

COMBINING KNOWLEDGE AND INFORMATION - GRAPH-BASED DESCRIPTION OF DRIVING SCENARIOS TO ENABLE HOLISTIC VEHICLE SAFETY

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ABSTRACT

Currently, vehicle safety is based on knowledge from injury values, crash pulses, and driving kinematics which leads to intervention strategies separated into isolated domains of active and passive safety. In this contribution, it is shown how vehicle safety can be approached holistically, allowing for human-centered and scenario-based safety decision-making. For this purpose, information from interior and exterior vehicle sensors can be linked by a mathematical framework, combining the knowledge that is already available in the individual domains. A universal graph representation for driving scenarios is developed to master the complexity of driving scenarios and allow for an optimized and scenario-based intervention strategy to minimize occupant injury values. This novel approach allows for the inclusion of sub-models, expert knowledge, results from previous simulations, and annotated databases. The resulting graph can be expanded dynamically for other objects or occupants to reflect all available information to be considered in case of urgency. As input, interior and exterior vehicle sensor data is used. Further information about the driving situation is subsequently derived from this input and the interaction between those states is described by the graph dynamically. For example, occupant attentiveness is derived from measurable eye gaze and eyelid position. From this quantity, reaction time can be estimated in turn. Combined with exterior information, it is possible to decide on the intervention strategy like e.g. alerting the driver. Physical or data-based functional dependencies can be used to represent such interactions. The uncertainties of the inputs and from the surrogate models are included in the graph to ensure a reliable decision-making process. An example of the decision-making process, by modeling the states and actuators as partially observable Markov decision process (POMDP), shows how to optimize the airbag efficiency by influencing the head position prior to an impact. This approach can be extended by additional parameters like driving environment, occupant occupancy, and seating positions in further iterations to optimize the intervention strategy for occupants. The proposed framework integrates scenario-based driving dynamics and existing knowledge from so far separated safety systems with individual activation logic and trigger points to enable holistic vehicle safety intervention strategies for the first time. It lays the foundation to consider new safety hardware, sensor information, and safety functions through a modular, and holistic approach.

INTRODUCTION

Since the 1950s, people have been thinking about how to make driving safer. In [1] and [2] a seatbelt was presented to prevent occupants from being thrown out of the car in the event of an accident. However, it became a legal requirement much later, see Fig. 1. Shortly afterward, it became clear how important it is to manage the energy of a crash properly. It was realized that high accelerations and impacts on the human body have to be prevented. Thus, a crumple zone and a solid passenger cabin became indispensable. In the 1990s, first series-produced airbags were introduced to increase crash mitigation and occupant protection, which further reduced peak loads on the human body by controlling the crash energy even better. With further inventions such as a belt force limiter, a knee airbag, a belt pretensioner, and an activated headrest, the mitigation of the impact, also described as passive safety, for the occupant was optimized.

The development of accident avoidance in addition to accident mitigation, began in the early 1990s to further reduce fatal and serious accidents. Whereas systems were previously only developed to be activated in the event of an impact, at the time of accident t_0 , from now on functions were also developed to be activated before a possible accident, i.e. $t < t_0$, also known as active safety systems and advanced driver-assistance system (ADAS). For the first time, systems intervened in the driving dynamics. Following anti-lock braking systems (ABS) and electronic stability control (ESC) other systems like the automated emergency brake and various warning functions were developed in the 2000s, [3]. The development of vehicle safety from crash mitigation to crash avoidance is shown in Fig. 1.

To make this possible and prevent collisions before they happen, it is necessary to sense the environment and collect information. The need for sensors on the vehicle grew. The further ahead of the impact the intervention is required, the more sensor information and knowledge about the current driving situation are necessary [4]. The availability of sensors on and inside the vehicle grows to make active safety functions and advanced driver-assistance systems possible. The development is going further and further in this direction, especially in the interior, sensors are the enabler for comfort functions. For example the interior sensors enable small functions such as gesture control. But also in conditional autonomous driving, changing seating positions is a function made possible by interior monitoring, or also an assessment of how distracted the driver is and whether he can react to a dangerous situation.

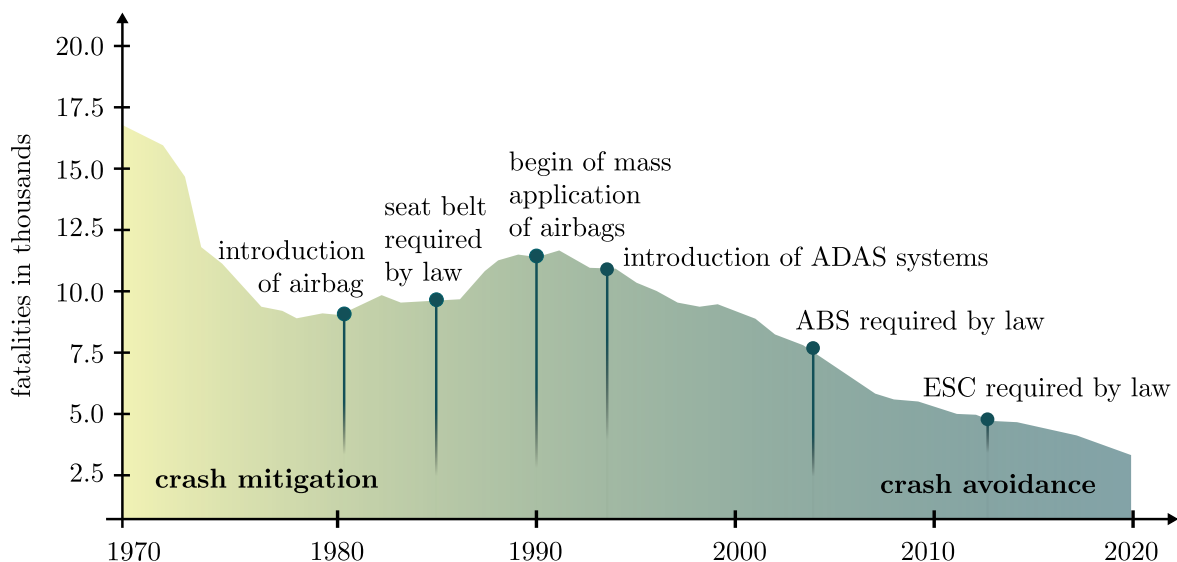


Figure 1: Development of vehicle safety [5] and fatalities on Japanese roads [6].

VISION

Despite those developments, developing safe vehicles and safe traffic is a challenge due to (i) changes in mobility solutions, (ii) the possibility to hand over driving tasks to the automated driver, or (iii) taking new seating positions. Covering the increased parameter space is not easy. Reasons are the increasing complexity but also because previous developments are based on minimizing harm to occupants in standard load cases. Those load cases are defined by national and international authorities and the safety functions are optimized to solve these load cases as reliably as possible.

To target the increased complexity and to increase vehicle safety to protect occupants better, the next step is the combination of all existing system knowledge with the information available due to the growth in sensor and data availability. In this work, a novel graph-based approach for such a holistic vehicle safety is presented. In Fig. 2 a small example scenario is depicted to show how complex decision-making can be facilitated in a presumably simple driving scenario. It also demonstrates what needs to be taken into account to protect the occupant in the best possible way in dangerous driving situations. The example exhibits a driving car with increasing road traffic. As time passes, the vehicle in front begins to decelerate and the driver would have to brake in order to prevent an accident. The driver misses the chance to brake because of distraction and the question is, how is it now possible to optimally protect the occupant in the given situation? The first step is to determine which variables are of interest at which point in time.

The vehicle under test (ego vehicle, E) is colored in green, whereas potential accident opponents ($B_{\{1,2,3\}}$), referred to as bullets hereinafter, are in black. Figure 2(a) describes the uncritical driving of the ego vehicle on an empty two-lane road with only one occupant, the driver, in the car. In the next time step Fig. 2(b) the road becomes more crowded. Besides a leading vehicle B_1 in front of the ego, another vehicle B_2 is driving in the second lane. If the B_1 decelerates, shown in Fig. 2(c), ideally detectable via the taillights, the ego vehicle needs to react and has different options. If the time-to-collision (TTC) is larger than the time-to-brake (TTB), it is still possible to stop before a collision occurs. However, if the TTC is smaller than the TTB, it has to be evaluated if a rear-end collision could be worse and should be avoided. For this evaluation, the head position and seating position of the driver is important because it determines how effective passive safety systems can be in the event of a collision. The driver in Fig. 2 is not in an ideal position for a full-frontal impact. As the left lane is blocked (B_3) and there is also an obstacle (O) on the right side, an evasive maneuver also influences the safety strategy. To exploit the safety potential of the airbag, the driver should be moved towards the center. This can be achieved by a collision on the passenger side of the vehicle, as shown in Fig. 2(d), which would be possible since the passenger seat is not occupied, making this particular scenario less critical.

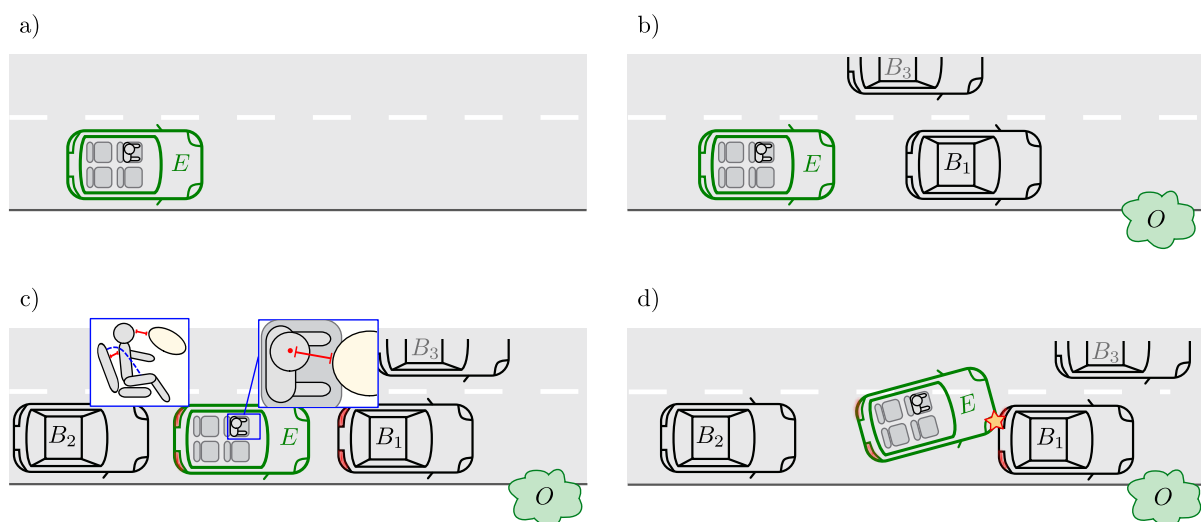


Figure 2: Evolution of a driving scenario (a)-(c) with ego vehicle (green) and bullet vehicles (black).

This small scenario shows that for each phase of the sequence specific input is required for safety decisions, both directly measurable and derived from sensor data, e.g. the TTB or the TTC are calculated from the ego vehicle dynamics and the bullet's velocity and position. However, there is a lot of research and development in the different domains, and scenarios like this can't be solved with separated and isolated systems and models. For example, there are already many physical approaches to describe and predict the movements of other vehicles [7]. Surrogate models exist to estimate the driver's head movement as a function of vehicle acceleration [8]. Furthermore, expert knowledge and data-based approaches exist to describe, for example, the driver's attention [9]. So a variety of mathematical descriptions of different contexts already exists at different levels of quality. But these models exist and many things can already be sensed, there is for example, no autonomous emergency braking system that takes the driver's drowsiness into account.

The challenge is combining the sensor information and all the knowledge already available in the domains to make an optimal safety decision for the occupant and tackle the problem holistically. The crucial point in such an approach is to know and take into account the uncertainties in the sensors and the limitations of the surrogate models so that the decision-making becomes reliable. In addition, the safety strategy must be traceable in retrospect, which precludes the use of black box models. To reach the goal and move towards a function that can optimally control driving safety actuators and fulfill the requirements, a simulation environment is set up that makes it possible to test different approaches. The simulation environment makes it possible to synthesize sensor signals and identify the most crucial parameters and relations. Furthermore, the virtual microscope simulation tells use which decision methodology is useful and how the gained information can be reused in further safety functions. Last but not least the information acquisition is highly dependent on available sensors and the proposed safety strategy on available safety actuators in the car, which requires a modular framework. Conversely, the modular approach makes it possible to integrate new security functions, new surrogate models, or new actuators directly into the framework.

FRAMEWORK

To achieve this a simulation framework is set up. The first step is to simulate and formulate a driving scenario. Based on the driving scenario and the vehicle, sensor models can then be used to simulate the data that the vehicle's sensors would provide. This augmented sensor data is then forwarded to the mathematical model of the driving scenario. Here, further information is derived from the sensor data, and the calculated variables and their dependencies are set in context. The mathematical description consists of several sub-models to calculate states. The exact structure is described in the following. The calculated states are then passed on to a decision-making process and a driving safety strategy is developed based on the mathematical description of the scenario. The safety intervention decision and its timestamp are then fed back to the driving scenario. The simulation framework described is depicted in Fig. 3.

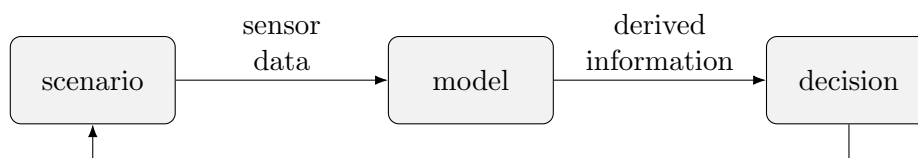


Figure 3: Simulation framework from scenario generation to decision-making.

Scenario Simulation and Input Data

When creating the driving scenario pool, scenarios in which only vehicles participate in traffic are currently being examined. For the dynamics of the vehicles, single-lane models are used. Subsequently, the ego vehicle is equipped with sensor models of cameras-, LIDAR-, or radar-modules. The use of sensor models is very important, as it is essential to take into account uncertainties that occur in reality

in order to make a reliable decision later. In addition to the external perception, it is also important to consider the information about the occupants based on interior sensors. The ability to simulate the behavior of occupants is still rather limited, which is why annotated data from databases are used and merged with the appropriate simulated driving scenario. In Table 1 the simulated and used quantities are listed. It is important to note that the sensors in the vehicle have different measuring frequencies. In this work, the signals are all sampled to the same frequency before further processing, and it is therefore assumed that all signals are in sync for the time steps.

Table 1:
Measurable quantities interior and exterior of the vehicle.

exterior data		interior data	
objects	· type · size · position · velocity · yaw	driver	· eyeglance · hands-On · eye closure · head position
setting	· environment	Other Occupants	· Seat occupancy

Mathematical Description of the Driving Scenario - Combining and Deriving Information

With the simulated sensor information now collected, the goal is to derive as much information as possible about the current driving situation. In doing so, it should be possible to use existing mathematical or data-based and expert knowledge to estimate or calculate non-measurable variables. To meet these requirements and in addition allow dynamic reaction to objects that appear in the driving scenario a graph description [10], [11] of the driving scenario is chosen. The graph description is based on the concept of a bipartite graph. The nodes of the graph represent the different states of the driving scenario. The edges represent the dependencies between these states. The edges are directed, which means that the direction of the edge indicates the direction of the dependency. In Fig. 4 the definition of the nodes and the edges are given. The state objects inherit attributes from the type objects. State objects are for example the position, velocity, and yaw of an object. Type objects are, e.g. a car, a pedestrian, or a bicycle. For example, if another car is detected by the sensor and the activation condition is met being a potential bullet, all states belonging to that bullet vehicle are added dynamically to the graph.

The edges contain the function that describes the relationship between two states. In the functions, already-known relationships are applied. For example, physical models, such as motion models [7], data-based models, such as behavioral models of road users [12], or models that reflect expert knowledge, such as the Karolinska sleepiness scale (KSS) [9], are used to define the edge functions.

In order to explain how the representation appears as a graph and what it actually can contain, the representation of the example scenario from Fig. 2 is shown in Fig. 5.

Starting with Fig. 5(a) on the top of the setting in which the vehicle is driving is represented, in (orange). It only has one sensor input describing if the car is driving in a rural environment, on a highway or in a highly dynamic city environment. Below the setting, the ego vehicle (green) is defined. The measurable input values for the ego vehicle are typically the acceleration and the steering angle. From those, other kinematic states can be calculated a trajectory can be derived afterward with the help of a motion model. Below is a description of the last object that occurs in the first frame of the driving scenario, the driver, in (red). The measurable input variables are head position, hands-on detection, head position, and eye closure detection, according to Table 1. It can be seen that the driver has a connection to both the vehicle and the environment. The connection to the vehicle is necessary to predict the head position, as this is strongly related to the trajectory being driven [8]. The connection to the environment describes the

relationship between the driver's attentiveness and the setting in which the driver is currently driving. For example, in highly dynamic urban traffic, a high level of attentiveness is more important than on a quiet country road. Those three objects (driver, ego vehicle, environment) are always active assuming we consider vehicles, where a driver is still needed. The small section in Fig.5 shows that a trajectory can be determined with the help of a constant turn rate and acceleration (CTRA) motion model, the accelerations and the velocities. In the next frame Fig. 2(b) another vehicle appears in front of the ego. This potential bullet vehicle is reflected in (blue), in Fig. 5(b). In the simulation, the activation e.g. the relevance of the bullet object is checked.

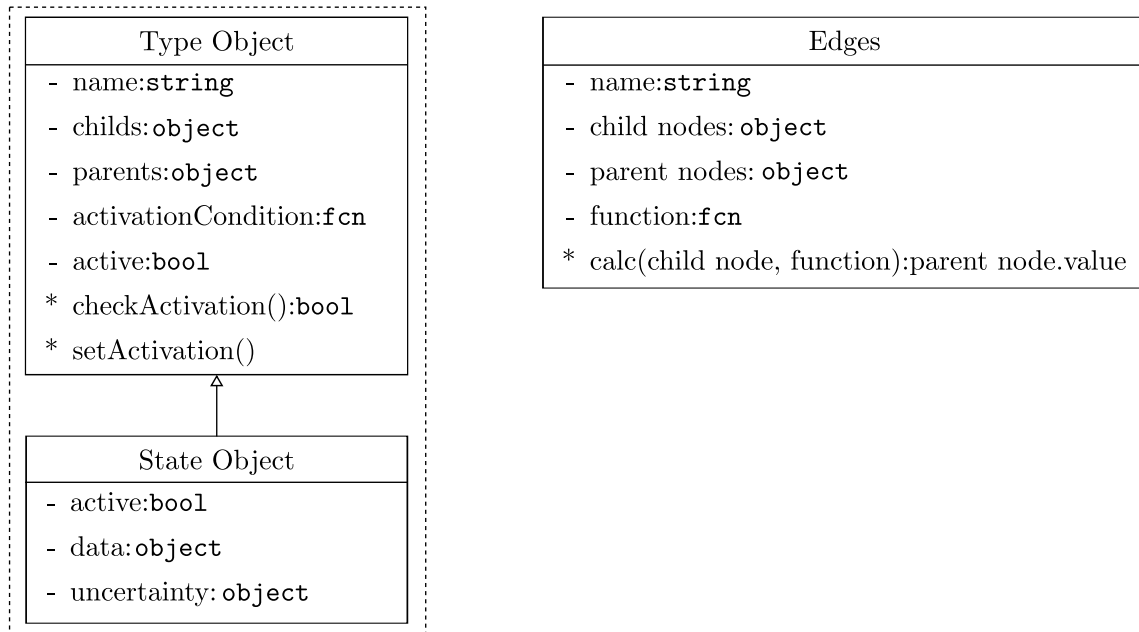


Figure 4: Definition of the node and edge objects of the graph including attributes and function.

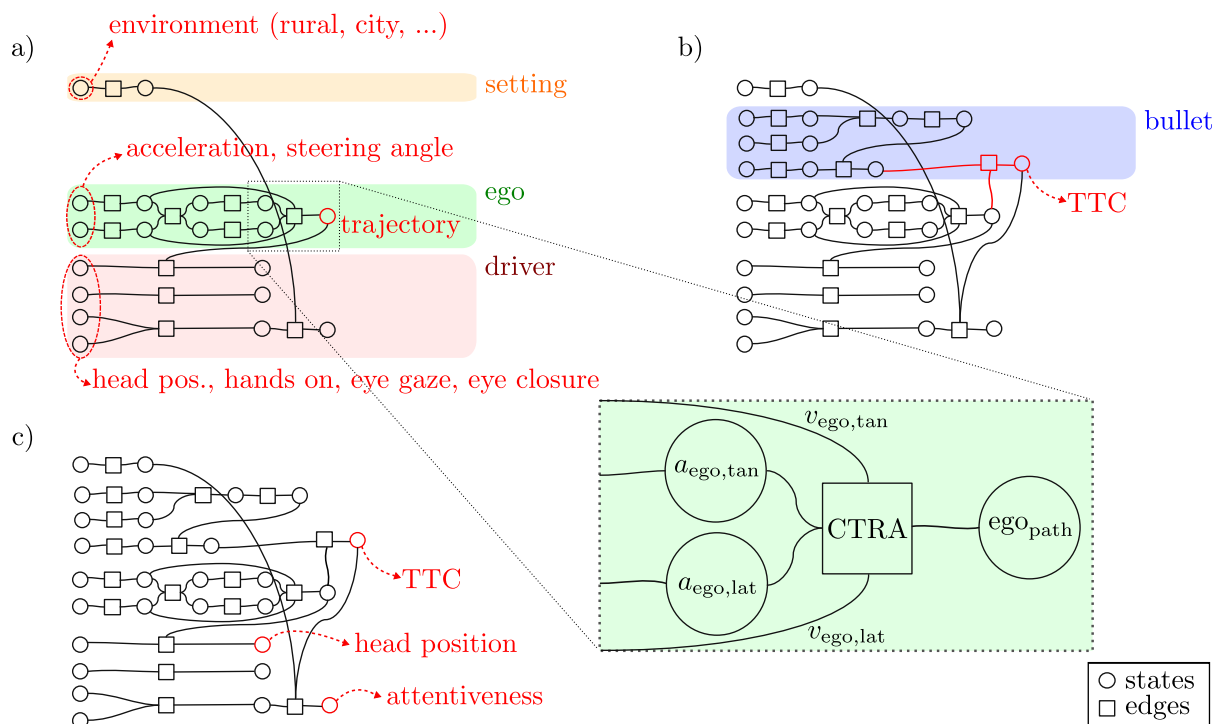


Figure 5: Graph representation of the evolving driving scenario presented in Fig. 2.

For this purpose, it is evaluated whether the activation condition is fulfilled, e.g. the bullet moves below a certain distance from the ego the object with its states is added to the graph. The basis of the kinematic characterization is the measurable states: position, velocity, and yaw angle. Similar to the ego vehicle, a potential driving trajectory can be determined from them with the help of a motion model. Together with the trajectory of the ego vehicle, a time to collision can now be calculated, which is highlighted in red. In the example scenario, the leading vehicle is now decelerating and states like the head position, the attentiveness, the surrounding area, and the TTC are important values to develop a safety strategy to optimally protect the occupants. Now the question is how to make a decision based on the information gained.

With this modeling and implementation, it is now possible to describe a scenario mathematically over time. It is possible to describe the development of the scenario dynamically and thus also take changes into account. Derived from this, the variables relevant to vehicle safety are calculated. With the information gained about the driving scenario, the next crucial step is reliable decision-making.

Decision-Making

In an attempt to make the right decision for the occupant, the problem is formulated as a Markov decision problem. A Markov decision process is a combination of states and actions but with a probability distribution over the outcome of the actions. For example, one wants to travel from state A to state B ($\{A, B\} \in \mathcal{S}$) in the shortest possible time and has the choice between the actions: taking the train, walking or cycling, ($\{\text{train, walk, bike}\} \in \mathcal{A}$). The probability of heavy traffic or of the train being late can be modeled as a probability distribution. The Markov decision process considers the probabilities and chooses the best action.

Since many important states of the safety decision problem cannot be measured directly, but are derived from measurable parameters, it is generalized as a partially observable Markov decision problem (POMDP) [13]. The POMDP is formulated as a tuple

$$\{\mathcal{S}, \mathcal{A}, \mathcal{O}, T, Z, R\}, \quad (1)$$

with the state space \mathcal{S} describing all states of the scenario and \mathcal{A} representing the possible actions, e.g. the actuators of the car that could be triggered. To reflect the uncertainties the decision are based on the so-called observation space \mathcal{O} , which describes the observable states. The observation function Z describes the probability of observing o when being in state s and taking action a . The transition function T describes the probability of transitioning from state s to state s' when taking action a . The reward function R describes the reward for being in state s and taking action a . The reward is a scalar value and is defined as the expected value of the reward function.

To make this approach a little more accessible, an example implementation is described below. In this example, the aim is to influence the driver's head position before the collision so that the airbag is as effective as possible. Figure 2 shows a sketch of the driver in bird's eye view.

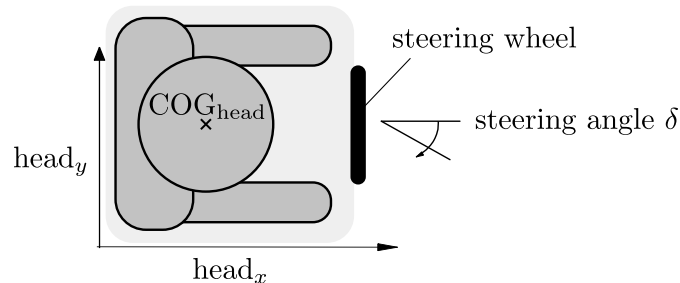


Figure 6: Outline of the POMDP scenario from a bird's eye view.

From this the state space is defined as $\mathcal{S} = \{\text{head}_x, \text{head}_y, a_{\text{ego}}, \delta_{\text{ego}}\}$ representing the head position of the occupant, the acceleration a of the vehicle, and the steering angle δ . The action space $\mathcal{A} = \{a_{\text{ego}}, \Delta\delta_{\text{ego}}, \text{BP}\}$ represents a deceleration a_{ego} , a steering intervention $\Delta\delta_{\text{ego}}$, and the activation of a two

staged belt pretensioner BP to restrain the occupant. Two-stage means that the belt pretensioner can activate an electronic retractor with a limited force and a pyrotechnic retractor with a stronger force. It is defined as $BP \in \{0, 1, 2\}$, with 0 = no activation, 1 = electronic retractor, 2 = pyrotechnic retractor. While the electronic retraction can be used several times, the pyrotechnic retraction is depleted after one firing. The steering intervention and the deceleration are also discretized, which makes the whole action space discrete. The observation space $O = \{\text{head}_{O,x}, \text{head}_{O,y}, a_O, \delta_O\}$ reflects the uncertainties between the actual states and the sensor data. Both, state and observation space are continuously defined. The transitions from state s and an action a to the new state s' are given by $T(s, a, s')$. The assumption is made, that the steering angle only influences the y -position of the head, whereas the belt pretensioner and the deceleration only influence the x -position. Pseudo-physical formulas are used for the transition function, which describes the relationship between vehicle acceleration, steering angle, and belt pretensioner to head position. To reflect the uncertainties in the sensors the observation probabilities $T(s, a, o)$ represent the dependency between the observation o and the real state s , which can be derived from sensor specifications or model validations. Finally, a reward function $R = (s, a, s')$ tells the algorithm which actions will bring the head closer to the desired target position, which is assumed to be $\text{head}_{\text{tar}} = [30 \text{ cm}, 35 \text{ cm}]$. Additionally, deceleration is always rewarded to minimize the crash energy. Choosing a feasible reward function and weighting the target correctly is not an easy task and requires further investigation, especially if more states and more actions are considered.

Figure 7 shows the results of the POMDP for the example scenario in the x -direction on the left column and in the y -direction on the right column. The black dashed line represents the desired head position, the blue line the actual head position, and the pink line the observation of the head position. The time decreases as it represents the time to collision, so the collision occurs at $t = 0 \text{ s}$.

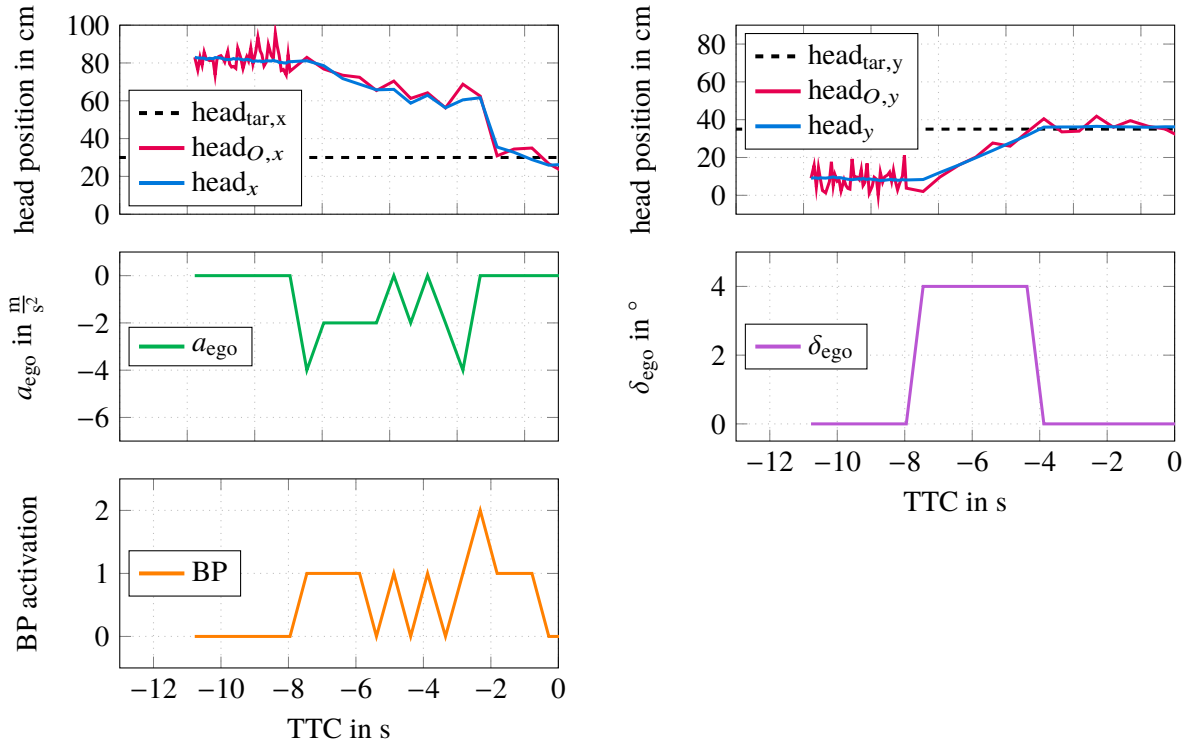


Figure 7: Results of a POMDP to optimize the head position before a collision.

The green line shows the chosen deceleration of the vehicle and the orange line the activation of the belt pretensioner. It is shown that the pyrotechnic belt tensioner is activated shortly before impact and the deceleration is set as high as possible. The algorithm is therefore able to find the solution. In y -direction the head position is influenced by a steering intervention, which is shown with the purple line. However, the methodology is not yet directly linked to the graph description, it is the goal to do this in the next

step. Then the observable states would come directly from the graph. Another challenge is currently the computation time. The solution of the POMDP was calculated with a sample-based approach, where 200 samples are calculated per time step. All samples search a tree to a predefined depth and look at which combination of actions maximizes the reward. The described model calculates about 0.2 s for a 50 ms time step, on a 4 x 3.4 GHz CPU, which is too slow for a real-time application. Especially when even more states and even more actions are to be considered, the computation time becomes even longer. However, it is necessary to optimize the POMDP for computational efficiency.

Tools

Within this framework, different tools are used. The scenario simulations run in Matlab using a proprietary toolbox in which single-track models are used for the vehicle dynamics and sensor models to augment the sensor data. The results are forwarded in csv-format to the graph which is implemented in Julia [14], to meet the more object-orientated requirements. The decision-making process, e.g. the POMDP, is also defined and solved in Julia using the POMDPs.jl [15] package and a DESPOT algorithm [16], an optimized tree search, as an online solver to grant fast evaluations and calculations.

INTEGRATION INTO CURRENT DEVELOPMENT PROCESSES

Currently, such a centralized approach can not be included in current vehicle with due to distributed control systems. However, in nearly all vehicle domains is an increase in electrically controlled functions and their complexity increases the number of electrical and electronic modules and the necessary communication requirements. New electrical/electronic (E/E) architectures that will keep the complexity of future vehicle systems manageable are constantly developed. The core element is the change from today's domain-specific to a cross-domain and centralized E/E architecture with a few but very powerful vehicle computers instead of many individual control units. That allows flexibility in the function development but also hardware independence. The developments in the other domains will enable such a holistic driving safety system becoming real. It will not only be possible but also the necessary step to achieve the goal of the software-defined vehicle. This holistic and modular approach allows for adding new functions as well as actuator-sensor combinations for future vehicle sensors and interior designs. The direction described above sets sensed quantities into context and a decision is made based on that. The aim is to be able to use the framework other directions in the future. In doing so, the insight should be used to determine which states have to be changed in order to make the situation safer for the occupant. This allows inferences to be drawn about which actuators are important in which driving scenarios and where there is still potential for actuator and function development.

CONCLUSION, CHALLENGES & OUTLOOK

In this work, an approach was presented to integrate vehicle safety from separated systems into a holistic decision-making process under inclusion of existing knowledge. The development of a holistic approach away from separate systems and load case optimization is the next logical step in the evolution of vehicle safety. A simulation framework was described which contains the whole chain from input data to decision-making. Starting with a simulation of driving scenarios and sensor data, it was described how this data can be further processed and how the existing knowledge and models can be used to link the variables in a meaningful way. Therefore, a graph representation was chosen which allows the application to a dynamically changing driving scenario. It was then described how a partially observable Markov decision process can be used to find a reliable, deterministic, safety strategy based on the mathematical description of the scenario and the available actuators.

However, the system is deterministic, the validation is very difficult for such an approach because of the high complexity. Covering the almost infinite variety of possible driving scenarios, and interactions between road users, humans, and machines and passing them on in a deterministic decision-making process is not an easy task. Another challenge is the computational efficiency of the microcontrollers

currently used in vehicles. Actuators in passive safety, such as the airbag, have activation times in the millisecond range. A decision computation in such a small time span would not be possible with current vehicle technology. Nevertheless, the computational performance in vehicles is constantly increasing, which also makes more complex evaluations possible. Last but not least, one has to think about what happens in case of a system failure. The dynamic modeling of the problem can well cover the failure of individual sensors and actuators, but not if the central system that makes the decisions and triggers the actuators is not working. It would be a possibility to have a redundant system running as a backup. Overcoming these challenges is the crucial step for the cross-system evaluation of driving scenarios and the further development of driving safety. When the software-defined vehicle with actuators becomes a reality, it will be the next step toward increased road safety for all stakeholders.

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