Multi-Stage Dynamic Programming for VAR Source Placement under Coal-Fired Power Plant Retirement

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***Abstract*—The increasing penetration of intermittent renewable energy sources and accelerating retirement of coal-fired power plants are exacerbating voltage stability issues in modern power systems, due to the reducing system inertia. This paper proposes a coordinated multi-stage framework for VAR source placement, formulated as a Markov decision process and solved through a dynamic programming-based path selection strategy. Dynamic multi-objective evolutionary algorithms are employed to generate stage-wise Pareto-optimal solution sets, which serve as year-by-year planning decisions for the multi-year planning trajectories. The trajectories are obtained by minimizing discounted cumulative costs across stages while maintaining desired voltage performance. Case studies on the IEEE 39-bus system show that, compared with conventional planning, the proposed approach achieves 0.2% and 4.9% improvement in steady-state and transient stability with a comparable investment, while ensuring temporal consistency of multi-year planning decisions.**

***Index Terms*-- Coal-fired power plant retirement; Dynamic programming; Dynamic multi-objective optimization; VAR source placement**

I. INTRODUCTION

Modern power systems are characterized by high penetration of inverter-based renewable resources and diverse integration of complex loads [1]. On the generation side, the displacement of conventional synchronous generators by electronically interfaced units has reduced system inertia, thereby weakening the ability to resist and recover from voltage disturbances. On the demand side, the widespread use of induction motors and air-conditionings increases reactive power consumption during voltage recovery, which further exacerbates instability risks [2]. The gradual evolution of the generation mix, exemplified by the retirement of coal-fired power plants (CFPPs), and the continuous growth of load demand amplify these challenges.

Flexible AC Transmission System (FACTS) devices, such as STATCOMs, are widely used in VAR planning [3]. While recent work has shifted toward multi-objective formulations [4], most still follow static target-year models, overlooking progressive load growth and CFPP retirement, which may yield infeasible long-term plans. This mismatch highlights the need for dynamic perspective: planning must address sequential decisions under evolving conditions, where early choices constrain later ones [5]-[6].

Dynamic programming (DP) offers a natural tool for multi-stage dynamic VAR planning [7]-[8], formulated as a dynamic multi-objective optimization problem (DMOP), where objectives and constraints evolve with temporal changes. Another critical challenge lies in selecting a coherent planning trajectory from the evolving Pareto fronts (PFs). Current selection rules, such as direct selection [9] and compromise-based approaches [10], often neglect inter-stage coupling, which may result in incompatible deployments and reduced long-term efficiency. Moreover, algorithms designed to handle DMOPs with temporal coupling constraints remain unexplored in the power system domain. The main contributions are as follows:

1) DP-based multi-stage planning framework: A dynamic programming framework is established to formulate VAR planning as a forward sequential decision process, explicitly capturing temporal couplings across stages to ensure long-term consistency in planning decisions.

2) Sequential multi-Pareto decision-making strategy: A forward DP-based technique is developed to achieve coherent path selection across time-varying PF, propagating cumulative value and enforcing inter-stage feasibility to avoid myopic choices and improve long-horizon robustness.

3) Improved dynamic optimization algorithm: An improved steady-state generational evolutionary algorithm (SGEA) is adopted to address the high dimensionality and time-varying objectives of DMOPs. Its convergence and diversity ensure robust and cost-effective planning solutions under evolving system conditions.

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II. Framework for Multi-Stage VAR Planning Based on Dynamic Programming

A. Limitations of Static Planning Approaches

In multi-stage VAR planning, system conditions evolve due to changing conditions such as load growth, renewable integration, and the retirement of CFPPs. As shown in Fig. 1, PFs at different stages are inherently interconnected, while conventional methods focus exclusively on the terminal stage, disregarding feasible options in earlier stages. This often results in inconsistent planning trajectories and infeasible outcomes as conditions change.

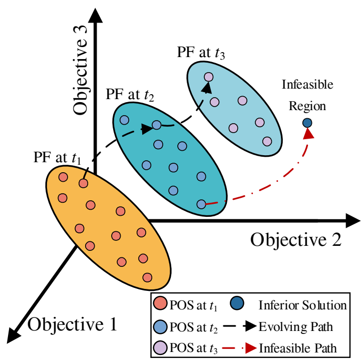

Figure 1. Evolving PFs in multi-stage planning.

To address the limitations of static approaches, multi-stage VAR planning can be reformulated as a sequential decision process, which is represented by a Markov decision process:

- S denotes the system configuration at a given stage. In the context of VAR planning, a state is defined by the installed capacities and locations of VAR sources, voltage stability indices, and the generator portfolio after staged CFPP retirements.
- A represents planning decisions such as installing new devices, expanding existing capacities, and the retirement of outdated equipment.
- $P(S_{k+1}|S_k, a)$ defines the evolution of the system from state S_k to S_{k+1} under action a, determined jointly by planner decisions and external developments such as load growth and CFPP retirements.
- $R(s,a)$ represents the stage cost associated with taking action a in state S_k, including investment and operation expenses as well as penalties for steady-state and transient stability violations.
- $\gamma\in[0,1]$ is the discount factor, which quantifies the trade-off between immediate gains and long-term planning benefits, thereby balancing short-term feasibility with long-term system resilience.

B. Dynamic Programming Formulation for VAR Planning

DP provides the foundation for solving the sequential decision problem of multi-stage VAR planning. At each stage k, multiple admissible system states can be defined, denoted as $S_{k,j}$, where $j=1,2,\ldots,n_k$ indexes the feasible states at stage k. The value of each state is evaluated by the recursive relation:

$$V_k\left(S_{k,j}\right)=\min_{u\left(S_{k,j}\right)}\left\{C\left(S_{k,j},u\left(S_{k,j}\right)\right)+\gamma V_{k+1}\left(S_{k+1}\right)\right\} \tag{1}$$

where $u(S_{k,j})$ denotes the action taken from state $S_{k,j}$, $C(.)$ is the immediate cost reflecting investment and stability indices.

This recursive formulation ensures that decisions at one stage incorporate both present performance and their long-term implications, thus avoiding locally optimal but globally inconsistent trajectories.

C. Forward-Looking VAR Planning Framework

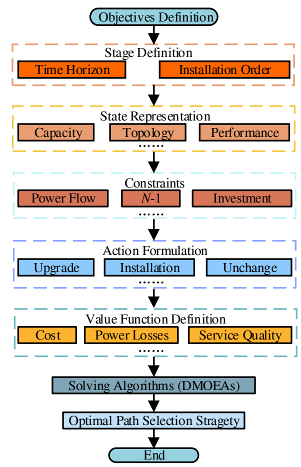

Figure 2. Framework of multi-stage VAR planning based on DP.

As shown in Fig.2, a forward-looking framework is established to implement multi-stage VAR planning. The proposed framework consists of the following parts:

1) Stage Definition: The planning horizon should be divided into sequential stages by time intervals or installation order, so as to reflect practicable construction sequences.

2) State Representation: States are characterized by VAR capacities, device locations, and corresponding voltage performance, subject to operational and economic constraints.

Each state must meet power flow, performance, and investment requirements, while some constraints can be relaxed at intermediate stages to preserve solution diversity.

3) Action Formulation: Actions specify feasible adjustments to the VAR configuration, including new installations, capacity reinforcements.

4) Value Function Definition: The stage cost is composed of investment, operation, and stability-related penalties, while the value function (1) guarantees that short-term decisions incorporate their long-term impacts.

5) Solution Strategy: Dynamic multi-objective evolutionary algorithms (DMOEAs) are employed to generate the Pareto-optimal solutions (POSs) at each stage under evolving constraints.

6) Path Selection: A sequential planning path through the stage-wise PFs should be determined using the path-selection method described. The general ideas of the optimal path selection strategy are detailed in Section IV.

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By treating time-varying PFs as interconnected rather than isolated, this framework aligns short-term feasibility with long-term resilience, ensuring consistent and cost-effective VAR planning decisions across the entire horizon.

It should be emphasized that VAR planning is not an independent planning problem, but a sub-planning task within the hierarchical structure of power system planning. In practical planning procedures, higher-level decisions, such as generation expansion, generation retirement, and network reinforcement, are typically determined first, while VAR planning is subsequently carried out based on these predefined system conditions.

Consequently, uncertainties associated with load growth, generation configuration evolution, and network development are mainly addressed at upper planning levels, whereas lower-level VAR planning commonly adopts representative operating scenarios derived from these decisions. This hierarchical treatment of uncertainty is widely adopted in both industrial practice and academic studies, as jointly optimizing all planning layers and uncertainties in a single framework is neither practical nor realistic.

Within this context, the proposed DP-based framework operates on successive system states along the planning horizon and focuses on ensuring temporal consistency and cost-effectiveness of VAR deployment decisions under given planning scenarios.

III. Dynamic Multi-Objective VAR Planning Model

A. Stage-Wise Dynamic Model Formulation

Based on the DP-based framework, the VAR planning problem is reformulated as a DMOP. The planning horizon is divided into successive stages that reflect evolving conditions such as load growth and CFPP retirements. The state variables describe cumulative deployments, while the decision variables specify stage-specific VAR placements. Stage transitions capture both physical system changes and sequential dependencies, ensuring that local decisions remain consistent with long-term objectives.

B. Objective Functions

Conventional static studies focusing on either steady-state or transient stability are inadequate, as practical VAR planning must account for both long-term evolution and short-term contingencies. To achieve this, the proposed framework integrates steady-state power flow with dynamic EMT analysis [4], as illustrated in Fig. 3.

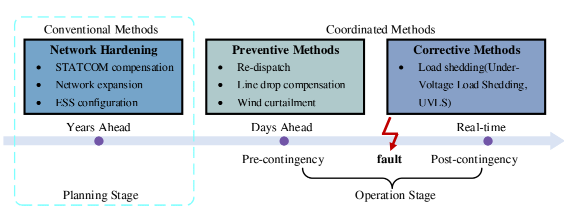

Figure 3. Multi-stage coordinated VAR control strategy.

The optimization aims to **minimize** a set of objective functions representing economic cost and voltage stability performance. The first sub-objective f_1, which quantifies the total VAR planning cost, is formulated as:

$$f_1^t = \sum_{i\in\Omega_c} C_{i,t}^{inv} + \sum_{i\in\Omega_w} (C_{w,t}^{cur} + C_t^{red}) + \sum_{i\in\Omega_l} C_{i,t}^{ls} \quad (2)$$

where $C_{i,t}^{inv}$ denotes the investment cost of STATCOM installation at bus i in stage t. Ω_c represents the candidate buses for deployment. $C_{w,t}^{cur}$ is the curtailment cost of wind power at bus w. Ω_w is the set of buses with wind farms. C_t^{red} is the re-dispatch cost incurred in stage t. $C_{i,t}^{ls}$ is the load-shedding cost at bus i in stage t while Ω_l is the set of load buses.

The second and third sub-objectives are defined as follows:

$$f_2^t = Risk^{VCPI}(t) = \sum_{k\in\kappa} VCPI_{k,t}^{p} \bullet p_{k,t} \quad (3)$$

$$f_3^t = Risk^{TVSI}(t) = \sum_{k\in\kappa} TVSI_{k,t}^{p} \bullet p_{k,t} \quad (4)$$

where $p_{k,t}$ denotes the probability of certain fault contingency k occurring in stage t. Comprehensive formulations and derivations of *VCPI* and *TVSI* can be found in [11].

IV. Solving Method

A. Improved SGEA

SGEA is effective for DMOPs due to its adaptability and population diversity. System transitions in VAR planning are deterministic, driven primarily by scheduled factors such as load growth and the retirement of CFPPs. This characteristic limit the direct applicability of the standard SGEA. To overcome this limitation, two enhancements are introduced:

1) Detection Module: Predefined stage transitions trigger adaptation, transferring elite solutions across stages to preserve consistency.

2) Dual-Source Mating: Offspring are generated from both the population and an elitist archive, accelerating convergence and ensuring reliability.

B. DP-Based Path Selection Strategy

The PFs generated at different stages represent snapshots of an evolving decision space shaped by load growth and CFPP retirements. Conventional selection methods, such as direct or compromise-based rules [9]–[10], neglect inter-stage dependencies and may lead to infeasible outcomes. To overcome these limitations, a DP-based path strategy is proposed, consisting of four steps:

1) Solution Set Filtering: At each stage, POSs are screened using voltage stability thresholds to retain only admissible candidates.

2) Path Initialization: Feasible solutions at the first stage are assigned initial costs to form the basis of path construction.

3) Recursive Construction: Extend candidate paths with transition costs defined by investment and adjusted by factor λ.

4) Global Path Determination: Select the path with minimum total discounted cost as the optimal trajectory.

This DP-based approach models inter-stage couplings, providing a systematic way to construct trajectories that remain feasible and cost-effective across the entire horizon.

C. Overall Solution Process

The proposed framework coordinates VAR allocation across stages, integrating preventive and corrective measures with both steady-state and transient assessments.

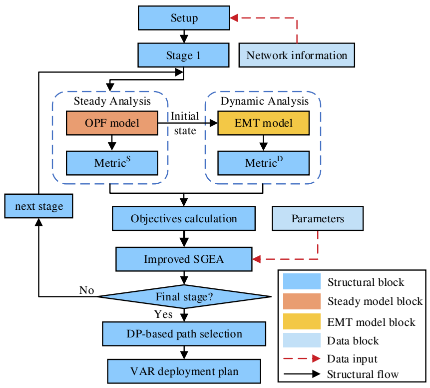

Figure 4. Framework of multi-stage VAR planning.

The overall solution process, shown in Fig. 4, consists of five parts:

1) Setup: Input network topology, control settings, contingency scenarios and algorithmic parameters.

2) Performance Analysis: Conduct steady-state evaluation using OPF and dynamic analysis by EMT models.

3) Objective Formulation: Calculate stage-wise investment costs and stability metrics based on the evaluation.

4) Optimization: Generate POSs by employing the customized SGEA under evolving conditions.

5) Path Selection: Select consistent deployment paths with the DP-based strategy.

This framework ensures trajectories are feasible, efficient, and temporally consistent, aligning near-term operation with long-term development.

V. SIMULATION RESULTS

A. Simulation Environment Construction

MATLAB serves as the integration platform, where SGEA is implemented through PLATEMO, OPF calculations are carried out using MATPOWER, and electromechanical transient analysis are performed in PSS®E with a Python API for automated extraction of voltage performance indices.

The IEEE 39-bus system is modeled as a three-stage VAR planning over 10 years, with a 5% annual load growth. CFPPs at buses 30, 32, and 34 are gradually replaced by wind farms, and contingencies and candidate bus selection follow the rules described in [4].

B. Validation on the Modifed IEEE 39 Bus System

The complete workflow was executed on a workstation equipped with an Intel Core i5-13600KF CPU and 32 GB RAM, requiring approximately 8.61 hours, which is intended for methodological validation rather than direct assessment of large-scale system performance. The IEEE 39-bus system is adopted in this study as a benchmark test system for voltage stability analysis, as recommended by the IEEE task force on test systems for voltage stability analysis and security assessment [12].

It should be noted that the primary computational burden in the present study arises from repeated EMT-based transient simulations, while the DP-based path selection itself operates on stage-wise Pareto solution sets and exhibits favorable scalability. The use of a standardized benchmark system allows the proposed multi-stage planning framework to be systematically validated and compared with existing approaches under well-established voltage stability test conditions.

As shown in Fig. 5, PFs shift toward higher cost and lower performance due to load growth and CFPP retirement, which reduce inertia and voltage support.

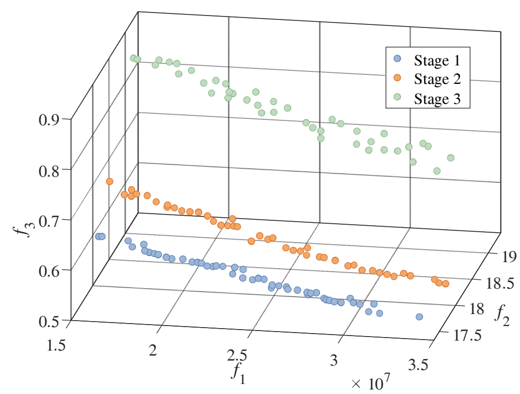

Figure 5. PFs obtained by the DP-based optimization.

The DP-selected path in Table I attains the minimum discounted cumulative cost of 3.13× 10^7, and increases total VAR capacity from 232.05 MVAR in Stage 1 to 517.35 MVAR in Stage 3. Allocations expand spatially, with bus 24 consistently receiving the largest share, reflecting its key role in voltage support.

TABLE I. DEPLOYMENT DECISIONS (IN MVAR)

Stage	18	24	27	30	32	34	f_1 ($\times 10^7$)	f_2	f_3
1	16.89	116.25	17.52	16.67	21.93	42.79	1.59	17.70	0.630
2	16.76	116.92	16.18	16.04	24.04	47.92	1.39	18.29	0.688
3	100.00	117.60	99.08	16.05	89.52	95.10	3.25	18.80	0.679

C. Comparative Analyses

1) Against Conventional Static Planning

The proposed method is benchmarked against a traditional strategy that optimizes only for the final target year, with both approaches tested under identical simulation settings. The approach adopts Constrained-Decomposition-based Multi-Objective Evolutionary Algorithm (C-MOEA/D), a commonly

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used algorithm known for reliable convergence in multi-objective problems.

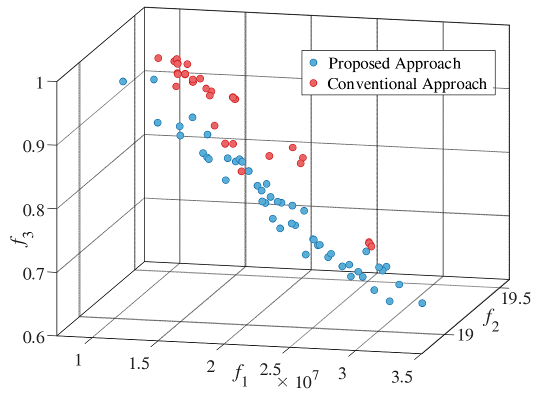

Figure 6. Comparison of PFs for DP-Based and Conventional Approaches.

As shown in Fig. 6, the DP-based results consistently dominate those of the conventional method. The DP technique produces improved voltage profiles with a comparable cost by capturing stage-wise evolution, whereas the conventional solutions are concentrated in some limited regions.

TABLE II. PERFORMANCE IMPROVEMENT OF DP-BASED PLANNING OVER CONVENTIONAL APPROACH

Methods	f_1 ($\times 10^7$)	f_2	f_3
Conventional Planning	1.64	19.30	0.836
Dynamic Planning	1.63	19.26	0.795

As shown in Table II, the DP-based method achieves almost the same investment cost but reduces f_2 by 0.2% and improves f_3 by 4.9% compared with conventional planning, demonstrating its superior long-term effectiveness.

2) Against Other Selection Strategies

Two methods are used: membership [9] and centroid-based [10]. As shown in Table III, both allocate excessive early capacity, boosting short-term support but raising overall cost and lacking coordination. In contrast, the DP-based path balances short- and long-term performance, ensuring efficiency and consistency.

TABLE III. DEPLOYMENT DECISIONS (IN MVAR)

Method	Stage	18	24	27	30	32	34	f_1 ($\times 10^7$)	f_2	f_3
[9]	1	96.98	119.67	98.76	16.41	80.16	95.10	3.20	17.34	0.547
	2	93.63	119.66	97.86	16.33	100.00	45.91	3.02	17.81	0.581
	3	100.00	117.60	99.08	16.05	89.52	95.10	3.25	18.80	0.679
[10]	1	99.50	117.53	18.11	16.33	72.29	38.58	2.48	17.48	0.586
	2	100.00	110.24	84.12	16.74	16.00	45.95	2.30	17.85	0.664
	3	17.44	117.10	92.81	16.20	21.82	44.25	2.21	19.03	0.792

3) Against Simultaneous Optimization

A static model with 18 variables optimized simultaneously is also tested, using a weighted-sum objective. Figure 7 shows the sparse PFs, indicating worse early-stage decisions. From the distribution of the three stage-wise PFs in Fig. 7, each stage produces an independent non-dominated set without guaranteeing inter-stage feasibility, leading to locally efficient but globally inconsistent solutions. In addition, the PFs of Stage 1 and Stage 2 show a clustering effect, suggesting limited diversity in the early stages and a reduced capacity to capture broader trade-offs at the system level.

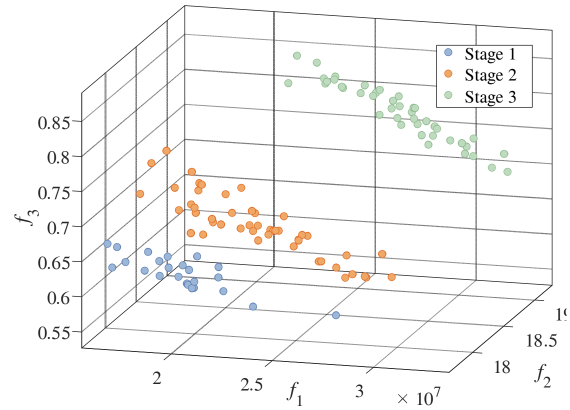

Figure 7. Stage-wise PFs Obtained under Static Multi-stage Optimization.

To ensure consistency, the PFs are further coordinated using the DP-based path planning method introduced in the previous section, and the results are reported in Table IV.

TABLE IV. COORDINATED INSTALLATION DECISIONS ACROSS THREE STAGES BASED ON DP PATH PLANNING. (IN MVAR)

Stage	18	24	27	30	32	34	f_1 ($\times 10^7$)	f_2	f_3
1	26.86	54.46	16.00	17.08	16.82	24.93	1.21	17.95	0.676
2	22.18	57.63	18.42	18.66	53.08	19.96	1.85	18.45	0.722
3	43.89	96.27	40.19	23.84	45.65	81.43	2.55	19.18	0.762

Table IV shows that the static optimization method over-investment in Stage 2, where f_3 (0.722) is even better than that of Stage 3 (0.762). This "over-performance" in the intermediate stage suggests unnecessary investment and weakens the rationality of the long-term plan.

By contrast, the DP-based selection with SGEA yields smoother results, with Stage 2 at f_3 (0.688) and Stage 3 converging to 0.679. This avoids unrealistic improvements in the middle stage and ensures more coherent trade-offs across the entire planning horizon.

VI. CONCLUSION

This paper presents a dynamic approach to multi-stage VAR planning that reflects temporal factors such as load growth and CFPP retirement. The framework treats planning as a sequential process and applies a customized algorithm to address stage dependencies. The main findings are:

1) Adjusted dynamic programming framework: Planning is described through successive state transitions, ensuring continuity between stages. Compared with conventional planning, the method achieves 0.2% and 4.9% improvement in steady-state and transient stability with a comparable investment.

2) Consistent multi-stage decisions: A MDP is used to link solutions from different Pareto fronts, producing coordinated and cost-effective deployment paths across stages, unlike traditional one-shot strategies.

3) Adapted optimization algorithm: A modified SGEA is developed to accommodate time-varying system conditions and evolving generation portfolios, delivering more reliable and well-distributed solutions than C-MOEA/D.

Although representative evolution trajectories are considered in this study, the proposed framework provides a flexible interface for incorporating uncertainty through scenario-based or stochastic extensions, which will be investigated in future work.

REFERENCES

[1] K. Yan, G. Li, R. Zhang, Y. Xu, T. Jiang and X. Li, "Frequency control and optimal operation of low-inertia power systems with HVDC and renewable energy: a review," *IEEE Trans. Power Systems*, vol. 39, pp. 4279-4295, Mar. 2024.

[2] Y. Chi et al., "Many-objective robust optimization for dynamic VAR planning to enhance voltage stability of a wind-energy power system," *IEEE Trans. Power Delivery*, vol. 36, pp. 30-42, Feb. 2021.

[3] I. Drovtar *et al.,* "Utilizing demand response in load modelling for voltage and reactive power control studies," *IEEE Trans. Power Systems*, vol. 40, no. 2, pp. 1389-1400, Mar. 2025.

[4] Y. Chi, and Y. Xu, "Multi-stage coordinated dynamic VAR source placement for voltage stability enhancement of wind-energy power system," *IET Gener., Transm. Distrib.*, vol. 14, no. 6, pp. 1104-1113, Feb. 2020.

[5] K. Sun, K. -J. Li, and Z. Zhang, "An integration scheme of renewable energies, hydrogen plant, and logistics center in the suburban power grid," *IEEE Trans. Ind. Appl.*, vol. 58, no. 2, pp. 2771-2779, Mar. 2022.

[6] J. Liu, Y. Xu, Z. Y. Dong and K. P. Wong, "Retirement-driven dynamic VAR planning for voltage stability enhancement of power systems with high-level wind power," *IEEE Trans. Power Systems*, vol. 33, no. 2, pp. 2282-2291, Mar. 2018.

[7] H. Xiong, Y. Shi, Z. Chen, C. Guo and Y. Ding, "Multi-stage robust dynamic unit commitment based on pre-extended -fast robust dual dynamic programming," *IEEE Trans. Power Systems*, vol. 38, no. 3, pp. 2411-2422, May. 2023.

[8] Z. Yan and Y. Xu, "Real-time optimal power flow: a lagrangian based deep reinforcement learning approach," *IEEE Trans. Power Systems*, vol. 35, no. 4, pp. 3270-3273, Jul. 2020.

[9] T. Han et al., "Surrogate modeling-based multi-objective dynamic VAR planning considering short-term voltage stability and transient stability," *IEEE Trans. Power Systems*, vol. 33, no. 1, pp. 622-633, Jan. 2018.

[10] Q. Wang et al., “Efficient coordinated VAR planning for enhancing voltage stability of wind-penetrated power system using hybrid surrogate technique,” *Sustain. Energy Grids Networks*, vol. 41, Mar. 2025.

[11] Y. Chi, and Y. Xu, " Zoning-based candidate bus selection for dynamic VAR planning in power system towards voltage resilience,". *IET Gener., Transm. Distrib.*, vol. 14, no. 6, pp. 1012-1020, Mar. 2020.

[12] T. Van Cutsem et al., "Test systems for voltage stability studies," *IEEE Trans. Power Systems*, vol. 35, no. 5, pp. 4078-4087, Sept. 2020.

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