AC-LORAKS: AUTOCALIBRATED LOW-RANK MODELING OF LOCAL K-SPACE NEIGHBORHOODS

Justin P. Haldar[1]
[1]*Electrical Engineering, University of Southern California, Los Angeles, CA, United States*

Introduction: Low-rank modeling of local k-space neighborhoods (LORAKS) is a recent constrained MRI framework that can enable accurate image reconstruction from sparsely- and unconventionally-sampled k-space data [1,2]. Specifically, Ref. [1] showed that the k-space data for MR images that have limited spatial support or slowly-varying image phase can be mapped into structured low-rank matrices, and that low-rank matrix regularization techniques can be applied to these matrices to yield high-quality reconstructions. This approach enabled novel k-space trajectories that are difficult to reconstruct using any other method, including calibrationless randomly undersampled half k-space trajectories. In addition, LORAKS is easily extended to incorporate parallel imaging constraints [2], and can also be easily combined with other regularization-based reconstruction penalties. Despite these advantages, previous LORAKS-based reconstructions have been relatively slow to compute because they depend on slow low-rank matrix recovery algorithms that repeatedly compute singular value decompositions (SVDs) of relatively large matrices [1,2]. In this work, we demonstrate that much faster linear least-squares reconstruction algorithms can be used if a fully-sampled autocalibration region of k-space has been acquired. We call this new approach autocalibrated LORAKS (AC-LORAKS).

Theory and Methods: Previous single-channel LORAKS work [1] showed that: (i) if an MR image has limited spatial support, then it is possible to form the fully-sampled k-space data into a structured low-rank matrix denoted by $\mathbf{C}$; (ii) if an MR image has slowly-varying spatial image phase, then it is possible to form the fully-sampled k-space data into a different structured low-rank matrix denoted by $\mathbf{S}$; (iii) if an MR image has slowly-varying spatial image phase, then it is possible to form the fully-sampled k-space into yet another rank-deficient structured matrix denoted by $\mathbf{G}$. In parallel imaging scenarios, the LORAKS matrices constructed from each individual channel can be combined into big matrices with even better low-rank characteristics [2]. In previous LORAKS work [1,2], the fully sampled data was estimated from undersampled k-space by simultaneously enforcing data consistency and the low-rank structure of these matrices. Enforcing the low-rank matrix structure required computationally demanding iterative algorithms.

In this work, we notice that if there exists a suitably large fully-sampled region of k-space data, then the $\mathbf{C}$, $\mathbf{S}$, and $\mathbf{G}$ matrices constructed from zero-filled k-space data will each have a number of fully-sampled rows. Similar to previous work [3,4], we observe that the existence of fully-sampled rows implies that it is possible to estimate the nullspaces of $\mathbf{C}$, $\mathbf{S}$, and $\mathbf{G}$ by applying an SVD to the submatrices formed from these rows. If the nullspace of a low-rank matrix is known in advance, then it is possible to estimate missing k-space data by imposing the fact that missing data must be consistent with the nullspace constraints [3,4]. This can be achieved by solving simple linear least-squares problems, instead of resorting to computationally-intensive matrix recovery algorithms. Specifically, let $\mathbf{C}_s$ denote the $\mathbf{C}$ matrix constructed from zero-filled measured data, and let $\mathbf{C}_u$ denote the $\mathbf{C}$ matrix constructed from zero-filled unsampled data. While the $\mathbf{C}_s$ can be constructed based on the measured data, the $\mathbf{C}_u$ matrix is unknown and needs to be estimated. However, if the rows of matrix $\mathbf{Z}$ form a basis for the nullspace of $\mathbf{C}$, then we know that $\mathbf{CZ} = (\mathbf{C}_s + \mathbf{C}_u)\mathbf{Z} \approx \mathbf{0}$, or that the unsampled data points obey the relationship $\mathbf{C}_u\mathbf{Z} \approx -\mathbf{C}_s\mathbf{Z}$. This is a simple linear system of equations that can be solved using standard linear least-squares methods for the values of the unmeasured samples contained in $\mathbf{C}_u$. Constructing $\mathbf{Z}$ and solving this problem is the general AC-LORAKS procedure for the $\mathbf{C}$ matrix. Similar AC-LORAKS procedures apply for the $\mathbf{S}$ and $\mathbf{G}$ matrices.

Results: Fig. 1 shows the results of AC-LORAKS applied to single-channel MRI data that was retrospectively undersampled, keeping 5/8ths of the fully-sampled k-space data. The acquisition was designed using a quasi half k-space random sampling scheme, which yields higher k-space sampling density in one half of k-space. Higher sampling density leads to improved reconstruction characteristics, and the half of k-space that was sampled with lower density can be recovered using $\mathbf{S}$- and $\mathbf{G}$-based phase constraints. Note that this kind of sampling scheme can be difficult to reconstruct using other methods. The AC-LORAKS reconstruction took ~6.5 minutes to compute (all computations used unoptimized MATLAB code). For comparison, conventional LORAKS-based iterative matrix-recovery algorithms took roughly 12 minutes (41 iterations) to converge when initialized with the AC-LORAKS result, and roughly 17 minutes (59 iterations) to converge when initialized using a zero-filled result (as used in previous work [1,2]). Clearly, AC-LORAKS leads to substantial improvements in reconstruction speed. Fig. 2 shows the results of AC-LORAKS applied to 32-channel MRI data that was retrospectively undersampled with 6× acceleration. As before, a quasi half k-space random sampling scheme was used. Compared to SPIRiT reconstruction [5], the AC-LORAKS reconstruction has a clear advantage in reconstruction quality.

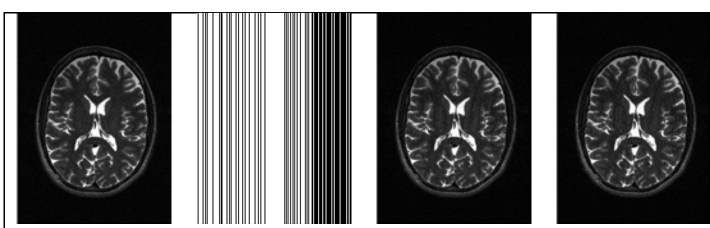

Fig. 1. Single-channel AC-LORAKS versus traditional LORAKS.

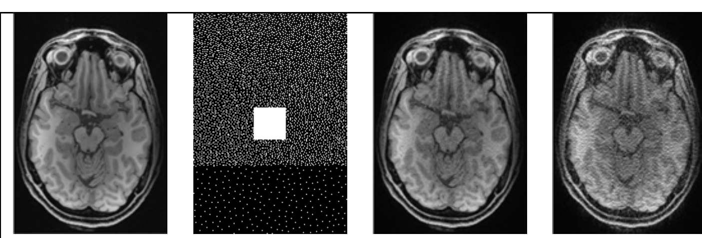

Fig. 2. 32-channel parallel imaging results with 6× acceleration.

Conclusions: This work proposed a fast algorithm for LORAKS-based reconstruction. The approach can be applied whenever a sufficiently large autocalibration region is present within the measured data. The results of AC-LORAKS can be used to replace or initialize the results of previous LORAKS-based reconstruction methods. Alternatively, AC-LORAKS can be used to obtain better results than those obtained with previous autocalibrated parallel imaging methods like PRUNO [4], SPIRiT [5], or GRAPPA [6]. AC-LORAKS is also easily augmented with other forms of regularization.

References: [1] J. Haldar, *IEEE Trans Med Imaging* 33, 2014. [2] J. Zhuo, *ISMRM* 2014, p. 745. [3] Z.-P. Liang, *IEEE ISBI* 2007. [4] J. Zhang, *Magn Reson Med* 66, 2011. [5] M. Lustig, *Magn Reson Med* 64, 2010. [6] M. Griswold, *Magn Reson Med* 47, 2002.