Multi-objective evolutionary algorithm for real-time estimation of voltage signal parameters

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***Abstract*—This paper presents the Multi-objective Evolutionary Algorithm with Tables and Regularization (MEATR), developed for accurate estimation of amplitude, frequency, and phase from voltage signals in an Electric Power System (EPS). The estimation process is formulated as a multi-objective optimization problem that minimizes several types of errors across different signal datasets. MEATR extends the Evolutionary Algorithm with Regularization (EAR), with multi-objective capabilities implemented in real time on FPGA hardware. The proposed algorithm can be parallelized in hardware to meet specific response-time demands and can handle a large number of objectives efficiently. The proposed method's performance was evaluated against a commercial relay implementing the Discrete Fourier Transform (DFT) and adhering to the IEEE C37.118 standard for Phasor Measurement Units, demonstrating promising results in the presence of transients.**

***Index Terms*—Evolutionary algorithms, Multi-objective functions, Power system parameters, FPGA.**

I. Introduction

Accurately estimating signal parameters is essential in Electrical Power Systems (EPSs) to ensure reliable monitoring, protection, and control. Parameters such as amplitude, frequency, and phase angle are fundamental for detecting faults, maintaining system stability, and enabling proper operation of relays and control devices. Considering the growing integration of renewable and converter-based sources, signals have become more non-stationary, making precise estimation even more critical for stable grid operation and coordination. In simple terms, a signal is considered non-stationary when its key characteristics, such as amplitude, frequency, or phase, change over time rather than remaining constant. An accurate signal parameter estimation underpins the reliability, efficiency, and resilience of modern power systems.

Nevertheless, signal processing in EPSs is an ill-posed problem, where small variations in certain variables can lead to significantly different solutions. Consequently, estimation methods may produce inconsistent results if this issue is not properly addressed. The increasing complexity of EPSs, driven by the installation of new equipment such as Phasor Measurement Units (PMUs), remote control switches and electronic devices generates complex signal characteristics of the so-called smart power grids [1].

Several methods have been proposed in the literature to estimate signal parameters from a power system signal. These methodologies can be classified as: phase estimation methods using the Discrete Fourier Transform (DFT) [2], [3], methods employing Kalman filters [4], Least Square Errors (LSE)-based methods [5], as well as nonlinear approaches based on the quadrature phase-locked loop concept [6], and adaptive filters [7].

In addition, techniques based on Artificial Intelligence have also been explored [8]. Evolutionary algorithms demonstrates that the quality of signal estimation can be improved by using additional information from the EPS, directly addressing the ill-posed nature of the problem [9].

This paper proposes MEATR, a novel multi-objective evolutionary algorithm for real-time estimation of electrical signal parameters (magnitude, frequency, and phase) explicitly designed for practical deployment in protection and monitoring devices. Its main contributions, compared to prior EAR-based approaches, include a multi-objective formulation that enhances estimation robustness during transients, a regularization strategy and lookup-table-based implementation that significantly reduce computational burden, and a hardware-oriented architecture suitable for FPGA realization. Extensive simulation and experimental results demonstrate that MEATR outperforms conventional DFT-based approaches compliant with IEEE C37.118, particularly in terms of accuracy, convergence speed, and resilience under dynamic, distorted, and unbalanced power system conditions.

II. Designing the Evolutionary Algorithm with Regularization (EAR) for FPGA

An evolutionary algorithm is a type of optimization algorithm inspired by the process of natural selection and evolution. It works by maintaining a population of candidate solutions that evolve over time through mechanisms such as selection, crossover (recombination), and mutation, aiming to improve performance according to a defined fitness function. The Multi-objective Evolutionary Algorithm with Tables and Regularization (MEATR) proposed is based on an EAR, which enables FPGA parallelization for real-time processing and incorporates regularization to handle ill-posed problems. Sections II-A and II-B detail, respectively, the hardware parallelization and the regularization technique.

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A. EAR in a Logic Circuit Implementation

The fundamental aspects for efficiently implementing an EAR on FPGAs are summarized as follows. The input data, corresponding to electrical signals digitized by an analog-to-digital converter, must be properly processed. The three-phase signals are sampled with a 16-bit resolution, one sample every 1.3 ms. Thus, 16 samples represent one complete cycle of a 60 Hz signal, and the EAR must perform analysis and update estimations within each 1.3 ms interval.

Based on this data, the EAR estimates the sinusoidal signal parameters (magnitude, frequency, phase), considering only the fundamental component to speed up candidate evaluation. Each set of parameters defines a candidate solution (individual) for the optimization problem. Each chromosome contains three indices linked to lookup tables for magnitude, frequency, and phase, mapping indices to quantized real values. This enables efficient FPGA parallelization without floating-point operations. The bit widths define quantization: magnitude 12 bits $[0.0, 2.0]$ pu, frequency 24 bits $[0.0, 96.15]$ Hz, and phase 16 bits $[0.0, 2\pi]$ rad intervals. The first set (population) of candidate solutions (individuals) are randomly generated. Then, individuals are evaluated using the error function described by Equation (1).

$$E(P_h) = \sum_{it=0}^{n-1} \left| S_{P_h}(i_t) - V_m \cdot \sin\left(2\pi\, i_t \frac{f}{F} + \phi\right) \right|, \tag{1}$$

where $S_{P_h}(i_t)$ is the measured voltage for phase P_h, and V_m, f, ϕ are the magnitude, frequency, and phase of the sinusoidal model. F is the sampling frequency, n the number of samples, and i_t the sample index ($t = i_t \cdot \frac{f}{F}$). The absolute error is used instead of the squared error for lower computational cost, enhancing EAR efficiency [9]. The sine function is precomputed in a lookup table to reduce FPGA resource use, alongside other digital circuit optimizations.

Selection uses binary tournament, BLX-(α) crossover is adapted for discrete values, and mutation increments/decrements indices. New individuals form the next generation in a standard genetic algorithm structure. The EAR operators are designed with simplicity for parallel execution [9], enabling thousands of evaluations with reduced processing time. The configuration uses 50 individuals, tournament size 2, 100% crossover, 20% mutation rate, and 500 generations.

B. Regularization Technique for Electrical Signal Processing

Regularization is a technique used in fields such as inverse problems, statistics, and machine learning to guide optimization toward practically relevant solutions. It penalizes overly complex solutions or those with highly oscillatory behavior, incorporating domain knowledge to avoid spurious results [10]. Regularization typically constrains variables within acceptable ranges or imposes smoothness on parameter variations.

In an EPS, the signal waveform during a transient event may differ from the model, therefore the optimal solution from Equation (1) can deviate from the actual system frequency. In such cases, the actual frequency often corresponds to a local optimum of the objective function. It can be determined using a regularization technique based on the generator's inertia. The swing Equation (2) describes frequency variation according to the generator's inertia constant.

$$\frac{2H}{\omega}\frac{d^2\delta}{dt^2} = P_m - P_e, \tag{2}$$

where P_m and P_e are mechanical and electrical power, δ is the rotor angle, and ω its angular velocity. Frequency variation Δf can be approximated as Equation (3).

$$\Delta f = f_0 \frac{P_m - P_e}{2H} \Delta t, \tag{3}$$

with $f_0 = \frac{1}{2\pi}\omega$ (assuming $f_0 = 60\,\text{Hz}$) and $P_m - P_e = 1$ to avoid dependence on the system load. Using this approximation and Equation (3), the frequency search space is restricted to $|f - \Delta f, f + \Delta f|$ over Δt, preventing high frequency variations and spurious solutions.

III. MULTI-OBJECTIVE MODELING OF THE PROBLEM

Many real-world problems involve conflicting objectives, and multi-objective modeling provides a realistic representation, where solutions are chosen from the Pareto-optimal set of non-dominated (efficient) solutions. Thus, the Multi-Objective Optimization (MOO) algorithm integrates domain knowledge to support the final decision-making process. In the estimation of multiple electrical signals, the error function (Equation (1)) for each phase of the EPS forms an objective, and these errors can conflict. Multi-objective analysis typically produces better estimations than using a single phase or a linear combination of all phases. Section (III-A) introduces the EARO, which informs the EAR the best phase to use at each instant, generating a reference of accurate estimations and showing that a careful analysis of available signals improves overall estimation performance.

A. Multiple Signal Analysis using an Oracle

Each component of a three-phase signal from an EPS provides distinct information for the estimations. Traditional strategies to handle multiple phases include: i) using a single-phase model via basis transformations (e.g., Clarke Transform); ii) aggregating phase errors into a single function. These approaches may lose information, as each phase contributes differently to the estimation process.

This paper shows that formally modeling conflicts between phase signals can improve estimation. According to the MOO theory, the error functions of the three phases form a conflicting objective space, producing a Pareto-optimal set from which the most efficient solutions can be selected.

Figure 1 illustrates an EPS with a synchronous generator, 80–150 km transmission lines, and an AG fault at the BGER busbar. Figure 2 shows frequency estimations by the EAR for each phase at GER busbar, with the red curve as the reference

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(actual generator frequency). After (t = 2 s) (phase A-ground fault), estimations diverge from each other, revealing distinct information contributions. Although all phases share exactly the same frequency, in this case the frequency is estimated by fitting a mathematical model to the signal of each phase individually. Traditional DFT-based relays fail in this scenario.

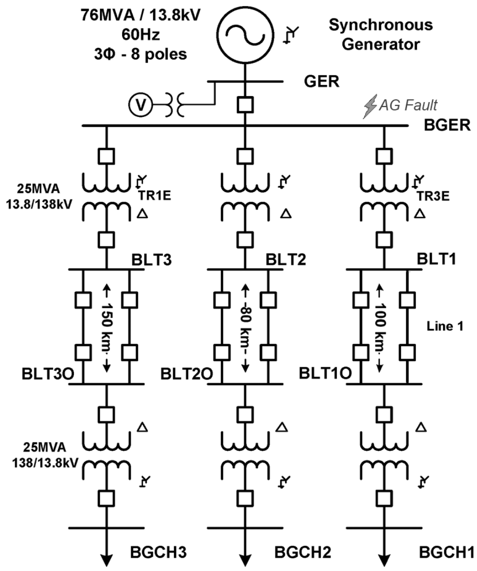

Fig. 1: EPS simulated in ATP, adapted from [9].

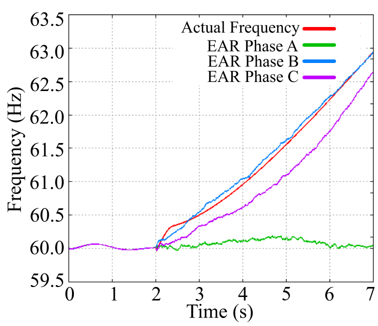

Fig. 2: Estimations obtained by the EAR for the signal from each phase at GER busbar.

A linear combination of the phase errors (EAR + Agg) is evaluated in Figure 3. Although it yields better results than the estimations from Phases A and C, its performance remains inferior to that obtained using Phase B alone.

To quantify distinct contributions, the EAR using an Oracle (EARO) is proposed. The EARO applies EAR independently to each phase and selects, at each instant, the estimation closest to the proper frequency. Figure 4(a) shows that EARO estimations closely match the reference and outperform Phase B estimations.

In practice, the EAR lacks an Oracle. Figures 4(b), 4(c), and 4(d) show how EARO performance decreases as the Oracle's correctness rate (r) drops from 1.0 to 0.5. Results confirm the MOO theory: better exploration of the objective space improves estimations. Even with a non-perfect Oracle (r = 0.9), EARO outperforms single-phase estimation, demonstrating that partial knowledge of phase contributions can yield high-quality estimations.

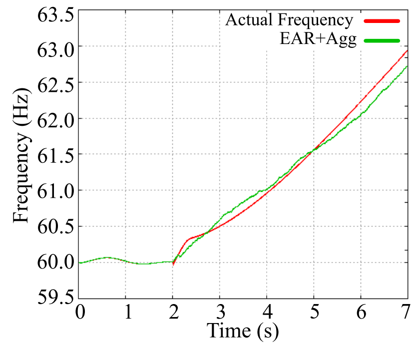

Fig. 3: Performance of the EAR using the Aggregation function - EAR+Agg.

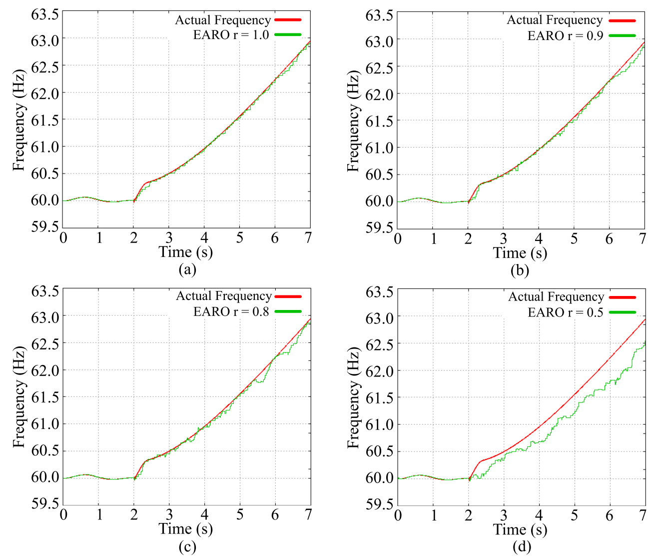

Fig. 4: EARO estimations using an Oracle with different correctness rate. (a) $r = 1.0$; (b) $r = 0.9$; (c) $r = 0.8$; (d) $r = 0.5$.

B. Multi-objective Evolutionary Algorithm with Tables and Regularization

The proposed MEATR aims to approximate the Oracle through a systematic exploration of the error function's objective space. Each new individual is evaluated with respect to multiple objectives and stored in dedicated subpopulation tables associated with each objective. When a table reaches capacity, the worst-performing individual is replaced if the new candidate shows superior performance. An individual may be included in multiple tables, and the selection process for reproduction involves randomly choosing a table and then selecting an individual from it.

The number of tables (T) can expand to store solutions for linear combinations of objectives, making MEATR behave

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similarly to MOEA/D [11]. Unlike other EAs, each individual survives if it enters a table, maintaining updated subpopulations and fast convergence. The final non-dominated set across all tables forms the MEATR solution set; the solution with the best aggregation function value is used as the output.

MEATR is simpler to implement on FPGA than multi-objective EAs, requiring fewer modifications than the mono-objective EAR. Its computational cost is higher due to multiple objectives but increases linearly with M. Evaluating $T > M$ criteria is efficient, as the additional $T-M$ criteria are combinations of the M objectives, avoiding redundant computations. In this work, MEATR uses $M+2$ tables, including aggregation and domain-knowledge criteria, whereby execution time increases linearly with M.

IV. EXPERIMENTAL EVALUATION

This section presents experimental results to: (i) show that MEATR can match the EARO Oracle for an appropriate r, effectively using multiple voltage signals, and (ii) evaluate its benefits in estimating electrical signal parameters according to IEEE C37.118. Tests used subpopulation tables of 10 individuals, with 40 in the $M+1$ tables and 50 in the $M+2$ tables, generating 25,000 individuals per execution. Other algorithm parameters were as previously described.

A. The MEATR working as an Oracle

The experiments refer to the same case study as Section III-A: a permanent Phase A–Ground fault on the BGER busbar of the EPS occurring at $t = 2\,\text{s}$ (Figure 1). Using $M+1$ tables, MEATR follows the correct frequency curve with a constant shift (Figure 5). Performance is improved with an additional domain-knowledge criterion, the Phase-Swap Heuristic (PH), which selects the phase with the lowest transient distortion, quantified by:

$$F_{V\,m} = \frac{1}{V_m} \times F, \tag{4}$$

$$F_{\text{PH}} = \min_{Ph\in\{A,B,C\}} F_{V\,m}(Ph), \tag{5}$$

where V_m is the maximum voltage magnitude of phase Ph and F is from Equation (1). Figure 6 shows the $M+2$ criteria MEATR, approximating the EARO Oracle for $r \geq 0.8$. Its mean square error is 3.09×10^{-3}, compared to 3.48×10^{-3} for EARO $r = 0.8$, 2.55×10^{-3} for $r = 0.9$, and 10.24×10^{-3} for EAR+Agg (Figure 3).

B. The MEATR performance according to the IEEE standard

The IEEE C37.118 standard defines performance requirements for PMUs in terms of magnitude, frequency, and phase. The Total Vector Error (TVE) measures estimation quality:

$$\text{TVE} = \frac{|X_{\text{est}} - X|}{|X|}, \tag{6}$$

where $X = V_m e^{i(\omega t+\phi)}$ is the actual voltage phasor and X_{est} the estimated phasor. Synthetic tests applying step changes to

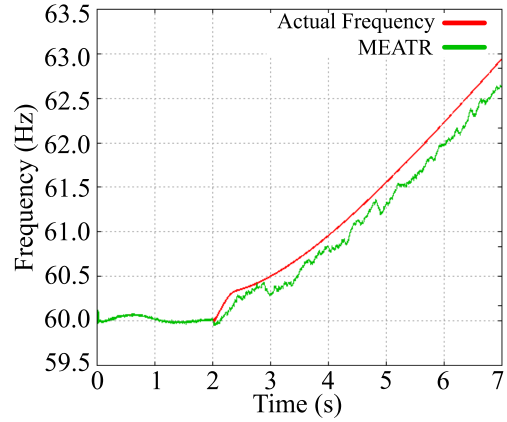

Fig. 5: The performance of the MEATR with $M+1$ tables.

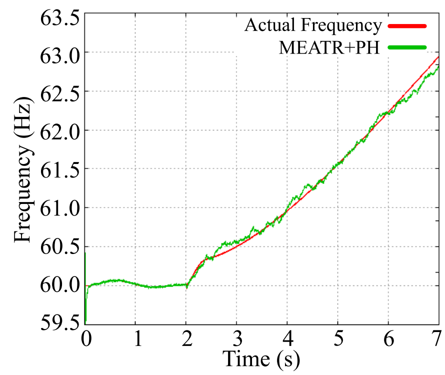

Fig. 6: MEATR perfomance with PH, $M+2$ criteria.

phase, magnitude, and frequency were used to evaluate the methods.

Figures 7a–c show the TVE responses. For sudden phase shifts (Figure 7a), both EAR and MEATR stabilize after one cycle, faster than a commercial DFT-based relay. For a 10% magnitude decrease (Figure 7b), EAR and MEATR recover within 0.02 s, while the DFT takes over 0.05 s. For frequency steps (Figure 7c), MEATR stabilizes within 0.04 s, outperforming EAR ($> 0.05\ s$) and the DFT ($0.10\ s$). Frequency estimation is more challenging due to the nonlinearity with respect to time and sensitivity to sampling errors.

Table I summarizes the transient recovery time to stabilize TVE below 0.2%. Both EAR and MEATR outperform the commercial relay, with MEATR consistently faster than EAR.

TABLE I: Recovering time of the estimators.

Test case	**DFT (s)**	**EAR (s)**	**MEATR (s)**
Phase	0.111979	0.026562	0.017708
Magnitude	0.043229	0.015104	0.012500
Frequency	0.095312	0.054166	0.038541

Explicit tests incorporating decaying DC offsets (using current signals) and harmonic components typically present in real power systems, as well as extending the number of objectives, were not included in this version but will be addressed

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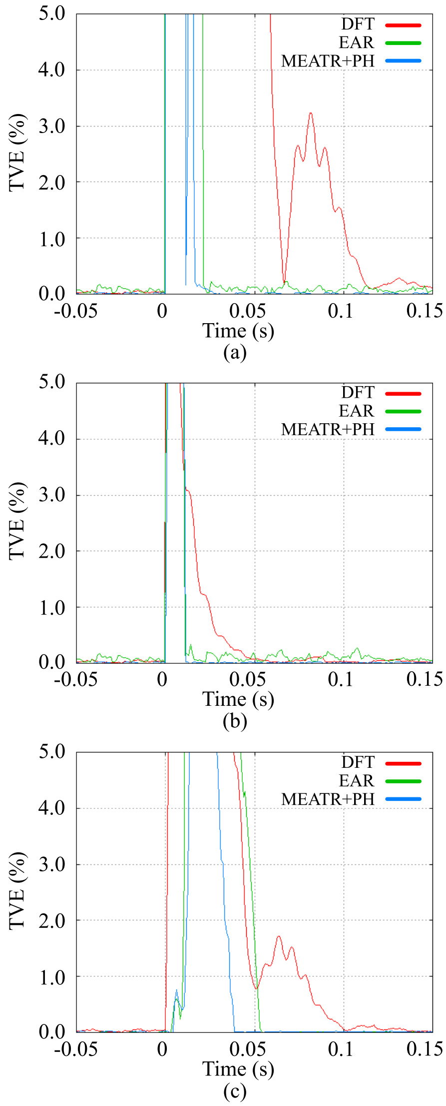

Fig. 7: TVEs of the tested methods for sudden (a) phase variation; (b) magnitude variation; (c) frequency variation.

in future work. Regarding computational efficiency, both EAR and MEATR were implemented on an Altera EPS260F672C3 FPGA with a sampling rate of 960 Hz. The MEATR approach preserves real-time capability and scales efficiently with the number of objectives when deployed in hardware. Despite its higher computational complexity, MEATR achieves real-time performance comparable to EAR, demonstrating its feasibility for real-time power signal estimation.

V. CONCLUSIONS

The paper introduces a MEATR algorithm for estimating voltage signal parameters (magnitude, frequency, and phase) optimized for efficient implementation on FPGAs. This design enables parallel execution of computations and data handling, allowing the evaluation of a larger number of individuals within the search space. The proposed multi-objective approach enhances estimation accuracy and robustness by comparing different error metrics and applying regularization to constrain frequency values within the typical range of synchronous inertial generators, thereby reducing variations between consecutive samples. The MEATR can simultaneously analyze multiple signals, including unbalanced phases and zero-magnitude cases, and exhibits faster transient recovery compared to a commercial DFT-based relay compliant with IEEE C37.118. These findings demonstrate that MEATR effectively supports fault detection in EPS, representing a promising methodology for next-generation smart grids by providing accurate and reliable analysis of both individual and combined electrical signals. It must be emphasized that, although the presence of inverter-based resources (IBRs) introduces faster dynamics, reduced system inertia, and increased harmonic distortion compared to conventional synchronous generation, the proposed MEATR algorithm is expected to remain effective in such environments. Moreover, the results demonstrate that the proposed method satisfies real-time constraints and can be feasibly integrated into practical monitoring and protection devices.

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