A New Linear Transform Approach for Estimating ODFs from Multi-Shell Diffusion Data

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Introduction: This work proposes a new linear transform method for estimating orientation distribution functions (ODFs) from multi-shell (MS) diffusion MRI data. MS data is becoming increasingly common, partly due to the fact that MS data often contains more information than standard single-shell acquisitions [1-7]. Existing methods for estimating ODFs from MS data have either used non-linear processing [1,4,6] or have used linear processing under specific modeling assumptions [3,5]. In each of these cases, the theoretical relationship between the estimated ODF and the original propagator has not been established for the common situation where modeling assumptions are violated. Since brain white matter characteristics can be quite complicated and can vary between different brain regions (e.g., axon radius, myelination, and packing density can all change), it is common for modeling assumptions to be at least partially violated in practice. Some approaches (particularly the nonlinear methods) can be sensitive to modeling violations, leading to inaccurate results [6,8].

In the context of single-shell ODF estimation, Ref. [8] introduced a new linear ODF estimation method called the Funk-Radon and Cosine Transform (FRACT) [8]. This method was based on novel theory that showed a direct relationship between ODFs estimated using linear methods and the true ensemble average diffusion propagator (EAP). This theory allowed the design of state-of-the-art linear ODF estimation methods that were competitive with nonlinear methods, but that also did not require any EAP modeling assumptions and exhibited predictable linear behavior. This work proposes a MS extension of FRACT (MS-FRACT), leveraging similar theoretical links between the ODF and the true EAP, and leading to a predictable model-free ODF estimation scheme for MS data.

(1)	$ODF(\mathbf{u}) = \int_{-\infty}^{\infty} EAP(r\mathbf{u})r^2dr = \int EAP(x)g_{\boldsymbol{ODF}}(\mathbf{u},\mathbf{x})dx$
(2)	$ODF_{est}(\mathbf{u}) = \int EAP(\mathbf{x})g(\mathbf{u},\mathbf{x})d\mathbf{x}$
(3)	$g^*(\mathbf{u},\mathbf{x}) = \text{argmin}\|g(\mathbf{u},\mathbf{x}) - g_{\boldsymbol{ODF}}(\mathbf{u},\mathbf{x})\|_{L_2}$

Theory: The ODF along orientation $\mathbf{u}$ is the marginal distribution obtained by integrating the EAP along radial lines, as shown in Eq. (1). Here, $g_{\boldsymbol{ODF}}(\mathbf{u},\mathbf{x})$ is the unique function that makes the second and third terms of Eq. (1) equal. By generalization of the FRACT theory [8], we can show that if we estimate an ODF from finitely-sampled MS q-space data using linear methods, the estimated ODF will be related to the true EAP according to Eq. (2). Here, $g(\mathbf{u},\mathbf{x})$ is an ODF response function that depends on the linear transforms that are applied to each shell of the MS diffusion data. Due to finite sampling of q-space, it is impossible to design a linear transform of the MS data so that $g(\mathbf{u},\mathbf{x}) = g_{\boldsymbol{ODF}}(\mathbf{u},\mathbf{x})$. However, by designing linear transforms that are optimal solutions to the optimization problem shown in Eq. (3), it is possible to find linear transforms of MS data that accurately approximate the ODF integral from Eq. (1). The solutions to Eq. (3) are what we call MS-FRACT transforms. Notice that, unlike many nonlinear methods, MS-FRACT does not require any modeling assumptions about the behavior of the EAP. In addition, due to linearity, MS-FRACT is quite stable and predictable, and can be computed efficiently using fast spherical harmonic transforms [8].

Methods: Optimal MS-FRACT transforms were computed for an imaging scenario with SNR=20, diffusion parameter $\Delta - \delta/3$ =40 ms, and multiple b-values (500, 1500, and 3000 s/mm^2) by solving (3). To account for noise, a small amount of regularization was incorporated when estimating the transform kernels. The derived transform was used to evaluate MS-FRACT with simulated data. Simulated acquisition used the analytic q-space signal expression for a mixture two identical diffusion tensors crossing at 90°. MS-FRACT which used 200 samples in each shell was compared against single-shell FRACT, where the single-shell FRACT acquisition used 600 samples on a single shell (3000 s/mm^2). *In vivo* MS data from the Human Connectome project [8] was also processed using MS-FRACT and single-shell FRACT, after estimating new response functions for the slightly different q-space acquisition.

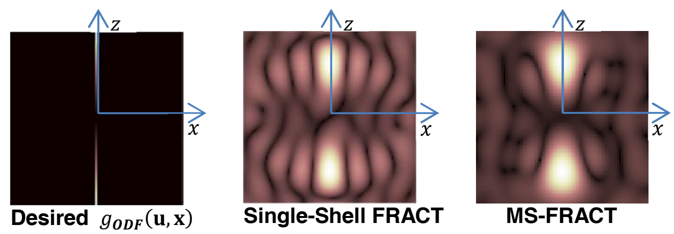

Fig. 1. ODF response functions (shown for $\mathbf{u}$ pointing along the z-axis and with y=0. Response functions are rotationally invariant).

Results: Fig. 1 shows the response function results. As expected, the MS-FRACT response function more closely follows the constant solid angle r^2 behavior from Eq. (1) and has reduced sidelobes relative to single-shell FRACT. Fig. 2 shows the mean ODFs estimated for the DTI simulation over 100 noise realizations. It can be seen that MS-FRACT follows the true ODF (computed analytically) more accurately than single-shell FRACT. Fig. 3 shows estimated ODFs for a section of the white matter core of the frontal lobe in real MR data. Unlike FRACT, which is incapable of representing isotropic diffusion [8], MS-FRACT robustly reconstructs ODFs in both anisotropic and isotropic regions. MS-FRACT also still robustly recovers crossing fibers.

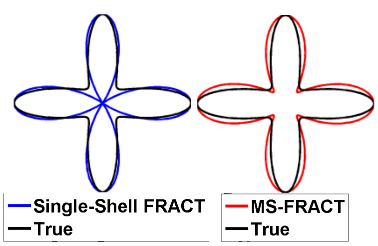

Fig. 2. Mean ODFs from the simulations.

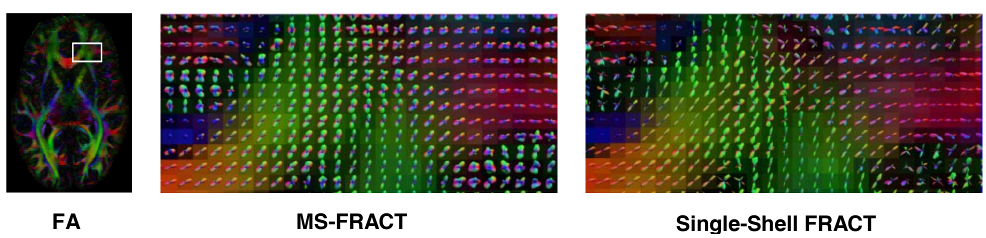

Fig. 3. Real data results.

Conclusion: We have proposed and investigated a novel model-free linear transform technique for ODF estimation from MS diffusion data. We have demonstrated that using multi-shell sampling can improve ODF estimation results compared to using single-shell FRACT estimation with the same number of measurements. In the future, we plan to compare MS-FRACT with other multi-shell ODF estimation techniques.

References: [1] Khachaturian, *MRM* 57, 2007. [2] Wu, *NeuroImage* 36, 2007. [3] Assemlal, *MIA* 13, 2009. [4] Aganj, *MRM* 64, 2010. [5] Descoteaux, *MIA* 15, 2011. [6] Jbabdi *MRM* 68, 2012. [7] Sotiropoulos, *NeuroImage* 80, 2013. [8] Haldar, *NeuroImage* 71, 2013.