft mechanism. Compared to traditional rotary compressors, the LOM offers several benefits, such as a simpler design, reduced noise, and precise piston stroke control [4-6]. Since the LOM piston stroke is unconstrained, adjusting the piston stroke directly will vary the gas emissions of the piston [7]– [10]. Nevertheless, the piston will strike the cylinder head if the piston stroke goes above the rated limit. Therefore, obtaining precise piston stroke control is important for the safe operation of the compressor system.

Beyond the compressor unit itself, the aggregation of millions of residential and commercial refrigerators represents a significant opportunity for demand-side management (DSM) in the modern power grid [11]. The inherent thermal storage in refrigeration systems allows for temporary load shifting without compromising their primary function. To participate in grid services like demand response and frequency regulation, these appliances must respond rapidly and reliably to external control signals. This requires a compressor motor drive that can adjust its stroke—and thus its power consumption—precisely and without the overshoot or slow settling times that characterize conventional PI controllers [12].

Generally, a PI regulator is implemented in the piston stroke control system. While simple to design, the PI controller has disadvantages such as slow response, steady-state fluctuations, overshoot, and inadequacy against load disturbances [13]. These limitations make it unsuitable for providing the high-quality grid services needed to support a power system with a high penetration of renewables. Recently, several robust control methods have been realized to enhance the dynamic control operation of electric machines and drive systems, for example adaptive control [14], back-stepping technique [15], model predictive control (MPC) [16], intelligent method [17], sliding mode control (SMC) [18]– [20], and so on. By comparing the aforementioned control techniques, the SMC is more suitable because of its simple design structure, ease of implementation, low computational burden, quick dynamic response, no overshoot, and lesser steady-state error under the whole dynamic process of the electric machines and drive systems. The SMC method is getting more attention from scholars because of its superior performance compared to the conventional PI controller, and it is widely implemented in electric machines and drive systems.

However, the LOM drive system has slowed dynamic response under the PI regulator, which is already implemented on the LOM drive system [21]– [23]. Hence, to resolve the

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issues of the conventional PI controller, the super-twisting sliding mode control (STSMC) has been designed and adapted for the piston stroke regulation of the LOM drive system. The STSMC can achieve faster convergence with no overshoot than that of the PI controller. Furthermore, a piston stroke control method based on STSMC has a smaller input current and a smaller total hormonic distortion (THD) value than the PI control method. This study presents a groundbreaking method that, to our knowledge, has not been employed in previous research to control the piston stroke of the LOM under any SMC methods. Hence, in this paper, the STSMC is designed and used for the piston stroke regulation of the LOM drive systems.

The paper is structured as follows: Section II presents the mathematical modeling of the LOM systems. Section III details the design and stability analysis of the STSMC. Sections IV and V provide simulation, respectively. Lastly, Section VI concludes this paper.

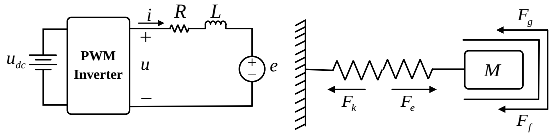

Fig.1(a) Fig.1(b)

Fig.1. The LOM system. (a) Equivalent circuit. (b) Kinetics model

II. Dynamic Modeling The LOM Drive Systems

The LOM drive system is designed and examined in this paper. The electrical and mechanical dynamic equations can be obtained by using the electrical circuit and kinetic model of the LOM as displayed in Fig. 1 (a) and (b), individually. In Fig. 1 F_e, F_k, F_g and F_f are the electromagnetic thrust, spring force, equivalent gas force, and the equivalent frictional force, respectively. Fig. 1(a) illustrates the electrical dynamic system of the LOM, which can be defined as

$$u = Ri + L\rho i + \mu i \tag{1}$$

$$m\rho\upsilon + l_m \rho x + \delta_m x + F_g = \mu i \tag{2}$$

where ρ represents the derivative operator, u is the voltage, i the motor current, R the motor stator resistance, L the motor stator winding resistance, and μ the electromagnetic thrust constant, separately. The piston speed, stroke displacement, the mass of the piston, the spring coefficient, mechanical damping coefficient and nonlinear gas force are denoted separately as υ, x, m, δ_m l_m, and F_g.

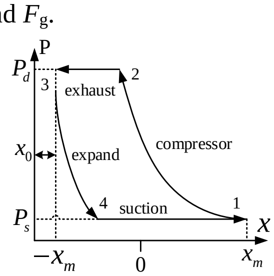

Fig. 2. Diagram illustrating the compressor's working cycle

The nonlinear gas force F_g can be linearized for the system characteristics [8], which can be illustrated as

$$F_g = l_g x + \delta_g \upsilon \tag{3}$$

where l_g and δ_g are represented as the corresponding gas spring coefficient and gas damping coefficient, respectively. These equivalent gas coefficients can nonlinearly change when discharge load of the compressor changes. The thermodynamic theoretical study of the gas compressor is displayed in Fig. 2, where, P_d is the discharge pressure, P_s is the suction pressure, n is the variable process index and x_m is the piston stroke. The gas equivalents (l_g and δ_g) coefficients can be described as

$$l_g = \pi D_p^2 (P_d - P_s) / 4x_m \tag{4}$$

$$\delta g = \left\{ \begin{array}{l} (n/n-1)P_s[2x_m + x_0 - (P_d/P_s)^{1/n} x_0] \\ [((P_d/P_s)^{n-1/n} - 1]D_p^2 \end{array} \right\} / 4\omega_0 x_m^2 \tag{5}$$

where the diameter of the piston and length of the clearance are represented in D_p and x_0, respectively. Therefore, the whole system is considered as linear when the piston stroke remains stable. The stable piston stroke can be achieved when l_g and n_g remain constant. Hence, the mechanical dynamics of the LOM is given as

$$m\rho\upsilon + (l_m + l_g)\rho x + (\delta_m + \delta_g)x = \mu i \tag{6}$$

$$m\rho\upsilon + l\rho x + \delta x = \mu i \tag{7}$$

where l and δ are equivalents spring and damping coefficient of the LOM drive system.

III. Piston Stroke Control System Based on STSMC for The Lom

The piston stroke controller based on the PI and super-twisting sliding mode control (STSMC) can be designed for the LOM drive system in the following sub-sections:

A. Design of Piston Stroke Controller Based on PI for LOM Drive system

The linear piston stroke of the LOM is regulated by PI regulator. The reference piston stroke is described as X_P^* and an actual piston stroke from the sensor is defined as X_P, which is expressed as

$$X_P(t) = \max\left(|X(\gamma)| : \gamma \in (t-T, t)\right) \tag{14}$$

where T=1/f represents the duration of one fundamental cycle. The error $e_x(t)$ of the reference and actual piston stroke can be written as

$$e_x(t) = X_P^*(t) - X_P(t) \tag{15}$$

The duty ratio (λ) of the LOM drive system can be obtained by regulating the piston stroke error ($e_x(t)$) through PI controller, which can be demonstrated as

$$\lambda = k_p e_x(t) + k_I \int e_x(t)\, dt \tag{16}$$

where the k_p and k_I are the proportional and integral constant gains values of PI regulator. Fig. 3 presents the block diagram of the PI-regulated piston stroke controller for the LOM drive system.

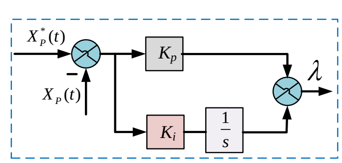

Fig. 3. Basic PI control scheme for piston stroke regulation.

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B. Design of Piston Stroke Controller Based on STSMC for LOM Drive system

The STSMC-based piston stroke control system utilizes the LOM's stroke error as the primary control input. The resulting closed-loop dynamics can be expressed in state-space form as follows:

$$x = \begin{cases} [\varepsilon_x(t)] = [X_P^*(t) - X_p(t)] \\ u = [\lambda] \end{cases} \tag{17}$$

where $\varepsilon_x(t)$ is the state space variable of the piston stroke. Meanwhile, u is the output control variable and λ is the duty ratio of the LOM. In the designing of the SMC based on super-twisting method the sliding mode surface s is chosen as error of the piston stroke $\varepsilon_x(t)$. Thus, the dynamic equation for duty ratio of the LOM for stroke control loop based on STSMC method can be expressed as

$$\begin{aligned} \lambda &= k_1 |\varepsilon_x|^{\alpha} + \partial \\ \dot{\partial} &= k_2 sign(\varepsilon_x) \end{aligned} \tag{18}$$

where k_1, k_2 and α are the positive switching gain constants. To mitigate chattering effects caused by the discontinuous *sign* function, we employ the boundary layer method which provides continuous output, defined as

$$sign(\varepsilon_x) = \begin{cases} 1 & \text{if} \quad \varepsilon_x > \beta, \\ \varepsilon_x/\beta & \text{if} \quad |\varepsilon_x| \le \beta, \\ \text{-1} & \text{if} \quad \varepsilon_x < -\beta \end{cases} \tag{19}$$

where β is a small constant value. The *sign* can be substituted by the *sat* function to reduce further the chattering of the state variable.

$$sat(\varepsilon_x) = \left\{ \frac{\varepsilon_x}{\varepsilon_x + \beta} \right. \tag{20}$$

The piston stroke control of the motor under STSMC is designed as follows:

$$\begin{aligned} \lambda &= k_1 |\varepsilon_x|^{\alpha} + \partial \\ \dot{\partial} &= k_2 sat(\varepsilon_x) \end{aligned} \tag{21}$$

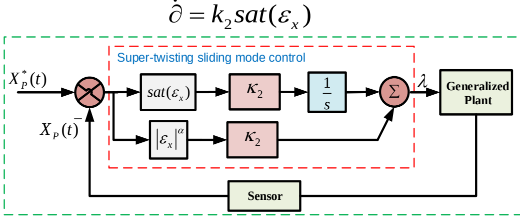

Fig. 4. The STSMC based piston stroke regulation for the LOM drive system.

Fig. 4 presents the block diagram of the STSMC based piston stroke controller for the LOM drive system.

C. Stability Check

The Stability of the LOM piston stroke closed-loop is confirmed via Lyapunov theory, using the sliding surface and its derivative:

$$\lim_{s \to 0} \dot{V} = \lim_{s \to 0} s\dot{s} \le 0 \tag{26}$$

and

$$\begin{aligned} \dot{V} &= s\dot{s} \\ &= -s\{-\kappa_2 sign(s)\} \\ &= -\kappa_2 |s| \le 0 \end{aligned} \tag{27}$$

where positive constants k_1 and k_2 denote controller gains. Equ. (27) being negative-definite ensures closed-loop stability of the STSMC for LOM piston stroke regulation. The corresponding control scheme is represented in Fig. 5.

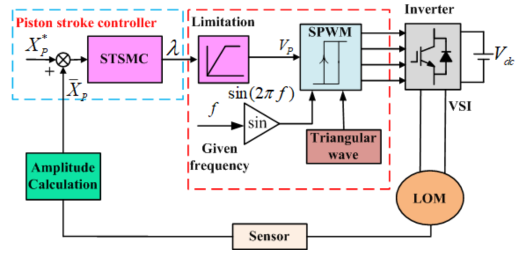

Fig. 5. Basic control diagram of the LOM based on STSMC method.

IV. SIMULATION AND ANALYSIS

In this part, simulation studies of the piston stroke have been conducted on the LOM drive system for the whole dynamic control operation under the proposed super-twisting sliding mode control (STSMC) and PI controllers. Table I shows the key parameters of the LOM drive system, and Table II represents the control gain values for the STSMC and PI controllers. The simulation of the LOM drive systems is fully analyzed on 6- and 7-mm piston strokes to check the superiority of the proposed STSMC technique over conventional PI controllers, respectively. The dynamic piston stroke, error of the actual and reference piston stroke, and input current responses of the LOM under PI and STSMC are presented in Fig. 6 separately.

TABLE I
KEY PARAMETERS OF LOM

Quantity	Symbol	Value	Unit
Thrust constant	α	30	N/A
Spring coefficient	δ_m	19.75	kN/m
Damping coefficient	l_m	16	N.s/m
Mass of mover	m	1.024	kg
Stator resistance	R	18.4	Ω
Stator inductance	L	0.787	H

TABLE II
CONTROL PARAMETERS OF THE STSMC AND PI CONTROLLERS

Name	Value
STSMC	α=0.5, k_1= 100, k_2=5.
PI	P=5, I=30.
Operating frequency	*23 Hz*
Input voltage	*200 V*

It can be understood from Fig. 6(a) that the piston stroke under STSMC has a faster dynamic response with a lesser steady-state error than that of the PI method. Fig. 6(b) shows the piston stroke error. It can be realized from this dynamic response that the piston stroke under STSMC has a quicker response with a smaller piston stroke error than the PI regulator. It is realized from Fig. 6(d) that the input phase current of the LOM drive system under STSMC has a smaller peak amplitude than that of the PI regulator. It can be determined from Fig. 6 that the dynamic piston stroke of the LOM drive system under STSMC is 25% faster with a smaller piston stroke error and a smaller input current amplitude than the PI controller, respectively. It means that the piston stroke response takes 0.2s and 0.8 s under STSMC and PI controller, respectively. Fig.7(a) demonstrates that the STSMC achieves faster dynamic response and lower steady-state error in piston stroke control compared to the PI method. Notably, while the PI controller exhibits overshoot, the STSMC response remains free from this behavior. The corresponding stroke error in Fig. 7(b) further confirms the superior performance of STSMC, with both quicker convergence and reduced error magnitude.

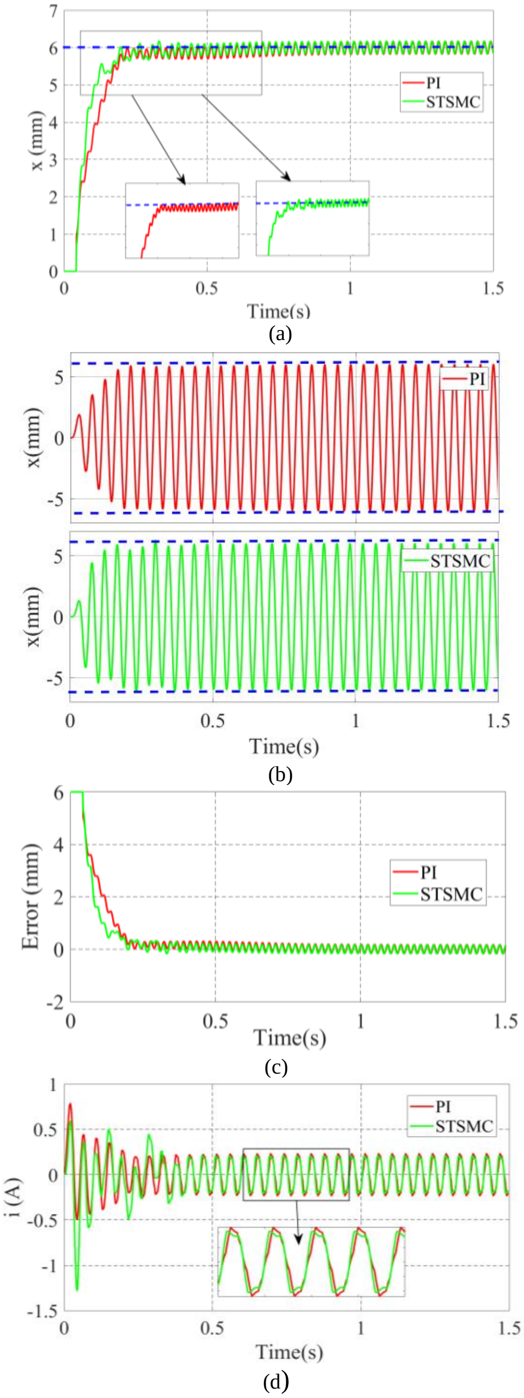

Fig. 6. The 6 mm dynamic piston stroke response of the LOM drive system. (a) Piston stroke under PI and STSMC. (b) Zoomed piston stroke under PI and STSMC. (c) Error of the stroke under PI and STSMC. (d) Current response under PI and STSMC.

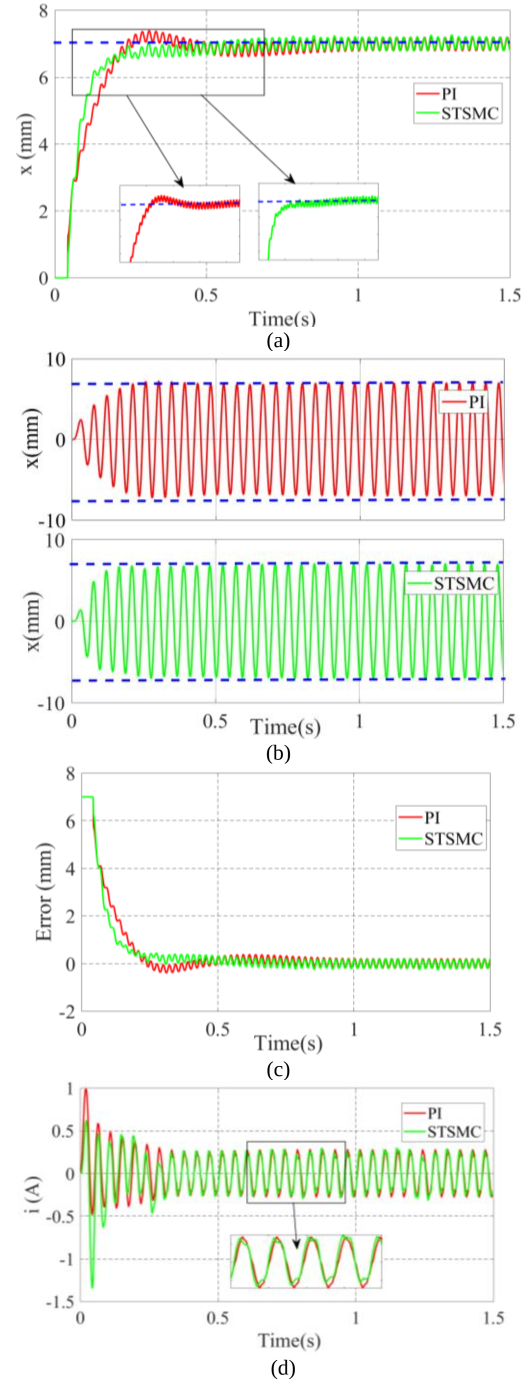

Fig. 7. The 7 mm dynamic piston stroke response of the LOM drive system. (a) Piston stroke under PI and STSMC. (b) Zoomed piston stroke under PI and STSMC. (c) Error of the stroke under PI and STSMC. (d) Current response under PI and STSMC.

Additionally, Fig. 7(d) reveals that the STSMC-regulated LOM drive system operates with lower peak phase current amplitudes than the PI-controlled system. It can be confirmed from Fig. 7 that the dynamic piston stroke of the LOM drive system under STSMC is 40% faster with a smaller piston stroke error and smaller input current amplitude than the PI controller, respectively. It means that the piston stroke response takes 0.48s and 0.8 s under STSMC and PI controller, respectively. Moreover, the implications of these improvements extend far beyond the compressor itself. The enhanced control performance directly enables the LOM compressor to function as a reliable and responsive grid-interactive appliance. The faster, more precise stroke regulation allows for rapid modulation of power consumption in response to grid signals, making it an ideal candidate for demand-side flexibility plans. The reduction in input current also improves overall system efficiency, contributing to reduced energy consumption at the point of use.

V. Conclusion

This paper has established a super-twisting sliding mode control (STSMC) strategy for the piston stroke control of a linear oscillatory machine (LOM) in a compressor application. The proposed method demonstrates significant advantages over the conventional PI controller, including a 35% faster dynamic response, the elimination of overshoot, a 2% reduction in steady-state error, and operation with a lower input current amplitude.

The stability of the proposed control system was rigorously proven using Lyapunov theory and validated through comprehensive simulations. By addressing the critical need for precise actuator control in flexible loads, this work paves the way for the widespread deployment of LOM-based compressors as a key technology for enhancing resource adequacy, integrating renewable energy, and building a more resilient and decarbonized power system.

Future work will focus on the experimental validation of this control method and the development of communication and aggregation frameworks to deploy these grid-responsive appliances at scale.

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